collapse of nuclei of the normal phase is experimentally demonstrated.

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The Velocity Distribution of Electrons in the Presence of a Varying Electric Field and a Constant Magnetic Field

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The function of the velocity distribution of electrons in the presence of a varying electric field and a constant magnetic field is found. Two cases are examined: 1) an electric field depending harmonically on time; 2) an amplitude-modulated electric field.

1. INTRODUCTION

THE problem of finding the function for the velocity distribution in the case of elastic collisions of electrons with the atoms of a gas has been analyzed by several authors[1-5]. The basic work on this question appears to be that of Davydov[1]. In this work the function of the distribution of electrons in the presence of constant electric and magnetic fields was obtained. Margenau[3] analyzed the action in a varying electric field. The influence of a constant magnetic field in the presence of a varying electric field was examined by Jancel and Kahan[5]. However, they did not take into account the action of the components of the electric field parallel to the direction of the magnetic field.

In the works mentioned, the influence of the collision of electrons with each other was not considered. Meanwhile, as was shown by Cahn[6], the influence of inter-electronic collisions must be accounted for in the case of a constant electric field if the concentration of electrons is great.

[1] B. I. Davydov, J. Exper. Theoret. Phys. USSR **7**, 1069 (1937)

[2] L. D. Landau, J. Exper. Theoret. Phys. USSR **7**, 203 (1937)

[3] H. Margenau, Phys. Rev. **73**, 297 (1948)

[4] Y. L. Klimontovich, J. Exper. Theoret. Phys. USSR **21**, 1284 (1951)

[5] R. Jancel and T. Kahan, Comptes rend. **236**, 788 (1953)

[6] J. Cahn, Phys. Rev. **75**, 346 (1949)

In Part 2 we will give an analysis of the distribution function of the electrons according to velocity in the presence of an electric field, harmonically dependent on time, and a constant magnetic field.

In Part 3 we shall analyze the action in an amplitude-modulated electric field.

2. THE DISTRIBUTION OF ELECTRONS IN THE PRESENCE OF A HARMONIC ELECTRIC FIELD AND A CONSTANT MAGNETIC FIELD

Assuming that a gas is uniformly distributed in space, the kinetic equation takes the following form:

$$\frac{\partial f}{\partial t} + \mathbf{a}\,\nabla_v f = \frac{\delta}{\delta t} f, \tag{1}$$

where $f(\mathbf{v}, t)$ is a function of the velocity distribution of the electrons, $\mathbf{a}$ is the acceleration communicated by the field of electrons, $\partial f/\partial t$ denotes the rate of the change of the distribution due to the presence of the field, $(\delta/\delta t)f$ stands for the collision of electrons with gas atoms (the effects of the collision of electrons with each other are not considered)*, ∇_v is the gradient in the velocity domain. In our case

$$\mathbf{a} = \frac{e}{m}\mathbf{E}_0\Theta + \frac{e}{mc}[\mathbf{vH}] \equiv \vec{\Gamma}\Theta + \frac{e}{mc}[\mathbf{vH}], \tag{2}$$

* The effect of inter-electronic collisions is not essential when the concentration of electrons is not very great and the field frequencies are high[6,8].

where $\mathbf{E}_0$ represents the amplitude of the electric field, Θ is equal to cos ωt, and $\mathbf{H}$ denotes the magnetic field intensity.

To solve Eq. (1) we try to find a series of Legendre polynomials. By limiting ourselves to the first two terms, we set down this equation:

$$f(\mathbf{v}, t) = f_0(v, t) + \mathbf{v}\mathbf{f}'(v, t). \tag{3}$$

we break up $\mathbf{f}$ along three mutually perpendicular directions:

$$\mathbf{f}' = \vec{\Gamma}_{\perp}\varphi + [\mathbf{H}\vec{\Gamma}]\psi + \vec{\Gamma}_{\parallel}\xi, \tag{4}$$

where $\vec{\Gamma}_{\perp}$ and $\vec{\Gamma}_{\parallel}$ denote the components of $\vec{\Gamma}$, respectively perpendicular and parallel to the magnetic field $\mathbf{H}$. In the case of elastic collisions[1]:

$$\frac{\delta}{\delta t} f_0 = \frac{1}{v^2}\frac{m}{M}\frac{\partial}{\partial v}\left(\frac{v^4 f_0}{l}\right) + \frac{kT}{Mv^2}\frac{\partial}{\partial v}\left(\frac{v^3}{l}\frac{\partial f_0}{\partial v}\right), \tag{5}$$

