Dependence of the Electrical Conductivity and Electron Emission on the Energy of a Metal in the Process of its Heating by a Current of High Density

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(Submitted to JETP editor August 1, 1953)
J. Exper. Theoret. Phys. USSR 28, 96-110 (January, 1955)

Upon heating of nickel wires by a current of 6×10^4 - 5×10^6 A/cm^2 phenomena were discovered which were of the same nature as those previously observed in tungsten [1-3]. Moreover, upon investigation of the dependence of the wire resistance R on the introduced energy E, discontinuities were discovered in the curve $R = R(E)$. The location of these with respect to resistance and energy does not change with a change in density of the heating current (Ni, W, Au, constantan). An investigation of emission showed that an anomalously high emission from unbroken wires can drop even if the rate of energy arrival in the wires exceeds the energy losses at melting point temperature in the case of stationary heating. Data are supplied, characterizing the speed of emission decay after interruption of the heating current.

IN the experiments considered below, the previously applied method [1-3] of recording and compilation of oscillograms was improved, thus increasing the accuracy of the data obtained.

We recall that $V_R(t) = i(t)R(t)$ - the voltage on the examined wire and $V_r(t) = i(t)r$ - the voltage at constant resistance r permit us to find the current $i(t) = V_r(t)/r$, the resistance of the examined wire $R(t) = [V_R(t)/V_r(t)]r$ and the energy entering in the time t

$$E(t) = \frac{1}{r}\int_{t_1}^{t} V_R(t)\, V_r(t)\, dt,$$

where t_1 is the time of switching on of the heating current i. It is assumed that the rate of energy arrival considerably exceeds its losses $i^2 \gg i_m^2$ (i_m is the maximum current, which the examined wire withstands on steady-state heating, i. e., the current which compensates for maximum losses). For nickel of 0.015 cm diameter in vacuum i_m = 1A, in air i_m = 3A.

Experiments on emission measurements were made according to the scheme shown in Fig. 1. The anode current I_a from the examined wire was determined from the oscillograms of the voltage $V_\rho(t) = \rho_a I_a(t)$, decreasing at a constant resistance ρ_a.

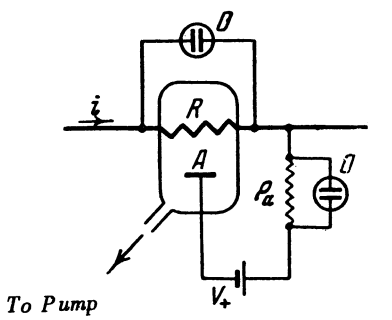

Fig. 1. Schematic of emission measurements. R - examined wire, A - anode, V_+ - anode battery, ρ_a - a constant resistance, O - oscillograph.

1. DEPENDENCE OF THE RESISTANCE ON ENERGY

Nickel wires were heated in transformer oil, air or vacuum by current pulses of high density. Qualitatively, the shape of the oscillograms $V_R(t) = i(t)R(t)$ obtained here, coincides with corresponding oscillograms taken on tungsten (Fig. 2). The presence of a region of small dR/dE followed by a jump R at the moment t_c - an explosion- is characteristic for these oscillograms (in our oscillograms a small dR/dE corresponds to a small dV_R/dt and a jump R corresponds toa jump V_R). Breaks in the oscillogram* as shown in Fig. 2 are designated by the numbers *1, 2, 3, 4, 5* and

[1] S. V. Lebedev and S. E. Khaikin, J. Exper. Theoret. Phys. USSR 26, 629 (1954)

[2] S. V. Lebedev and S. E. Khaikin, J. Exper. Theoret. Phys. USSR 26, 723 (1954)

[3] S. V. Lebedev, J. Exper. Theoret. Phys. USSR 27, 605 (1954)

* The presence of sharp breaks in the oscillograms $V_R(t)$ shows that the changes dR/dt take place simultaneously through the whole volume of the wire or, in any case, in the major part of its volume. Therefore, the oscillograms characterize a change in the specific resistance of the metal under investigation.

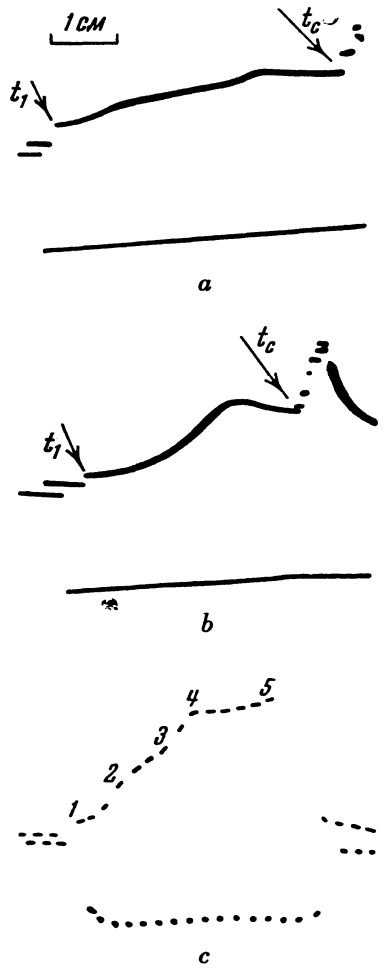

Fig. 2. Oscillograms $V_R(t) = R(t)i(t)$ (top curves) and $V_r(t) = ri(t)$ (bottom curves) for wires of length l and diameter d in oil.

a - Nickel: $d = 0.008$ cm; $l = 1.1$ cm; $j(t_1) = 10^6$ A/cm^2; $r = 2\Omega$, sensitivity of the oscillograph $a = 75$ V/cm ($a_R \sim a_r = a$), time scale $b = 7.9 \times 10^{-5}$ sec/cm.

b - Tungsten: $d = 0.006$ cm; $l = 0.46$ cm; $j(t_1) = 1.8 \times 10^6$ A/cm^2; $r = 2\Omega$; $a = 75$ V/cm; $b = 7.9 \times 10^{-5}$ sec/cm.

c - Nickel: $d = 0.015$ cm; $l = 2.15$ cm; $j = 5 \times 10^6$ A/cm^2; $r = 0.5\Omega$; the time markings are made with a frequency $\nu = 7.7 \times 10^{-5}$ sec^{-1}. At the start of the photograph $a_R = 470$ and $a_r = 425$ V/cm, at the end $a_R = 440$ and $a_r = 385$ V/cm. The zero lines are parallel straight lines forming a continuation of horizontal segments at the start of the oscillograms.

In tests 2c; 3a; 3b; 3c, the zero lines for $V_r(t)$ also go through a horizontal part at the end of the oscillogram $V_r(t)$ [the end of the oscillograms $V_R(t)$ in these tests is above the zero line].

they correspond to time intervals t_c, t_2, t_3, t_4, t_5. The Figure 5 (or t_5) marks the moment preceding t_c in cases where the jump R is not visible or not clearly expressed. The character of the change in the shape of the oscillograms of nickel and tungsten as a function of a change in current density is the same: the ratio of the time interval $t_c - t_4$, during which dR/dE is small, and the total heating time $t_c - t_1$ decreases with a decrease in current density j (Fig. 3). When j decreases below 2×10^5 A/cm^2 in the oscillograms for nickel (Figs. 3*d*, 3*e*) the region of small dR/dE is not visible. Unlike the region $t > t_4$, the dependence of R on E in the region $t < t_4$ is not altered by a change of j from 6×10^4 to 5×10^6 A/cm^2.

