Relativistic Corrections to the Two Body Problem

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RECENTLY wide application has been made of the method of finding propagation functions in quantum electrodynamics by means of variational derivatives at the sources [1]. This method, in particular, has been used for obtaining the relativistically covariant equation of motion of electron and positron, interacting through the electromagnetic field [2*] It has been shown also [4], that the Salpeter-Bethe [3] type of equation contains corrections to the energy levels of the hydrogen atom which are not contained in equations of the Berit [5] type. In the positronium atom these corrections have special importance since in this case, as distinguished from that of hydrogen, they have the same order of magnitude as the vacuum (radiative) corrections.

In the present communication we wish to point out the principal significance of taking into account the corrections due to the relativistic character of the two bodies in the case of interaction of positronium with external fields. Before taking up this question, we consider the method of finding the energy of interaction between particles by means of the relativistic equation for bound states [2,3]. In this equation there appears the interaction operator I.

$$(F_2^+ F_1^- - iI)\,\psi = 0, \tag{1}$$

$$F_2^+ = c\mathbf{p}_2\vec{\alpha}_2 + \beta_2 M_2^+ c^2 + e\alpha_{2\mu} A_\mu \,(2),$$

$$F_2^- = c\mathbf{p}_1\vec{\alpha}_1 + \beta_1 M_1^- c^2 - e\alpha_{1\mu} A_\mu \,(1)$$

where M_1^- and M_2^+ are mass operators.

Let us assume that we have somehow solved Eq. (1) for instantaneous interaction, and are interested in the energy of interaction of the electron and the positron between themselves and with the external field, conditioned by the perturbation ΔI in the quantity I appearing in (1), $I = I_0 + \Delta I$. To find the matrix elements of the perturbation energy we must determine the "scattering matrix for ΔI". For this it is sufficient to expand the wave function, governed by Eq. (1) with the inclusion of ΔI, in terms of the unperturbed wave functions [governed by (1) without ΔI]. It is more convenient, however, to deal with the solutions of the equation of type (1) since the expansion of the Green's function G of Eq. (1) in terms of the unperturbed G_0 has an extremely simple appearance:

$$G = G_0 + G_0\, i\Delta I G_0 + G_0\, i\Delta I G_0\, i\Delta I G_0 + \ldots \tag{2}$$

The relativistically covariant equation (1) contains different times for the particles, t_1 and t_2, so that if the external field is stationary we may go over to relative time $t = t_2 - t_1$ and common time $T = (t_1 + t_2)/2$. The scattering matrix, describing the scattering of bound systems from $T = -\infty$ to $T = +\infty$ during adiabatically isolated interaction ΔI, equals

$$S_{-\infty,\,+\infty} = G\,(T = +\infty,\ T = -\infty).$$

With the aid of $S_{-\infty,\,+\infty}$ we can find the transition amplitude from state n to state n', induced by the perturbation ΔI. We will regard as the effective perturbation energy V that energy in the equation for instantaneous interaction

$$(E - H_1 - H_2 - V)\,\varphi = -(\Lambda_+^{(1)} - \Lambda_-^{(2)})\,K(\mathbf{r})\,\varphi \tag{3}$$

(where $\phi = \psi|_{t=0}$, H_1 and H_2 are Hamiltonians for the free electron and positron, $K(\mathbf{r})$ is the potential of the instantaneous interaction, Λ_+ and Λ_- are projection operators), which leads to the same values of the scattering amplitudes as are obtained from $S_{-\infty,\,+\infty}$. In the first approximation of perturbation theory, as can be shown, the matrix elements of the effective perturbation energy have the form

$$V_{nn'} = \frac{1}{\hbar}\int \tilde{\psi}_{n'}(\mathbf{x}_1\mathbf{x}_2 t)\,\Delta I \psi_n(\mathbf{x}_1\mathbf{x}_2 t)\,d^3x_1\,d^3x_2\,dt, \tag{4}$$

where

$$\psi_n = \theta_n(t)\,\varphi_n;\quad \tilde{\psi} = \theta_n(-t)\,\varphi_n^+;\quad \varphi_n = \psi_n|_{t=0}. \tag{5}$$

Integration with respect to the relative time in (4) is easily performed with the aid of operator $\theta_n(t)$ describing the causal development in relative time. Its properties will be considered in a separate communication.