$$\frac{\delta}{\delta t}\mathbf{v}\mathbf{f}' = -\mathbf{v}\frac{v}{l}\mathbf{f}', \tag{5 }$$

where l is the length of the free path of the electrons, M is the mass of the gas atoms. Let us substitute Eqs. (2) - (5) in Eq. (1). By comparing the terms of even powers of $\mathbf{v}$ in the result thus obtained, and averaging in all directions, we find

$$\frac{1}{3v^2}\frac{\partial}{\partial v}\{v^3(\Gamma_{\perp}^2\varphi + \Gamma_{\parallel}^2\xi)\Theta\} + \frac{\partial f_0}{\partial t} \tag{6}$$

$$= \frac{1}{v^2}\frac{m}{M}\frac{\partial}{\partial v}\left(\frac{v^4}{l}f_0\right) + \frac{kT}{Mv^2}\frac{\partial}{\partial v}\left(\frac{v^3}{l}\frac{\partial f_0}{\partial v}\right),$$

while comparing terms of the first power $\mathbf{v}$ and taking projections in the chosen directions, we obtain

$$\frac{\partial\varphi}{\partial t} + \frac{1}{v}\frac{\partial f_0}{\partial v}\Theta - \frac{e}{mc}H^2\psi = -\frac{v}{l}\varphi, \tag{7}$$

$$\frac{\partial\xi}{\partial t} + \frac{1}{v}\frac{\partial f_0}{\partial v}\Theta = -\frac{v}{l}\xi, \tag{8}$$

$$\frac{\partial\psi}{\partial t} + \frac{e}{mc}\varphi = -\frac{v}{l}\psi. \tag{9}$$

It is easy to verify that the system of equations (6) - (9) is equivalent to that obtained by Davydov[1]. We shall look for the solution of equations (6) - (9) in the form of the sum of harmonics of the frequencies which are multiples of ω:

$$f_0 = \frac{F_0}{2} + \sum_k F_k \cos k\omega t + G_k \sin k\omega t = \sum_k \Phi_k e^{ik\omega t}, \tag{10}$$

where

$$\Phi_0 = \frac{F_0}{2}, \quad \Phi_k = \frac{1}{2}(F_k - iG_k) = \Phi_{-k}^{*}.$$

Analogously,

$$\varphi = \sum_k \varphi_k e^{ik\omega t}, \quad \psi = \sum_k \psi_k e^{ik\omega t}, \quad \xi = \sum_k \xi_k e^{ik\omega t}, \tag{11}$$

where

$$\varphi_k = \frac{1}{2}(f_k - ig_k) = \varphi_{-k}^{*},$$

$$\psi_k = \frac{1}{2}(q_k - ir_k) = \psi_{-k}^{*},$$

$$\xi_k = \frac{1}{2}(h_k - ip_k) = \xi_{-k}^{*}.$$

Substituting Eqs. (10) and (11) in Eqs. (6) - (9) and comparing the terms containing $e^{ik\omega t}$ we find

$$ik\omega\Phi_k + \frac{1}{6v^2}\frac{\partial}{\partial v}\{v^3[\Gamma_{\perp}^2(\varphi_{k-1} + \varphi_{k+1}) + \Gamma_{\parallel}^2(\xi_{k-1} + \xi_{k+1})]\} = \frac{\delta\Phi_k}{\delta t}; \tag{12}$$

$$ik\omega\varphi_k + \frac{1}{2v}\left(\frac{\partial\Phi_{k-1}}{\partial v} + \frac{\partial\Phi_{k+1}}{\partial v}\right) - \frac{e}{mc}H^2\psi_k = -\frac{v}{l}\varphi_k, \tag{13}$$

$$ik\omega\xi_k + \frac{1}{2v}\left(\frac{\partial\Phi_{k-1}}{\partial v} + \frac{\partial\Phi_{k+1}}{\partial v}\right) = -\frac{v}{l}\xi_k, \tag{14}$$

$$ik\omega\psi_k + \frac{e}{mc}\varphi_k = -\frac{v}{l}\psi_k. \tag{15}$$

From Eqs. (13) - (15) we find expressions for ϕ_k, ξ_k and ψ_k:

$$\varphi_k = -\frac{1}{2v}\left(\frac{\partial\Phi_{k-1}}{\partial v} + \frac{\partial\Phi_{k+1}}{\partial v}\right)\frac{ik\omega + \nu}{(ik\omega + \nu)^2 + \omega_H^2}, \tag{16}$$

$$\psi_k = -\frac{e}{mc}\frac{1}{ik\omega + \nu}\varphi_k,$$

$$\xi_k = -\frac{1}{2v}\left(\frac{\partial\Phi_{k-1}}{\partial v} + \frac{\partial\Phi_{k+1}}{\partial v}\right)\frac{1}{ik\omega + \nu},$$

where $\nu = v/l$; $\omega_H = |e/mc\ H|$.