A break in the curve $R(E)$ can also be seen in oscillograms for nickel as well as at point *4* at point *3*. Up to this point no peculiarities in the dependence of R on E can be seen, while $R(t_3) = R_3 \approx R^s_{mp}$ and $E(t_3) = E_3 \approx W_h$, i. e., the values of resistance and energy in point *3* are such as would normally correspond to a start of fusion (R^s_{mp} and R^l_{mp} are the resistances of the wires at the melting point temperature in the solid and in the liquid state, W_h - the energy to heat it to melting point temperature, W_{fus} the heat of fusion). The energy entering the wire in the time interval $t_4 - t_3$ is approximatley equal to W_{fus}, but the change in resistance during the transition from point *3* to point *4*, R_4/R_3, is found to be smaller than the tabulated value[4] of R^l_{mp}/R^s_{mp}. Actually, according to tabulated data for nickel wire of 0.015 cm diameter, per centimeter length, $W_H = 1.39$ joule, $W_{fus} = 0.46$ joule, $R^l_{mp}/R^s_{mp} = 1.94$ (the value $R^l_{mp}/R^s_{mp} = 1.94$ is quoted in reference 4 on p. 295. Data on the resistance of liquid nickel are shown below in Table 2. Values of W_H and W_{fus} are taken from Seitz[5], p. 30, and reference 6, v. 1, p. 129. But, according to data obtained in our experiments,

[4] Encyclopedia of Metalphysics, edited by Masing, v. 1., Moscow, 1937

[5] F. Seitz, *Modern Theory of Solid State*

[6] *Technical Encyclopedia*, Tables of phys. - chem. and technological data, Moscow, (1931)

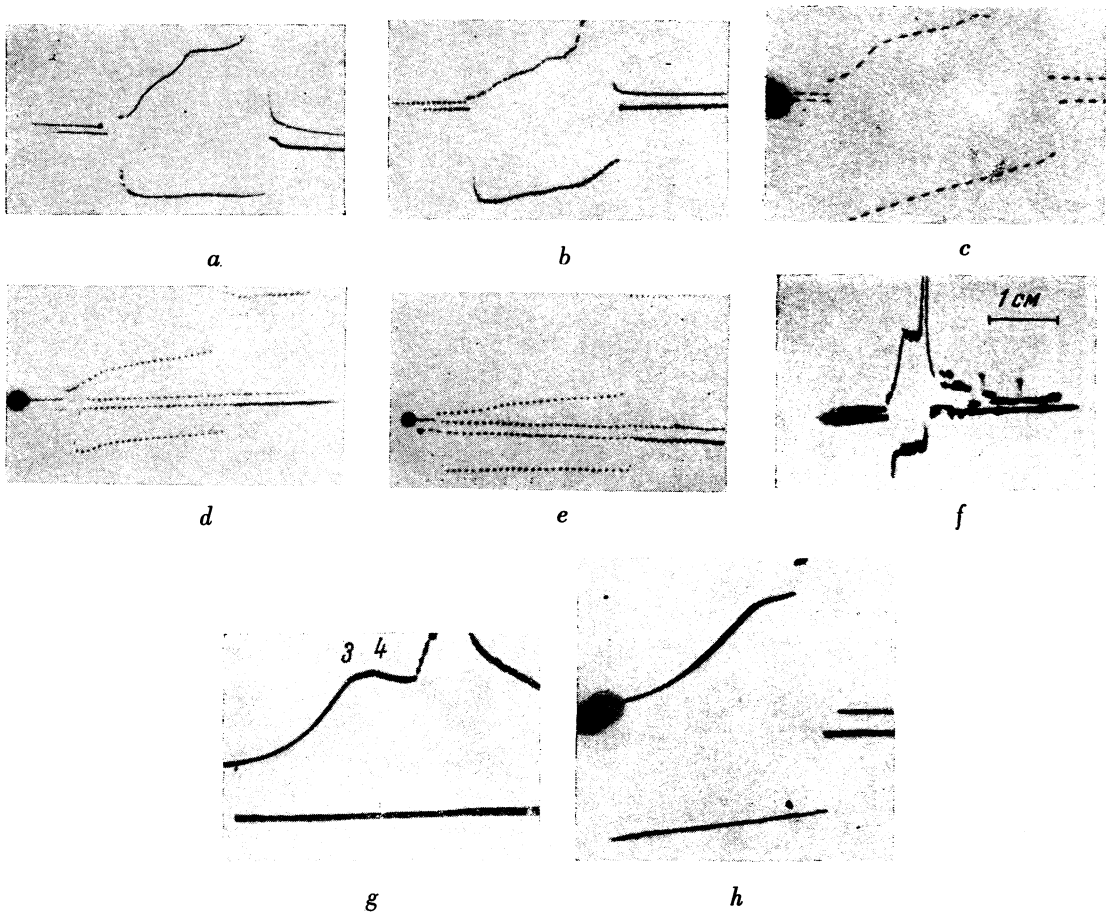

Fig. 3. Oscillograms $V_R(t)$ (upper curves) and $V_r(t)$ (bottom curves) for nickel and tungsten.

a - Nickel: $j = 5 \times 10^6$ in oil ($d = 0.015$; $l = 1.8$; $r = 0.5$; $a = 560$; $b = 2.6 \times 10^{-5}$).

b - Nickel: $j = 2 \times 10^6$ in oil ($d = 0.015$; $l = 2.35$; $r = 2$; $a = 160$, $\nu = 2.6 \times 10^5$).

c - Nickel: $j = 7.10^5$ in oil ($d = 0.015$; $l = 2.85$; $r = 1.22$; $a = 117$; $\nu = 2 \times 10^4$)

d - Nickel: $j = 2 \times 10^5$ in air ($d = 0.015$; $l = 2.85$; $r = 0.72$; $a = 64$; $\nu = 5 \times 10^3$).

e - Nickel: $j = 6 \times 10^4$ in air ($d = 0.015$; $l = 5.25$; $r = 2$; $a = 50$; $\nu = 5 \times 10^2$).

f - Tungsten: $j = 7 \times 10^6$ in water ($d = 0.0077$; $l = 0.75$; $r = 0.225$; $a_R = 650$; $a_r = 155$; $b = 24.5 \times 10^{-6}$).

g - Tungsten: $j = 1.2 \times 10^6$ in oil ($d = 0.006$; $l = 1.07$; $r = 2$; $a = 110$; $b = 1.2 \times 10^{-5}$).

h - Tungsten: $j = 3.5 \times 10^5$ in oil ($d = 0.006$; $l = 3$; $r = 9.5$; $a = 70$; $b = 9.6 \times 10^{-4}$). j - A/cm^2 - average current density; l and d centimeters; ν - sec^{-1}; r - Ω; a - V/cm and b sec/cm - average value of sensitivity and time scale. In experiments 3d and 3e the zero lines were traced during the writing of the oscillogram. In all other oscillograms the location of the zero lines is explained in the caption of Fig. 2.