The energy of interaction between electron and positron which we have found by means of (4) exactly coincides with the energy found by Landau and Berestetskii [6] in the particular case of the absence of external field and with limitation to terms of order α^2.

Let us consider now the interaction of the positronium atom with the external electromagnetic field. We consider the case of weak fields when

the interaction with the external field can be treated as a perturbation. We regard the external fields as constants, i. e., we regard the potentials as linear functions of the coordinates. If only terms that are linear in the field are retained, then the operator for interaction with the external field ΔI_{ex} has the form

$$i\Delta I_{\text{ex}} = -e\alpha_{1\mu}A_{\mu}(1)\left(-\frac{\hbar}{2i}\frac{\partial}{\partial T}+\frac{\hbar}{i}\frac{\partial}{\partial t}-H_2\right) \quad (6)$$
$$+e\alpha_{2\mu}A_{\mu}(2)\left(-\frac{\hbar}{2i}\frac{\partial}{\partial T}-\frac{\hbar}{i}\frac{\partial}{\partial t}-H_1\right).$$

The energy of interaction with the external field found by means of (6) in the first approximation exactly coincides with the energy found by us by the usual method in the first approximation of perturbation theory. It is important, however, to emphasize that by means of (6) corrections are obtained to this energy, connected with the relativistic two body problem and its influence on the interaction of positronium with external fields. For $n = 1$ they have the magnitude

$$(V_{\text{ex}}^{\text{rel}})_{ss'} = -\mu_0\frac{\alpha^2}{12}<\sigma_{1z}-\sigma_{2z}>_{ss'}H_z, \quad (7)$$

where s characterizes the spin states σ_{1z} and σ_{2z} are Pauli matrices, $\alpha = e^2/\hbar c, \mu_0 = e\hbar/2mc$, H_z is the intensity of the magnetic field. This energy produces additional mixing of ortho and para states in positronium.

Since the operator for relative energy enters into operator (6), interest exists in the question to what degree the corrections which have been found are related to the operator $(\hbar/i)(\partial/\partial t)$ and, consequently, with the different times of the particles. If the energy of interaction associated with the part of operator (6) containing $(\hbar/i)(\partial/\partial t)$ is found separately, then exactly (7) is obtained.

The corrections found in this manner are of principal interest, since they are all related to the different times of the particles.

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14

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*The difference between this equation and the Salpeter-Bethe equation [3] is connected with calculation of the specific exchange interaction between electron and positron determined by their virtual annihilation.

Investigation of the Anisotropy of the Surface Resistance of Tin at Low Temperature

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RESULTS of investigations of the anisotropy of the surface resistance of tin at helium temperatures in normal and superconducting states are reported in the papers of Pippard [1-3]. In these studies the properties of the coaxial resonator (at a frequency of approximately 9400 mcs/sec) were experimentally investigated; the specimen under investigation serves as the internal conductor-cylindrical monocrystalline tin, 14 mm in length and 0.25 to approximately 1 mm in diameter, having a different angle of inclination of the major crystalline axis relative to the axis of the specimen. The dependence, arrived at in reference 1, of the active surface conductivity (at normal state) on the angle θ is shown in Figure 1 (solid curve; vertical dashes--Experimental points). A similar dependence of the reactive surface conductivity on the angle θ as well as the penetration depth of the electromagnetic field calculated from it is arrived at in reference 1. These experiments led Pippard to a conclusion as to the non-tensorial character of the anisotropic effects noted.

The principal significance of the above mentioned work is the necessity for careful and detailed consideration of these investigations as to the methods employed. As a first step it may be of interest to determine the experimental results one should expect if we assign a normal tensorial character to the anisotropy of surface conductivity of tin at helium temperatures. This article deals with this important question.

In the coaxial resonator the electrical oscillations may be produced along the axis (usual electromagnetic wave) as well as in the perpendicular direction. Keeping in mind the fundamental frequencies associated with these two types of resonance one