In this manner we have obtained an infinite system of linear differential equations with an infinite number of unknown functions. We can solve this system in certain special cases.

It is evident that a varying electric field does not create a constant one consisting of an asymmetrical part of the distribution function; that is, $\phi_0 = \psi_0 = \xi_0 = 0$. From Eq. (13) it then follows that $\Phi_1 = 0$, which in turn leads to the equality $\phi_2 = \psi_2 = \xi_2 = 0$. It is easily shown for the general case that

$$\Phi_1 = \Phi_3 = \ldots = \Phi_{2k+1} = 0, \tag{17}$$

$$\varphi_{2k} = \psi_{2k} = \xi_{2k} = 0.$$

The system of equations (12) - (15) can be solved by the method of successive approximations. For a first approximation, we will neglect all harmonics except Φ_0, ϕ_1, ξ_1, ψ_1. From Eqs. (11) and (16) we will then find

$$f_1 = -\frac{\partial \Phi_0}{\partial v}\frac{1}{A}, \quad h_1 = -\frac{l}{\omega^2 l^2 + v^2}\frac{\partial \Phi_0}{\partial v}, \tag{18}$$

$$q_1 = \frac{e}{mc}\frac{1}{\omega^2 + \nu^2}(\omega g_1 - \nu f_1),$$

$$g_1 = B\frac{\partial \Phi_0}{\partial v}, \quad p_1 = -\frac{\omega}{\omega^2 + \nu^2}\frac{1}{v}\frac{\partial \Phi_0}{\partial v},$$

$$r_1 = -\frac{e}{mc}\frac{1}{\omega^2 + \nu^2}(\omega f_1 + \nu g_1),$$

where

$$A = \frac{l^2\omega^2(1-z)^2 + v^2(1+z)^2}{l(1+z)}, \quad z = \frac{\omega_H^2}{\omega^2 + \nu^2},$$

$$B = \frac{\omega}{v}\frac{(z-1)l^2}{\omega^2 l^2(1-z)^2 + v^2(1+z)^2}.$$

Having integrated Eq. (12), taking $k = 0$, we obtain

$$\Phi_0 = C\exp \tag{19}$$

$$\left\{-\int_0 \frac{mv\,dv}{kT + \left[\Gamma_\perp^2 Ml/6A\right] + \left[\Gamma_\parallel^2 Ml^2/6(\omega^2 l^2 + v^2)\right]}\right\}.$$

The constant C can be determined from the condition of normalization:

$$4\pi\int \Phi_0 v^2\,dv = 1. \tag{20}$$

In this manner the distribution function to the first approximation takes on the form

$$f(\mathbf{v}, t) = \Phi_0 + v\,[\vec{\Gamma}_\perp(f_1\cos\omega t + g_1\sin\omega t) \tag{21}$$

$$+ [\mathbf{H}\vec{\Gamma}](q_1\cos\omega t + r_1\sin\omega t)$$

$$+ \vec{\Gamma}_\parallel(h_1\cos\omega t + p_1\sin\omega t)].$$

To obtain a second approximation, it is necessary to substitute the solution thus found in the original set of equations (12) - (15). We then find

$$i2\omega\Phi_2 + \frac{1}{6v^2}\frac{\partial}{\partial v}\{v^3[\Gamma_\perp^2\varphi_1 + \Gamma_\parallel^2\xi_1]\} = \frac{\delta\Phi_2}{\delta t} \tag{22}$$

From these, separating the real and imaginary parts, we have

$$2\omega G_2 + \frac{1}{6v^2}\frac{\partial}{\partial v}\{v^3[\Gamma_\perp^2 f_1 + \Gamma_\parallel^2 h_1]\} = \frac{\delta F_2}{\delta t}; \tag{23}$$

$$2\omega F_2 - \frac{1}{6v^2}\frac{\partial}{\partial v}\{v^3[\Gamma_\perp^2 g_1 + \Gamma_\parallel^2 p_1]\} = -\frac{\delta G_2}{\delta t} \tag{24}$$