$E_3 = 1.40$ joule $\approx W_H$; $E_4 = 1.82$ joule $\approx W_H + W_{fus}$; $R_3 = 0.36 \approx R^s_{mp}$; $R_4 = 0.46$. Consequently $R_4/R_3 = 1.3$ (Table 1). The value of $R^s_{mp} = 0.35$ was obtained at the instant of fusion of the investigated wire on slow heating in vacuum. The suitability of tabulated values of W_H and W_{fus} for the types of nickel used by us can be checked in the following way. In vacuum tests with $j \approx 10^5$ A/cm^2 (Fig. 3*e*, Table 1) the arrival of energy is stopped at the instant t_4, when $E = W_H + W_{fus}$, the wire being completely liquified as a result of the test (being totally converted into little balls). It can be seen directly from this that at the instant t_4, the energy $E = W_H$

$+ W_{fus}$ was actually introduced into the metal. However, the tabulated value* $R^l_{mp}/R^s_{mp} = 1.94$ for normal fusion could not be checked with our method, because on slow heating the wire disintegrates before it has time to melt completely.

The similarity in the behavior of tungsten and nickel is completed by the fact that it is also possible to separate , for tungsten, a region similar to the region (3,4) of nickel. For example, in the experiment with a tungsten wire of 0.006 cm diameter, 1 cm length at $j = 1.2 \times 10^6$ A/cm^2, the oscillogram of which is shown in Fig. 3*g*, values are obtained for $R_3 = 3.72\,\Omega$; $R_4 = 4.02\,\Omega$; $E_4 - E_3 = 0.15$ joule. On the other hand, it was measured at the instant of fusion of the tested wire upon slow heating in vacuum that $R^s_{mp} = 3.74\,\Omega$ per centimeter length and the tabulated value $W_{fus} = 0.137$ joule per centimeter length. Thus for tungsten too**, $R_3 \approx R^s_{mp}$ and $E_4 - E_3 = W_{fus}$.

The points *3* and *4* can also be seen on the oscillograms of experiments with gold. The measurements in these experiments were less precise than in those made with nickel, but qualitatively the results were the same: the values of R_3, R_4 E_3 and E_4 do not change when the current density is changed from 2.5×10^5 A/cm^2 to 3.6×10^6 A/cm^2 and $E_3 \approx W_H$, $E_4 - E_3 \approx W_{fus}$. The value R_4/R_3 was found in this case to be equal to 1.94, whereas, according to tabulated data for gold, $R^l_{mp}/R^s_{mp} = 2.28$.

The break in the oscillograms $V_R(t)$, observed on nickel at point *2* corresponds to the Curie point and is naturally absent in the case of tungsten and gold.

The results of tests with nickel are shown in Fig. 4. The usual dependence of resistance on the energy for solid and liquid nickel (see reference 4, p. 295) is shown as a solid curve. The dotted line, initially coinciding with the full line, corresponds to data obtained at large *j*. For current

Table 1*

Nickel, diameter 0.015 cm, length 1 cm

i A	*j* A/cm^2	$t_2 - t_1$ sec	R_2 Ω	E_2 joule	$t_3 - t_1$ sec	R_3 Ω	E_3 joule	$t_4 - t_1$ sec	R_4 Ω	E_4 joule	$t_5 - t_4$ sec	$E_5 - E_4$ joule	E total joule
900	5×10^6	5×10^{-6}	0.18	0.37	1×10^{-5}	0.35	1.42	1.1×10^{-5}	0.46	1.77	6.5×10^{-6}	1.98	3.75
350	2×10^6	2×10^{-5}	0.17	0.31	5×10^{-5}	0.35	1.30	5.6×10^{-4}	0.48	1.76	1.2×10^{-5}	0.66	2.42
125	7×10^5	1.5×10^{-4}	0.18	0.34	4×10^{-4}	0.35	1.38	5×10^{-3}	0.45	1.77	0.6×10^{-4}	0.27	2.04
42	2×10^5	1.3×10^{-3}	0.20	0.32	4×10^{-3}	0.36	1.46	5×10^{-2}	0.46	1.90	—	—	1.90
17	9×10^4	1.1×10^{-2}	0.20	0.40	2.5×10^{-2}	0.36	1.46	3×10^{-2}	0.48	1.96	—	—	1.96
11	6×10^4	2.5×10^{-2}	0.20	0.38	6×10^{-2}	0.36	1.38	7×10^{-2}	0.42	1.75	—	—	1.75
			0.19 ±0.01	0.36 ±0.03		0.36 ±0.01	1.40 ±0.05		0.46 ±0.02	1.82 ±0.08			

* The value and the density of the current are indicated as an approximation, since the current changed during the experiment (see for example, Fig. 3). In each line are recorded average results of several tests. (The change of a_R, a_r and *b* throughout the oscillogram was taken into account for the calculations.)

* *Note added in proof:* [see N. P. Mokrovskii and A. R. Regel, Zh. Tekhn. Fiz. 23, 2121 (1953)] A value of $R^l_{mp}/R^s_{mp} = 1.3$ is given for nickel, which is different from the tabulated value[4] $R^l_{mp}/R^s_{mp} = 1.94$ and coincident with the value of R_4/R_3 measured by us.

** *Note added on proof:* If it is assumed that the ratio R_4/R_3 is characteristic for the change in resistance on melting (as it can, seemingly, be concluded for nickel from the values of E_3, E_4, R_3 and their comparison with the data [see N. P. Mokrovskii and A. R. Regel, Zh. Tekhn. Fiz. 23, 2121 (1953)],) we obtain, for tungsten, $R^l_{mp}/R^s_{mp} = 1.08$.

densities from 6×10^4 to 2×10^5 A/cm^2 the graph is valid only for $E \leq W_H + W_{fus}$. [At a current $6 \times 10^4 - 2 \times 10^5$ A/cm^2 in vacuum experiments, the current stops at the instant t_4 ($E_4 \approx W_H + W_{fus}$), which indicates a destruction of the wire. In the case of experiments in air an arc is started between parts of the wire upon destruction, after which the measurements of R and E become unreliable*. The oscillograms of the experiments are the same in air and in vacuum up to the moment of destruction of the wire. The experiments were not conducted in oil at these j, because it was impossible to measure the energy without accounting for the losses**.] The complete dotted curve corresponds to the process up to the instant of explosion for $j = 5 \times 10^6$ A/cm^2. In case of smaller current density the explosion occurs at lower energy.

Let us note that the value of energy at which an explosion takes place for a given j was changing from test to test more than other measured values. Sometimes the region of explosion had the appearance of individual steps, probably corresponding to a non-simultaneous explosion of different segments of the wire.

2. EMISSION

Emission measurements were made on nickel wires with $j = 1 \times 10^5$ A/cm^2, according to the scheme shown in Fig. 1. The anode current I_a, measured by means of oscillograms, and divided by the surface area of the wire S, reaches a magnitude of 1.6×10^{-4} A/cm^2 whereas the normal emission of pure nickel at the melting point temperature $I_{mp} = 3 \times 10^{-6}$ A/cm^2.

The normal emission is calculated by means of the formula

$$I = AT^2 e^{-\phi/kT}$$

with $\phi = 7.4 \times 10^{-12}$ erg, $A = 30$ A/cm^{2}deg^2, $T = 1725^0$K.

Thus I_a/S is fifty times larger than I_{mp}.