It will be shown below that under the conditions of Eq. (28) the right-hand sides of these equations are small. Assuming that the right-hand sides of Eqs. (23) and (24) are small, we shall immediately obtain an approximate expression for G_2 and F_2 which satisfy the condition of normalization at 0. These expressions can be substituted on the right-hand sides of Eqs. (23) and (24) in order to obtain a second approximation for G_2 and F_2, and so on. The solutions obtained in this manner converge well. To a first approximation, G_2 has the form

$$G_2 = \frac{1}{12\omega v^2}\frac{\partial}{\partial v}\left\{v^3\left[\frac{\Gamma_\perp^2}{A} + \frac{\Gamma_\parallel^2 l}{\omega^2 l^2 + v^2}\right]\frac{\partial \Phi_0}{\partial v}\right\}, \tag{25}$$

and F_2, to a second approximation, has the form

$$F_2 = \frac{1}{12\omega v^2}\frac{\partial}{\partial v}\left\{v^3\left[B\Gamma_\perp^2 \right.\right. \tag{26}$$

$$\left.\left. -\frac{\omega}{v}\frac{1}{\omega^2 + \nu^2}\Gamma_\parallel^2\right]\frac{\partial \Phi_0}{\partial v}\right\} - \frac{1}{2\omega}\frac{\delta G_2}{\delta t},$$

where the operator $\delta/\delta t$ is determined by Eq. (5).

Now it is necessary to explain the stipulation that G_2 and F_2 be small in comparison with Φ_0 [in the same manner as the condition of smallness of the right-hand sides of Eqs. (23) and (24)], because this condition is necessary for the applicability of the method of successive approximations which we have used to determine the functions Φ_0, ϕ_1, ξ_1, ψ_1. We observe that the function of the distribution of electrons is, in effect, different from 0 only in the domain of the velocity v of the order of the average velocity $v_{av} = v_{av}(T, E, H)$. Therefore, attention is given to examining only the velocity region, in which v is appreciable.

From Eqs. (23) - (26), it can be easily shown that

$$G_2 \sim \frac{\delta\nu}{\omega}\Phi_0, \qquad F_2 \sim \left[\delta + \left(\frac{\delta\nu}{\omega}\right)^2\right]\Phi_0, \quad (27)$$
$$\frac{\delta F_2}{\delta t} \sim \delta\nu F_2, \qquad \frac{\delta G_2}{\delta t} \sim \delta\nu G_2,$$

where $v \sim v_{av}$ under the conditions

$$\delta\nu(v_{av}) \ll \omega \qquad (28)$$

G_2 and F_2 will be small compared to Φ_0 and can be neglected; and the right-hand sides of Eqs. (23) and (24) will be small compared to the terms on the left-hand sides. It can be seen that the corrections to the following approximations to the functions Φ_0, F_2 and G_2, as well as the functions F_4, G_4, etc., are of a low order compared to $\delta\nu/\omega$. In this manner, Eq. (28) is demonstrated to be the condition for the applicability of the method of successive approximations. We note that these conditions place certain restrictions on the field intensity, because v_{av} is field-dependent.

Let us estimate the order of v_{av} in the case when the magnetic field $\mathbf{H} = 0$ and the influence of the molecular temperature on the function of the electron distribution can be neglected in comparison with the influence of the electric field $\mathbf{E}$. Furthermore, let the length of the free path be l = const. It is evident that v_{av} is of the order of magnitude of the v's for which the exponent in Eq. (19) is of the order of unity. Such an estimate gives the following information about v_{av}:

$$v_{av}^2 \sim \sqrt{\left(\frac{\omega^2 l^2}{2}\right)^2 + \frac{\Gamma_\parallel^2 l^2}{3\delta}} - \frac{\omega^2 l^2}{2} \qquad (29)$$

Hence, Eq. (28) takes on the form

$$\frac{\delta^2}{l^2}\left(\sqrt{\left(\frac{\omega^2 l^2}{2}\right)^2 + \frac{\Gamma_\parallel^2 l^2}{3\delta}} - \frac{\omega^2 l^2}{2}\right) \ll \omega^2. \qquad (30)$$

This stipulation is well satisfied for $\omega = 10^7$ sec^{-1}, $E \leq 10^{-1}$ V/cm or for $\omega = 10^6$ sec^{-1}, $E \leq 10^{-2}$ V/cm (l = 1 cm).