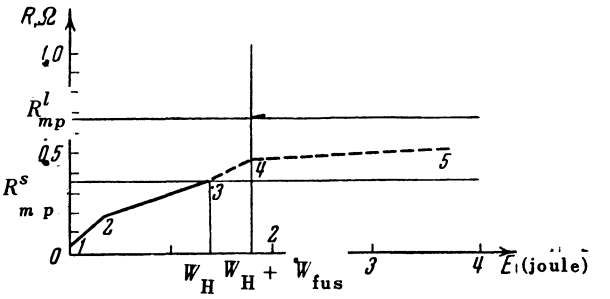

Fig. 4. Comparison of the normal dependence of R on E (solid line) with the dependence between these values at large j (dotted line coinciding with the solid line up to point 3). The numbers 1, 2, 3, 4, 5 on the curve mark the same states as on the oscillograms Fig. 2c. Nickel, diameter 0.015 cm length - 1 cm.

The vacuum in these tests was approximately 10^{-6} mm Hg. The wires were degassed at yellow heat for several hours. The lead-in connections and the anode were degassed at red heat. A typical oscillogram of the current I_a under these conditions is shown in Fig. 5*a*. As it can be seen from the oscillograms of Fig. 5, the current I_a, having reached a maximum, decreases, even though the energy of the wire continues to increase. Actually the current $i = 17$ A, while losses at the melting point temperature are compensated by the current $i_m = 1$ A. The interruption of the current i occurred here automatically (because of destruction of the wire) at the moment t_4. The oscillogram $V_R(t)$ shows that the wire is not destroyed up to the instant t_4, and that the drop of I_a with increasing energy takes place in the region 3, 4. (The second peak of I_a is related to the destruction of the wire at the instant t_4.)

The magnitude of the current and the character of its dependence on time remain also approximately the same in those cases when the current i is interrupted somewhat before the instant t_4 and the wire is not destroyed. In such cases one can succeed in making several tests with the same small piece of wire. The results of repeated tests are similar. Consequently, the observed drop I_a cannot be explained by the cleaning of the wire surface during the test. In Fig. 5*b* are shown two oscillograms, obtained in repeated tests on the same wire. The reproducibility of the results was

* The electric resistance to breakdown of the gap between the parts of a disintegrating nickel wire is larger in vacuum than in air. The reverse can be observed for tungsten wires of high emission.

** For nickel of 0.015 cm in diameter in oil, $i_m = 6$A, while for $j = 10^5$ A/cm^2, $i = 20$A. While measuring i_m in oil, convection currents were created which changed the cooling conditions of the wire as compared to the conditions in pulsed tests. At low j this circumstance makes it difficult to evaluate energy losses in pulsed tests from the value i_m.

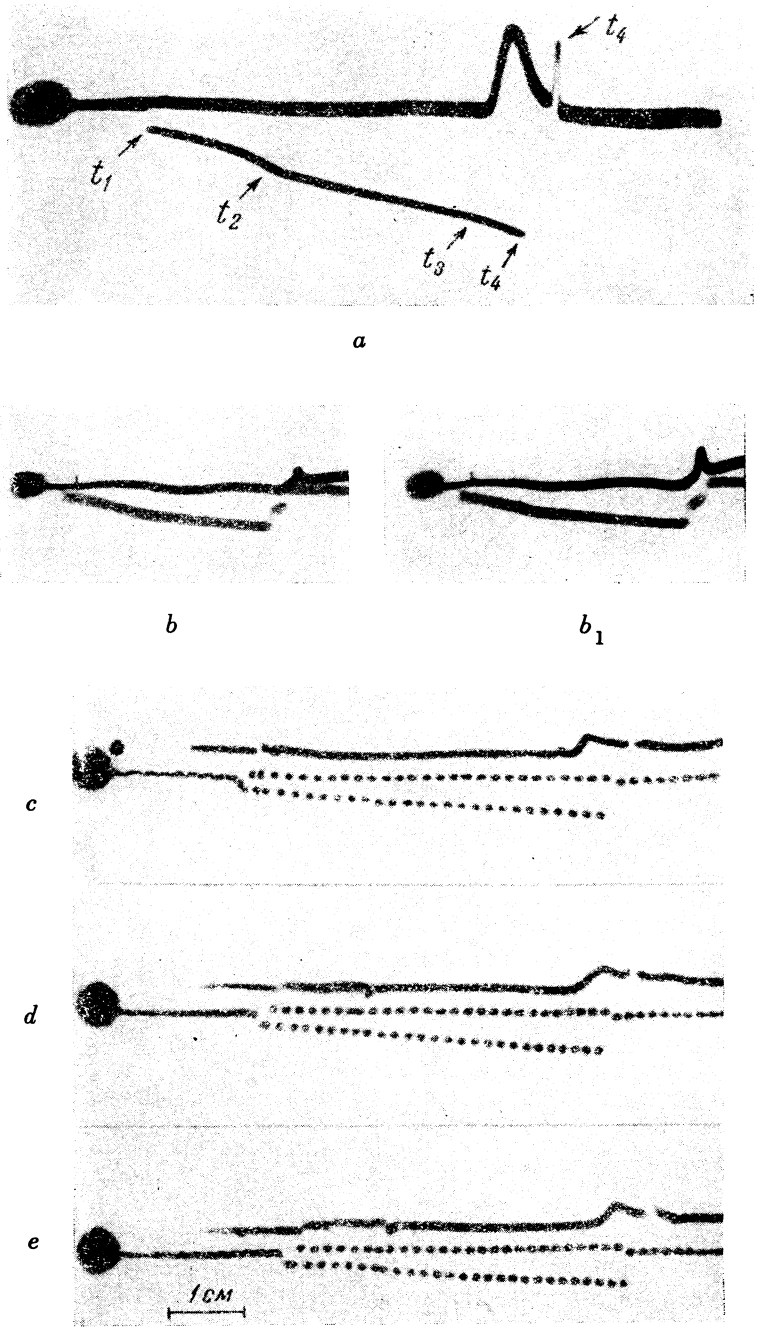

Fig. 5. Oscillograms $V_\rho(t) = \rho_a I_a(t)$ (upper curves) and $V_R(t)$ (lower curves) of nickel wire $d = 0.015$ cm (the upper curves are displaced to the right relative to the lower curves). $j = 10^5$ A/cm^2, j decreasing 3 - 5% from the time of starting t_1 to the instant of interruption t_i. In experiments *c*, *d*, $e_2 l \approx 1$ cm, $V_+ = 70$ V; $\rho_a = 10^6\ \Omega$; $a = 24.5$ V/cm; $\nu = 10^3$ sec^{-1}. In experiments *a*, *c*, *d*, *e*, the wires are destroyed and in experiments *b* and b_1 they are not destroyed. The horizontal part of the oscillograms $V_\rho(t)$ up to the rise coincides with the zero line. In Fig. 5 *a*, *b*, b_1, the zero lines of the oscillograms $V_R(t)$ and $V_\rho(t)$ are almost coinciding straight lines. In Fig. 5, *c*, *d*, *e* the location of the zero lines of the oscillograms $V_R(t)$ is recorded during the photography.