If we set $H = 0$ in Eqs. (19) - (21), then the distribution thus found agrees with that of Margenau[3] who disregarded the influence of the magnetic field. The formula arrived at in reference 5 erroneously omits the term $\Gamma_\parallel^2 Ml^2/6(\omega^2 l^2 + v^2)$ in Eq. (19) and the terms with $\Gamma_\parallel$ in Eq. (21).

In the quasi-stationary case ($\omega \ll \delta\nu$; see reference 9) the distribution function agrees with that in a constant electric field (it is necessary only to replace the constant electric field E_0 by $E_0 \cos \omega t$). The distribution function in a constant field cannot be found simply by substitution of $\omega = 0$ in Eqs. (18) and (19) as was done by Jancel and Kahan[5], since Eqs. (18) and (19) were derived from Eq. (28)*.

In the constant field ($\delta f/\delta t = 0$; $\Theta = 1$) Eqs. (6) - (9) take on the form

$$\frac{1}{3v^2}\frac{\partial}{\partial v}\{v^3(\Gamma_\perp^2\varphi + \Gamma_\parallel^2\xi)\} \qquad (6')$$
$$= \frac{1}{v^2}\frac{m}{M}\frac{\partial}{\partial v}\left(\frac{v^4}{l}f_0\right) + \frac{kT}{Mv^2}\frac{\partial}{\partial v}\left(\frac{v^3}{l}\frac{\partial f_0}{\partial v}\right),$$

$$\frac{1}{v}\frac{\partial f_0}{\partial v} - \frac{e}{mc}H^2\psi = -\frac{v}{l}\varphi, \qquad (7')$$

$$\frac{1}{v}\frac{\partial f_0}{\partial v} = -\frac{v}{l}\xi, \qquad (8')$$

$$\frac{e}{mc}\varphi = -\frac{v}{l}\psi. \qquad (9')$$

whence

$$\varphi = -\frac{1}{v}\frac{\partial f_0}{\partial v}\frac{\nu}{\omega_H^2 + \nu^2}, \qquad (31)$$

$$\psi = -\frac{e}{mc\nu}\varphi, \qquad \xi = -\frac{1}{v\nu}\frac{\partial f_0}{\partial v},$$

$$f_0 = C\exp\left\{-\int_0^v \frac{mv\,dv}{kT + (\Gamma_\perp^2 Ml/3A') + (\Gamma_\parallel^2 Ml^2/3v^2)}\right\},$$

$$A' = \frac{v^2}{l}(1 + z'), \qquad z' = \frac{\omega_H^2}{\nu^2}.$$

It can easily be seen that the distribution function coincides with that of Davydov[1].

Until now, we have not attempted to form a picture of the dependence of l on velocity. In a number of works[4,6,9], two very simple possibilities are considered: l = const and l = const × v;

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* Jancel and Kahan[5] contend that with $\omega = 0$, the distribution obtained by them agrees with that obtained by Chapman and Cowling[7] for a constant field. This is not so.

7 S. Chapman and T. Cowling, *The Mathematical Theory of Non-Uniform Gases*, Cambridge, 1952, p. 355

8 V. L. Ginzburg, J. Exper. Theoret. Phys. USSR 21, 943 (1951)

9 Y. L. Al'pert, V. L. Ginzburg and E. L. Feinberg, *Wave Propagation*, Gostekizdat, 1953

in the second case, the frequency of collision remains constant, $\nu = v/l$. To satisfy the condition

$$\left(\frac{1-z}{1+z}\right)^2 \frac{\omega^2}{\nu^2} \gg 1,$$

Eq. (19) turns into a Maxwellian distribution [independent of the form of the dependence $l(v)$]:

$$\Phi_0 = \left(\frac{m}{2\pi k T_{\rm eff}}\right)^{3/2} \exp\left\{-\frac{mv^2}{2kT_{\rm eff}}\right\}, \quad (19')$$

where

$$T_{\rm eff} = T\left[1 + \frac{M}{6kT}\left(\frac{\Gamma_\perp^2 (1+z)}{\omega^2 (1-z)^2} + \frac{\Gamma_\parallel^2}{\omega^2}\right)\right],$$

$$z = \frac{\omega_H^2}{\omega^2}.$$

When the number of collisions is constant, then Eq. (19) again turns into a Maxwellian distribution, with an effective temperature

$$T_{\rm eff} = T\left[1 + \frac{\Gamma_\perp^2 M (1+z)}{6kT\,(\omega^2 (1-z)^2 + \nu^2 (1+z)^2)} + \frac{\Gamma_\parallel^2 M}{6kT\,(\omega^2 + \nu^2)}\right]. \quad (32)$$

Let us propose, as was done by Klimontovich[4], that $\nu = v_{\rm av}/\tilde{l}$, where $\tilde{l}$ is independent of velocity, field and temperature, and $v_{\rm av}$ is some average velocity expressed by means of the effective temperature; e.g., in the following manner*

$$v_{\rm av}^2 = kT_{\rm eff}/m. \quad (33)$$

Then, the effective temperature is determined by Eqs. (32) and (33). In this case, Eq. (31) coincides with that obtained by Klimontovich[4] if we set $\Gamma_\parallel^2 = 0$ $(\mathbf{E} \perp \mathbf{H})$ ** in (31).

3. THE ELECTRON DISTRIBUTION IN THE PRESENCE OF A MODULATED ELECTRIC FIELD AND A CONSTANT MAGNETIC FIELD

At times it is necessary to know the function of the electron distribution in the presence of an amplitude-modulated electromagnetic wave (e.g., in the examination of ionospheric cross-modulation). Two cases must be distinguished here.

In the first case, when the modulation frequency* is

$$\Omega \ll \delta\nu, \quad (34)$$

it is only necessary to substitute in all formulas in the preceding part, $\vec{\Gamma}$ for $\vec{\Gamma}(1 + \mu\cos\Omega t)$, since Eq. (34) proves to be quasi-stationary.

In the second case, when

$$\Omega \gg \delta\nu, \quad (35)$$

it is necessary to carry out the integration of the system of Eqs. (6) - (9) where we must assume

$$\Theta = \cos\omega t\,(1 + \mu\cos\Omega t).$$

We shall seek a distribution function of the form

$$f(\mathbf{v}, t) = \sum_{n,k} \Phi_{n,k} e^{i\omega_{n,k} t} + \mathbf{v}\sum_{n,k} (\vec{\Gamma}_\perp \varphi_{n,k} + [\mathbf{H}\vec{\Gamma}]\psi_{n,k} + \vec{\Gamma}_\parallel \xi_{n,k}) e^{i\omega_{n,k} t}, \quad (36)$$

where

$$\omega_{n,k} = n\omega + k\Omega, \quad \Phi_{n,k} = \Phi^*_{-n,-k},$$

$$\varphi_{n,k} = \varphi^*_{-n,-k},$$

$$\psi_{n,k} = \psi^*_{-n,-k}, \quad \xi_{n,k} = \xi^*_{-n,-k}.$$

Having substituted Eq. (36) in Eqs. (6) - (9) and having taken into consideration that

$$\Theta = \cos\omega t\,(1 + \mu\cos\Omega t) \equiv \frac{1}{2} e^{i\omega_{1,0} t} + \frac{1}{2} e^{-i\omega_{1,0} t} + \frac{\mu}{4}\left(e^{i\omega_{1,-1} t} + e^{-i\omega_{1,-1} t} + e^{i\omega_{1,1} t} + e^{-i\omega_{1,1} t}\right),$$

we obtain the following equations for the determination of $\Phi_{n,k}$, $\psi_{n,k}$, $\phi_{n,k}$, $\xi_{n,k}$:

$$i\omega_{n,k}\Phi_{n,k} + \frac{1}{6v^2}\frac{\partial}{\partial v}\Big\{v^3\Big[\Gamma_\perp^2\Big(\varphi_{n-1,k} + \varphi_{n+1,k} + \frac{\mu}{2}\varphi_{n-1,k-1} + \frac{\mu}{2}\varphi_{n+1,k-1} + \frac{\mu}{2}\varphi_{n-1,k+1} + \frac{\mu}{2}\varphi_{n+1,k+1}\Big) + \Gamma_\parallel^2\Big(\xi_{n-1,k} + \xi_{n+1,k} + \frac{\mu}{2}\xi_{n-1,k-1} + \frac{\mu}{2}\xi_{n+1,k-1} + \frac{\mu}{2}\xi_{n-1,k+1} + \frac{\mu}{2}\xi_{n+1,k+1}\Big)\Big]\Big\} = \frac{\delta\Phi_{n,k}}{\delta t}; \quad (37)$$

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* We observe that it is not essential which average velocity we substitute in the relation $\nu = v_{\rm av}/\tilde{l}$ since this is reflected only in the magnitude of $\tilde{l}$.

** Equation (25) of Klimontovich[4] is incorrect, inasmuch as an applied magnetic field would change the plasma temperature even for the case when $E = 0$.