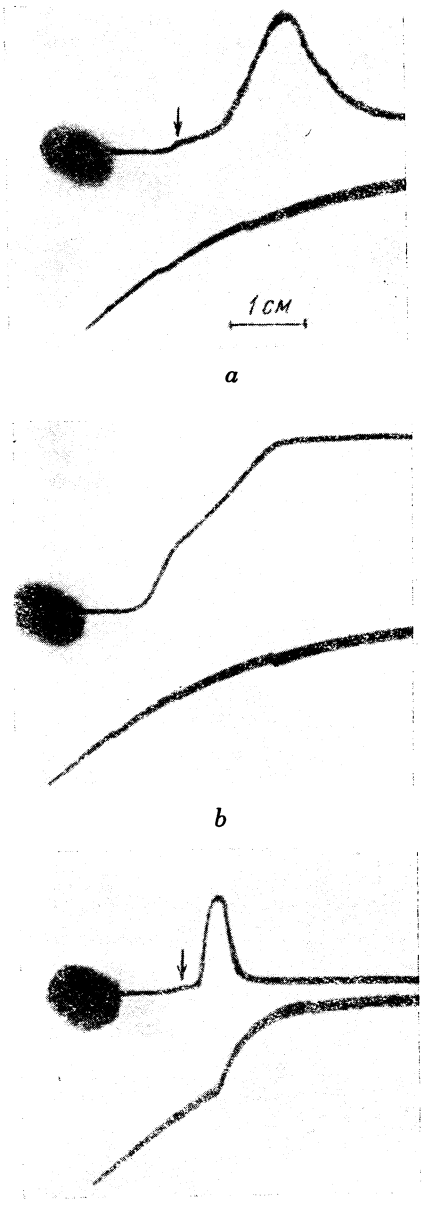

Fig. 6. Oscillograms $V_\rho(t)$ (upper curves) and $V_r(t)$ (lower curves). Tungsten: $d = 0.01$ cm, $r = 2\Omega$; $a = 52$ V/cm; $b = 5.7 \times 10^{-4}$ sec/cm. The wires are not destroyed during the experiment. The configuration of the electrodes in experiments 6 *a*, *b*, *c*, is different. The upper oscillogram is displaced with relation to the lower by 0.3 cm to the left.
a - $i(t_1) = 59$ A; $l = 0.75$ cm; $\rho_a = 200\,\Omega$; $V_+ = 300$ V; $I_a^{max} = 5.7$ A/cm^2;
b - $i(t_1) = 56$ A; $l = 0.2$ cm; $\rho_a = 2600\,\Omega$; $V_+ = 1000$ V; $I_a^{max} = 5$ A/cm^2;
c - $i(t_1) = 63$ A; $l = 0.5$ cm; $\rho_a = 200\,\Omega$; $V_+ = 300$ V; $I_a^{max} = 12.7$ A/cm^2. The zero lines of the oscillograms $V_\rho(t)$ and $V_r(t)$ are parallel straight lines. For $V_\rho(t)$ the zero lines are directed lengthwise along the horizontal segments at the start of the oscillograms $V_\rho(t)$. For $V_r(t)$ the zero lines are about 0.15 cm below the zero lines of the oscillograms $V_\rho(t)$.

also good in tests with destroyed wires. These tests (5 *c*, *d*, *e*) were made under the same conditions, the variation in the value of I_a approximately corresponding to the difference in length of the wire. The drop of I_a with an increase in wire energy also shows that the large value of I_a is not connected with the appearance of a discharge between wire and anode. In fact a discharge could not be extinguished when the energy of the wire was increased, because the conditions for its formation become more and more favorable.

Thus the measured anode current appears to be a current of abnormal emission, the characteristics of which are its large value, and the possibility of a decrease with increasing energy of the metal.

The drop in emission with rising energy is also observed in the case of tungsten, although the emission current density is here several orders of magnitude larger than in nickel. Examples of such tests are shown in Fig. 6 *a* and *c*. In test 6 *a*, the drop of I_a begins at a current value $i = 13$ A, while the current which compensates the losses is $i_m = 3$ A. The areas of the oscillograms marked by arrows correspond to a current I_a^*, limited by space charge (region of validity of the law of Boguslavskii - Langmuir[†]). It was previously clarified that I_a large compared to I_a^* is caused by the state of the cathode and not by secondary currents[7]. Therefore, a drop in I_a indicates a drop in emission.

If emission is determined from the formula $I = AT^2 e^{-\phi/kT}$ large values ($I > I_{mp}$) can be explained by the overheating of the metal above melting point temperature for regular values of A and ϕ or by overheating of the electron gas alone. However, the drop in emission with an increase in the energy of the metal can only be explained by a change of $A = A_0(1 - K)$ and ϕ (which under ordinary circumstances practically do not change with a change of T) or a decrease of the electron

[†] In our experiments with nickel deviations from the law of Boguslavskii - Langmuir cannot be observed, since $I_{mp} < I_a^*$

[7] S. V. Lebedev, J. Exper. Theoret. Phys. USSR 27, 487 (1954)

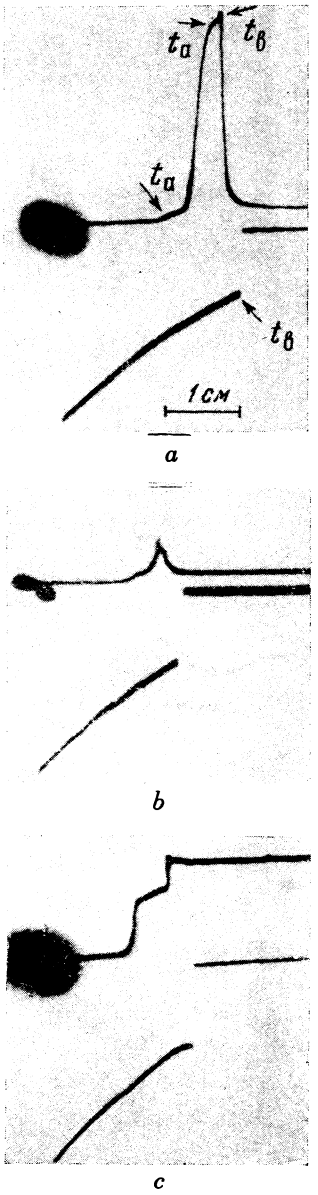

Fig. 7. Oscillograms $V_\rho(t)$ (upper curves) and $V_r(t)$ (lower curves). Tungsten: d =0.01 cm; l =0.5 cm; $i(t_1)$ =63 A; r =2Ω; a =52 V/cm; b=5.7 × 10^{-4} sec/cm. The wires are not destroyed. The configuration of the electrodes is different in experiments *a*, *b*, *c*. The upper oscillogram is displaced with relation to the lower by 0.3 cm to the left.

a - ρ_a =500Ω; V_+=200 V; I_a^{max}=10A/cm^2.
b - ρ_a =200Ω; V_+=300 V; I_a^{max}=5.3A/cm^2.
c - ρ_a =7000Ω; V_+=200 V; I_a^{max}=0.38A/cm^2

The zero lines for $V_\rho(t)$ are straight lines, directed lengthwise along the horizontal segments at the start of the oscillograms $V_\rho(t)$. The zero lines for $V_r(t)$ are straight lines directed along the horizontal segments at the end of the oscillograms $V_r(t)$.

temperature for $i > i_m$. In this manner, if this formula is valid, the experiments under study confirm the assumption that the observed anomalies are caused by a change in the properties of the metal and not by its overheating to a higher than melting point temperature.