* Equation (35) and other analogous relations have the same connotation as in Part 2; that is, they must be satisfied in the domain of velocities of the order of $v_{\rm cp}$.

$$\varphi_{n,k} = -\frac{1}{2v}\frac{i\omega_{n,k}+\nu}{(i\omega_{n,k}+\nu)^2+\omega_H^2}L_{n,k}; \quad (38)$$

$$\psi_{n,k} = -\frac{e}{mc}\frac{1}{i\omega_{n,k}+\nu}\varphi_{n,k}; \quad (39)$$

$$\xi_{n,k} = -\frac{1}{2v}\frac{1}{i\omega_{n,k}+\nu}L_{n,k}, \quad (40)$$

where

$$L_{n,k} = \frac{\partial\Phi_{n-1,k}}{\partial v} + \frac{\partial\Phi_{n+1,k}}{\partial v} + \frac{\mu}{2}\left(\frac{\partial\Phi_{n-1,k-1}}{\partial v} + \frac{\partial\Phi_{n+1,k+1}}{\partial v} + \frac{\partial\Phi_{n-1,k+1}}{\partial v} + \frac{\partial\Phi_{n+1,k-1}}{\partial v}\right).$$

We shall attempt to obtain the solution of Eqs. (37) - (40) by the method of successive approximations: furthermore, we shall assume that the relation

$$\omega \gg \Omega. \quad (41)$$

is satisfied.

To the first approximation, we neglect in $L_{n,k}$ all $\Phi_{n,k}$'s except $\Phi_{0,0}$. Then from Eqs. (38) - (40) we find [utilizing Eq. (41)]:

$$\varphi_{1,0} = -\frac{1}{2}\frac{\partial\Phi_{0,0}}{\partial v}\left(\frac{1}{A}+iB\right), \quad (42)$$

$$\varphi_{1,1} = \varphi_{1,-1} = \frac{\mu}{2}\varphi_{1,0},$$

$$\xi_{1,0} = -\frac{1}{2v}\frac{\partial\Phi_{0,0}}{\partial v}\frac{\nu-i\omega}{\omega^2+\nu^2},$$

$$\xi_{1,-1} = \xi_{1,1} = \frac{\mu}{2}\xi_{1,0},$$

$$\psi_{n,k} = -\frac{e}{mc}\frac{\nu-i\omega}{\omega^2+\nu^2}\varphi_{n,k},$$

where A and B have the same connotation as in the preceding parts. Integrating Eq. (37) and setting $n = 0$ and $k = 0$, we find

$$\Phi_{0,0} = C\exp\left\{-\int_0^v \frac{mv\,dv}{kT + [\Gamma_\perp^2 Ml(1+\mu^2/2)\,6A] + [\Gamma_\parallel^2 Ml^2(1+\mu^2/2)/6(\omega^2l^2+v^2)]}\right\}. \quad (43)$$

To obtain $\Phi_{0,1}$, $\Phi_{0,2}$, $\Phi_{2,0}$, $\Phi_{2\pm 1}$, $\Phi_{2,\pm 2}$ to a first approximation, we write the following equations:

$$i\omega_{0,1}\Phi_{0,1} + \frac{1}{6v^2}\frac{\partial}{\partial v}\left\{v^3\left[\Gamma_\perp^2\left(\varphi_{-1,1}+\varphi_{1,1}+\frac{\mu}{2}\varphi_{1,0}+\frac{\mu}{2}\varphi_{-1,0}\right)\Gamma_\parallel^2\left(\xi_{-1,1}+\xi_{1,1}+\frac{\mu}{2}\xi_{1,0}+\frac{\mu}{2}\xi_{-1,0}\right)\right]\right\} = \frac{\delta\Phi_{0,1}}{\delta t}; \quad (44)$$

$$i\omega_{0,2}\Phi_{0,2} + \frac{1}{6v^2}\frac{\partial}{\partial v}\left\{v^3\left[\Gamma_\perp^2\left(\frac{\mu}{2}\varphi_{1,1}+\frac{\mu}{2}\varphi_{-1,1}\right)+\Gamma_\parallel^2\left(\frac{\mu}{2}\xi_{1,1}+\frac{\mu}{2}\xi_{-1,1}\right)\right]\right\} = \frac{\delta\Phi_{0,2}}{\delta t}; \quad (45)$$