The large value of current I_a (50 I_{mp}, 100 I_{mp} [2]) cannot be explained by a small value of the average reflection coefficient $\bar{K}$, and indicates an abnormally low value of the work function ϕ, or an overheating of the electron gas with respect to the lattice. The surplus of electron energy connected with a rise of their temperature above the temperature of the lattice or with their excitation [3] can be sufficient to cause a large increase in emission and, at the same time, be insufficient to lead to a considerable change in the dependence of R on E. Therefore, the absence of noticeable anomalies of the dependence of R on E for $j \leq 5 \times 10^6$ A/cm^2 (part 1) does not disprove the assumption of an energy excess of the electrons. This fact can be cited against the supposition of an overheating of the electron gas, namely, that the abnormal anode current decreases too slowly after the decrease of current i: as was pointed out in reference 7, $I_a > I_a^*$ for $\sim 10^{-5}$ sec after an interruption of the current i.

One can try to explain the drop in emission with the rise in energy by a fusion of the metal, assuming that the emission of the liquid metal is smaller than that of the solid metal at the same temperature*. However, whereas the drop in emission with a rise in energy is observed in experiment 6 *c*, it is not observed in experiment 7 *a*** although the heating current $i(t)$, initially the same in both tests, decreases more steeply in test 6 *c* from a certain moment on. This signifies that, beginning at that instant, the rate of energy intake becomes larger in 7 *a* than in 6 *c*. Therefore the

* *Note added on proof:* It was observed in [see V. G. Bol'shov and L. N. Dobretsov, Doklady Akad. Nauk SSSR 98, 193 (1954)] that the thermal emission current does not show a jump during transition through the melting point.

** In experiment 6 *c*, the drop of I_a is observed at $i \approx 5 i_m$. In test 7 *a* the drop occurs only after complete interruption of current i.

Table 2

	ρ_{20} Ωcm	ρ^{s}_{mp} Ωcm	ρ^{l}_{mp} Ωcm	ρ^{l}_{1500} Ωcm	Bibliography
Tabulated values for nickel	6.9×10^{-6}	55×10^{-6}	107×10^{-6}	—	[4] p. 295
	6.9×10^{-6}	—	—	109×10^{-6}	[6] v. I, p. 127, 129
	7.24×10^{-6}	—	108×10^{-6}	—	[6] v. IV, p. 335
	8.7×10^{-6}	—	—	—	[8] p. 86
Our wire * Diameter 0.015 cm	8.5×10^{-6}	63×10^{-6}	—	—	Measured under usual conditions
	ρ_1 Ωcm	ρ_3 Ωcm	ρ_4 Ωcm		
Our wire ** Diameter 0.015 cm	—	64×10^{-6}	81.5×10^{-6}		Measured at large j's
		$\pm2\times10^{-6}$	$\pm4\times10^{-6}$		

*All data, given in the present paper (except experiment 2*a*) were obtained on this type of nickel.

** The start of the oscillogram is distorted.

[8] Espe and Knoll, *Technology of Electro-vacuum Materials*, Moscow (1939)

melting in 7*a* must occur earlier than in 6*c*, and the drop in anode current caused by it should have been observed in 7*a* even before the time of interruption of the current *i*. (It must also be remembered that in 7*a*, as well as in 6*c*, the wire remains intact after the end of the experiment.)

The oscillograms of Fig. 7*a*, *b*, *c* show that the drop in anode current $|dI_a/dt|$ after interruption of the current is the faster, the larger I_a/I_a^*. For example it can be seen in Fig. 7*a* that $|dI_a/dt|$ decreases when I_a/I_a^* is approaching unity. The same can be seen by comparing various oscillograms: in test 7*a* at $I_a^{max}/I_a^* = 30$, I_a drops almost by a factor of 30 in a time interval not exceeding 2×10^{-4} sec, whereas in test 7*c* with $I_a^{max}/I_a^* = 1.5$, a drop in I_a is almost unnoticeable.

The character of the dependence of the decrease of I_a on the value of I_a/I_a^* remains the same in the case when the current *i* is not instantly interrupted but drops sufficiently fast. Thus, for example, in test 6*b* with $I_a^{max}/I_a^* = 2.7$, there is no drop of I_a. (The value of $|dI_a/dt|$ after complete interruption is found to be larger than under incomplete interruption*.)

The speed of decrease of current I_a is not given by the magnitude of this current. For example, in experiments 6*a*, *b*, the speed of decay of current I_a is completely different, although its maximum value is almost the same in both cases and is equal respectively to 5.7 and 5 A/cm^2. (Experiment 6*b* was conducted with $V_+ = 1000$ V but it was also repeated with $V_+ = 500$ V and $V_+ = 250$ V. The value of I_a^{max} was thereby changed, but the ratio I_a^{max}/I_a^* remained equal to 2.7, and in all these cases no noticeable drop in I_a was observed. Therefore the difference in the speed of decay of I_a in tests 6*a* and 6*b* cannot be explained by a difference in V_+.)

Thus the disappearance of anode current anom-

* In reference 7 [e.g., see S. V. Lebedev, J. Exper. Theoret. Phys. USSR 27, 487 (1954)] (Fig. 3), I_a does not drop during $t_2 - t_1 \sim 10^{-5}$ sec after interruption of the current j. It seems that the drop of I_a is hindered by a weak repeated pulse $j' \sim j/150 \sim 7\times10^4$ A/cm^2 given by the network L, C at $t_1 \leq t \leq t_2$ (see $V_r(t)$ in reference 7, Fig. 3, *zh*, upper curve).

Table 3

	ρ_{20} Ωcm $\times 10^6$	ρ^l_{mp} Ωcm $\times 10^6$	ρ_4 Ωcm $\times 10^6$	
Pure Au	2.4	30.82	—	[6] v. IV, p. 335
Our wire, diameter 0.01 cm	3.45	—	—	Measured under usual conditions
Same	—	—	26.2 ± 1.2	Measured at large j's

alies (i. e., the disappearance of an increase in I_a as compared to I^*_a) takes place the faster, the larger is I_a/I^*_a, independently of the magnitude of the anode current.

3. CONSIDERATION FOR POSSIBLE IMPURITIES IN THE METAL

The question can arise whether the peculiarities discovered in the behavior of metals can be explained by impurities contained in them. As it is known, impurities strongly influence the electrical conductivity of a metal. It can be seen on the top part of Table 2 that the specific resistance ρ_{20} of our wire at 20° C is very little different from the specific resistance of pure nickel. In emission anomalies we considered not only the absolute value of anode current, but also the unusual character of the curve $I_a = I_a(E)$. However the drop in emission with increasing energy cannot be explained by the presence of impurities. It is obvious that the dependence of the value of energy $E(t_c)$ (at which the metal explodes) on the density of the heating current also cannot be explained by the presence of impurities.

The situation is different as far as the difference of R_4 and the tabulated value of R^l_{mp} is concerned. Insofar as the difference between these values is not changed with a change in j and the value of R^l_{mp} for the investigated wire was not rechecked by us, it is possible that the tabulated value of R^l_{mp} (which we used) is not applicable to the type of wire investigated. It is known, however, that small additions to nickel of other metals (copper in particular) increase its specific resistance in the liquid state[4]. Consequently, for nickel with an additive, R^l_{mp} must be rather somewhat larger but not smaller than in pure nickel (Table 2).

Let us note that for nickel with $\rho_{20} = 14 \times 10^{-6}$ Ω cm, known to contain an added amount of Mn, the shape of the oscillograms and the value of the ratio R_4/R_3 are found to be the same as for pure nickel (Fig. 2a). Moreover, the transition of the type $R_3 - R_4$ and the region of small dR/dE following it, which terminates in an explosion, are observed also constantan, in which case $R_4/R_3 = 1.3$. On the other hand, the tabulated value for nickel is $R^l_{mp}/R^s_{mp} = 1.94$; for constantan* it seems that this value is also larger than 1.3.

Thus the difference between the value of R_4 determined by us and the value of R^l_{mp} listed in the tables cannot be explained by the presence of impurities. According to our measurements on gold $R_4/R_3 = 1.94$ while according to tabulated values $R^l_{mp}/R^s_{mp} = 2.28$. However, the break in the curve $R(E)$ at point 3 on the oscillograms for gold is not expressed very definitely and the determination of the value R_3 may be unreliable. As a control let us compare the value of ρ_4 found in our oscillograms with ρ^l_{mp} (Table 3). Judging from the value ρ_{20}, the investigated type of gold contained an impurity. But, as it was already indicated, an impurity should not decrease the value ρ^l_{mp}. Therefore, in this case also, ρ_4 is somewhat smaller than the tabulated value of ρ^l_{mp}.

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* The constantan wire investigated by us had $\rho_{20} = 48 \times 10^{-6}$ Ωcm, $\rho^s_{mp} = 58 \times 10^{-6}$ Ωcm. According to tabulated data for the alloy 40% Ni and 60% Cu for which $\rho_{20} = 50 \times 10^{-6}$ Ωcm, in the liquid state $\rho_{1500} = 85 \times 10^{-6}$ Ω cm. For other concentrations of the alloy Cu - Ni for which $\rho_{20} \approx 50 \times 10^{-6}$ Ωcm, $\rho^l_{mp} > 85 \times 10^{-6}$ Ωcm [see *Encyclopedia of Metalphysics*, edited by Masing, v. 1, Moscow (1937), p. 306]. For example, for the alloy 65% Ni and 35% Cu with $\rho_{20} = 50 \times 10^{-6}$ Ωcm and $T_{mp} = 1358°$ C it is known that $\rho^l_{mp} = 136.7 \times 10^{-6}$ Ω cm, and $\rho_{1400} = 138.5 \times 10^{-6}$ Ωcm [see *Technical Encyclopedia*, Tables of phys - chem. and technological data, Moscow (1931), v. V, p. 50].

4. COMPARISON OF RESULTS WITH DATA OBTAINED BY OTHER AUTHORS

1. In the paper of Khaikin and Bene[9] results are shown of an investigation of the melting of tin rods, heated by a current $j \approx 10^3 - 1.4 \times 10^3$ A/cm^2 and cooled on the surface by a jet of air. It was found that polycrystalline rods begin to melt from the inside and single crystal rods from the surface.

The conditions of these tests differed basically from the conditions of the discussed experiments made with $j > 6 \times 10^4$ A/cm^2 only in current density.

The results of these experiments coincide qualitatively with the results of our experiments with interrupted impulses for tungsten[3] in the sense that the metal does not melt, although it should be liquid according to the energy introduced into it. (In their paper, Khaikin and Bene observed that the single crystal is molten only at the surface, where the temperature is lower than inside, and in reference 3 the wires did not melt with $E = W_H + W_{fus}$.) It is true that, from the qualitative point of view, the difference here is quite considerable. If, for example, we recalculate the excess of energy, $E - W_H \approx W_{fus}$, observed in the experiments of reference 3, at a temperature for the usual value of specific heat, we obtain a value of $\sim 1000^0$ and not $1 - 2^0$ as estimated in reference 9 (for small j). However, it is possible in our experiments to interrupt the heating current earlier; then, obviously, one can obtain as small an excess of energy as desired.

Khaikin and Bene[9] concluded that they observe an overheating of a solid body. It seems to us, that if one starts with such a concept, one can not explain all the phenomena which we observe. In particular, for the case of an overheating of the metal, a drop in emission with an increase in energy is incomprehensible (see Sec. 2). One could assume that the losses in energy increase as a result of the overheating and that the drop in emission is tied to a cooling of the wire. Then in tests 6 *a, c* the losses should exceed the losses at the melting point by more than 16 times ($i = 4.3 i_m$ and $i = 5 i_m$). The order of magnitude of the necessary temperature can be estimated by assuming that the wire is heated uniformly and emits as a black body. Then a temperature is obtained which is twice as high as the melting point temperature of tungsten, i. e., it exceeds its boiling temperature ($T_{mp} \approx 3650^0$ K, $T_{bp} \approx 6200^0$K*). However, the overheating of a non-melting metal over its boiling point temperature appears to be unlikely. Moreover a large overheating probably would have caused a large increase in the resistance of the wire, because the dependence of R on E upon overheating should remain of the same character as in a metal below the melting point. Actually, the resistance of the wire during the drop in emission differs from R^s_{mp} by not more than 10%, while the wire remains intact after the experiment. We note that in experiments with tungsten we cannot definitely indicate the start of the drop in current I_a with relation to points *3* and *4* since these points do not appear in experiments 6 and 7, and such experiments were not repeated with an improved technique of taking the oscillograms V_R.

2. Ignat'eva and Kalashnikov[10] studied the dependence of $R = R(E)$ in metals with j as high as 5×10^6 A/cm^2, using the same method as in our experiments. It is stated in this paper that the resistance of transition metals, measured at high current density, is much larger than the resistance at ordinary current, i. e., there is a violation of Ohm's law. In this case the increase in R has its maximum value at the instant of switching the high current on, i. e., when the metal has not yet received any appreciable energy from the current (for example, with $j \approx 5 \times 10^{-6}$ A/cm^2 for platinum, $[V_R(t_1)/V_r(t_1)]r = 5R_{20}$).

In our experiment such an effect was much smaller (for example, from oscillogram 4 *c* for nickel at $j = 5 \times 10^6$ A/cm^2, $[V_R(t_1)/V_r(t_1)]r = 1.15 R_{20}$). Here the experiment shows that the increase of $[V_R(t_1)/V_r(t_1)]r$ over R_{20} at the start of the oscillograms is considerably modified by a change in the configuration of the wires of the segment of the network, located between the terminals of the plates of the oscillograph. Therefore we believe that the difference between $[V_R(t_1)/V_r(t_1)]r$ and R_{20} cannot be considered as characteristic of the condition of the metal and the statement about the dependence of resistance on the current density seems erroneous to us. The error, in our opinion, is due

* When the length of the tungsten wire is changed from one to several centimeters, the current i_m is changed only by a small amount. Consequently, the losses of energy due to thermal conduction through the ends of the wire are small compared to radiation losses even at the melting point of tungsten.

[9] S. E. Khaikin and N. P. Bene, Doklady Akad. Nauk SSSR 23, 31 (1939)

[10] L. A. Ignat'eva and S. G. Kalashnikov, J. Exper. Theoret. Phys. USSR 22, 385 (1952)

to the fact that, during some time after the start of the current i, V_R and V are considerably different from iR and ir, because of the influence of the inductance L of the segments of the network between the connection points of the oscillograph plates[1,3] ** We believe that it is not possible to determine R and E from the initial parts of the oscillograms from the formulas

$$R = \frac{V_R}{V_r} r$$

and

$$E(t) = \frac{1}{r} \int_{t_1}^{t} V_R(t)\, V_r(t)\, dt.$$

In the later regions of our oscillograms, in which these formulas are valid, there are no signs of deviation from Ohm's law.

It was also concluded in reference 10 that, for a sufficient increase in current density, the energy necessary to start melting is decreased, which is confirmed by the course of the curve IV (see reference 10, Fig. 9, p. 395). However, the displacement of the entire curve along the energy axis is observed only for the curve taken at $j = 4.2 \times 10^6$ A/cm^2. Its course is very different from the course of all other curves shown in Fig. 9, including that of the curve taken at $j = 3.9 \times 10^6$ A/cm^2. For all these curves the values of R and E at point *3* (in our terminology) are equal, and do not differ to any extent from the usual relationship between R and E, before the onset of melting. The values of R and E are also equal in these curves at the point *4*. These results coincide with our results obtained at $6 \times 10^4 \leq j \leq 5 \times 10^6$ A/cm^2 for nickel.

The decrease in energy E_3 observed by the authors in reference 10 at the transition to large j's was never observed by us*.

** Upon cooling the wire with liquid air while keeping the other test conditions the same, we observed an increase in the deviation of $[V_R(t_1)/V_r(t_1)]r$ from the value of R_{20}. This can be explained by the fact that at the start of the oscillogram, when di/dt is large, the difference between $V_R/Ri = (L\frac{di}{dt} + Ri)/Ri$ and unity increases with a reduction in R. The increase in j at constant i increases the rate of heating and displaces the working regions of the oscillograms to the region of distortions of V_R and V_r.

* Our early experiments in measuring the dependence $R = R(E)$ [e.g., see S. V. Lebedev and S. E. Khaikin, J. Exper. Theoret. Phys. USSR **26**, 629 (1954)] rather indicate an increase in this energy with increasing j above 5×10^6 A/cm^2 than its decrease.

CONCLUSIONS

1. At current densities $6 \times 10^4 \leq j \leq 5 \times 10^6$ A/cm^2 the character of the dependence of R on E for nickel appears to be the same as for tungsten. The breaking point *4* separates the curve $R = R(E)$ into two parts, so that for $t \leq t_4$ the function $R(E)$ does not change with a change in j, but for $t > t_4$ the shape of the curve $R(E)$ depends on j. The value of the energy $E(t_c)$ at which R increases abruptly, decreases when j is decreased.

2. At the breaking point *3* of the curve $R = R(E)$ the values of energy and resistance correspond to the start of melting of the metal under ordinary conditions. The value of energy in point *4* corresponds to the end of the melting of the metal under ordinary conditions. A comparison of the measured value of the resistance of the metal in the liquid state at the melting point R^l_{mp} is difficult, because the value R^l_{mp} has not been checked by us.

3. An increased electron emission is observed on nickel as well as on tungsten. The anode current, measured upon heating the nickel wire by a current pulse $\sim 10^5$ A/cm^2, is 50 times larger than the nickel emission at the melting point and the nickel wire is not destroyed.

4. A drop in emission is observed on nickel and on tungsten, while the intake of energy into the observed wire is many times larger than energy losses, measured during stationary heating in vacuum at the melting point temperature.

5. In experiments with tungsten, after an abrupt interruption of the heating current, the large anode current I_a decreases, coming close to the value I_a^*, corresponding to the law of Boguslavskii - Langmuir. Here the rate of decrease of I_a/I_a^* appears to be larger, the larger the value of I_a/I_a^* independently of the value of I_a.

It was not possible to discern the breaking points in the curve $R = R(E)$ (points *3* and *4*) in our former experiments on tungsten, because the oscillograms were not sufficiently sharp. The presence of these points shows that the drop of dR/dE observed on tungsten near $R = R^s_{mp}$, does not occur smoothly, but by jumps. These jumps correspond to the start and the end of the melting process, if it is assumed that there is only 8% change in the resistance of tungsten on melting. (I.e., the change is very small in comparison with other metals.) This assumption is more natural, as the ratio R_4/R_3 does not change with a change of the current density in large limits. Thus our former supposition[1,3] that the drop of dR/dE in tungsten near $R = R^s_{mp}$ is anomalous, is probably wrong.

Other phenomena*, observed in our previous experiments with tungsten at large current densities and not observed under ordinary conditions, depend considerably on the current density. These phenomena were considered by us as indications of an anomalous state of the tungsten caused by a flow of current of high density. The dependence of energy at the instant of explosion - $E(t_c)$ - on j and emission anomalies is observed in the experiments treated in present paper, not only on tungsten, but also on nickel. New peculiarities of emission (see point 4 of the Conclusions) were discovered which we also could not explain by means of a supposition that the metal is in a normal state. We note that the conditions of the experiments on nickel and on tungsten were materially different because of the considerable difference in melting point temperature (for example the measured anode currents were different by several orders of magnitude). The identical character of the phenomena observed in these metals near $R = R^s_{mp}$ shows that they are caused by processes in metals independent of the value of their melting point temperature.

* The dependence on the energy $E(t_c)$ on j(t) [e.g., see S. V. Lebedev and S. E. Khaikin, J. Exper. Theoret. Phys. USSR 26, 629 (1954) and S. V. Lebedev, J. Exper. Theoret. Phys. USSR 27, 605 (1954)], anomalies of emission [e.g., see S. V. Lebedev and S. E. Khaikin, J. Exper. Theoret. Phys. USSR 27, 487 (1954)], peculiarities of melting in experiments with interrupted pulses [e.g., see S. V. Lebedev, J. Exper. Theoret. Phys. USSR 27, 605 (1954)].

Translated by S. Paskwer
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SOVIET PHYSICS-JETP VOLUME 1, NUMBER 1 JULY, 1955

The Wave Function of the Lowest State of a System of Interacting Bose Particles

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(Submitted to JETP editor June 13, 1954)
J. Exper. Theoret. Phys. USSR 28, 129-139 (1955)

The wave function and the energy of the lowest state of a slightly non-ideal Bose gas are determined by means of the method of "auxiliary variables", with accuracy to terms of second order relative to the smallness of the parameter of the energy of interaction.

I. INTRODUCTION

THE problem of the investigation of the wave of a system of a large number of interacting Bose particles arose in connection with attempts at the formation of a microscopic theory of the superfluidity of He II. In spite of a series of successes in this direction[1,2] we are today still far from the completion of such a microscopic theory.

If we select as a model for the liquid helium a slightly non-ideal degenerate Bose gas, then it is possible, as one of us has shown[2], to explain the phenomenon of the superfluidity of He II by the properties of the energy spectrum of such a system. However, inasmuch as the slightly non-ideal Bose gas cannot be regarded an entirely satisfactory model of liquid helium, the necessity arises of improving the theory of the non-ideal Bose gas, taking into account interactions that are not small. Up to the present time only such systems with weak interactions between particles have been studied theoretically.

Wave functions of the lowest state of a system consisting of a large number of weakly interacting Bose particles have been determined by Bijl[3]. However the results of his work are in error because of the lack of validity of the approximations used (i.e., terms that are not small in magnitude have been neglected).

In the present work the correct wave functions of the lowest state of a Bose system with weak interaction have been determined by means of the method of "auxiliary variables" with accuracy to terms of second order of smallness.

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[3] A. Bijl, Physica 7, 869 (1940).