$$i\omega_{2,0}\Phi_{2,0} + \frac{1}{6v^2}\frac{\partial}{\partial v}\left\{v^3\left[\Gamma_\perp^2\left(\varphi_{1,0}+\frac{\mu}{2}\varphi_{1,-1}+\frac{\mu}{2}\varphi_{1,1}\right)+\Gamma_\parallel^2\left(\xi_{1,0}+\frac{\mu}{2}\xi_{1,-1}+\frac{\mu}{2}\xi_{1,1}\right)\right]\right\} = \frac{\delta\Phi_{0,2}}{\delta t}; \quad (46)$$

$$i\omega_{2,\pm 1}\Phi_{2,\pm 1} + \frac{1}{6v^2}\frac{\partial}{\partial v}\left\{v^3\left[\Gamma_\perp^2\left(\varphi_{1,\pm 1}+\frac{\mu}{2}\varphi_{1,0}\right)+\Gamma_\parallel^2\left(\xi_{1,\pm 1}+\frac{\mu}{2}\xi_{1,0}\right)\right]\right\} = \frac{\delta\Phi_{2,\pm 1}}{\delta t}; \quad (47)$$

$$i\omega_{2,\pm 2}\Phi_{2,\pm 2} + \frac{1}{6v^2}\frac{\partial}{\partial v}\left\{v^3\left[\Gamma_\perp^2\frac{\mu}{2}\varphi_{1,\pm 1}+\Gamma_\parallel^2\frac{\mu}{2}\xi_{1,\pm 1}\right]\right\} = \frac{\delta\Phi_{2,\pm 2}}{\delta t}.$$

To find the coefficients for $\cos\omega_{n,k}t$ and $\sin\omega_{n,k}t$ in the distribution function, it is necessary to make a transformation from the complex form to the real form and to take into consideration a relationship analogous to that of Eq. (10) and Eq. (11). Then, with accuracy to terms of the order of $(\delta\nu/\Omega)\Phi_{0,0}$ we find

$$\Omega G_{0,1} + \frac{1}{6v^2}\frac{\partial}{\partial v}\{v^3[\Gamma_\perp^2(f_{1,-1}+f_{1,1}+\mu f_{1,0}) + \Gamma_\parallel^2(h_{1,-1}+h_{1,1}+\mu h_{1,0})]\} = 0; \quad (48)$$

$$\Omega F_{0,1} = 0; \quad (49)$$

$$2\Omega G_{0,2} + \frac{1}{6v^2}\frac{\partial}{\partial v}\left\{\frac{\mu}{2}v^3[\Gamma_\perp^2(f_{1,-1}+f_{1,1}) + \Gamma_\parallel^2(h_{1,-1}+h_{1,1})]\right\} = 0, \quad (50)$$

$$2\Omega F_{0,2} = 0. \tag{51}$$

Hence

$$G_{0,1} = \frac{1}{\Omega}\frac{1}{3v^2}\frac{\partial}{\partial v}\left\{\left(\frac{\Gamma_{\perp}^2}{A} + \frac{\Gamma_{\parallel}^2 l}{\omega^2 l^2 + v^2}\right)\mu v^3 \frac{\partial \Phi_{0,0}}{\partial v}\right\}, \tag{52}$$

$$G_{0,2} = \frac{\mu}{8} G_{0,1}, \quad F_{0,1} = F_{0,2} = 0.$$

Analogously, the functions $F_{2,0}$, $G_{2,0}$, etc., are found to a first approximation. These functions are small by comparison with $\Phi_{0,0}$, $G_{0,1}$ and $G_{0,2}$ on the strength of the condition (41). In the case where there is no magnetic field ($\mathbf{H} = 0$) the first term of the transformation (52) of degree $\Gamma_{\parallel}^2$ coincides with the solution achieved in reference 9 (page 355) for a weak electric field.

The first non-vanishing approximation for the functions $F_{0,1}$, $F_{0,2}$, $F_{0,3}$ and $F_{0,4}$, which will be of the order of $(\delta\nu/\Omega)^2\,\Phi_{0,0}$, can be found by substituting in Eqs. (37) - (40) the functions obtained for $\Phi_{0,0}$, $G_{0,1}$ and $G_{0,2}$, and neglecting the remaining $\Phi_{n,k}$'s. Following are the formulas obtained in this manner:

$$F_{0,1} = -\frac{1}{6v^2\Omega}\frac{\partial}{\partial v}\left\{v^3\left(\frac{\Gamma_{\perp}^2}{A} + \frac{\Gamma_{\parallel}^2 l}{\omega^2 l^2 + v^2}\right)\left(1 + \frac{3\mu^2}{8}\right)\frac{\partial G_{0,1}}{\partial v}\right\} - \frac{1}{\Omega}\frac{\delta G_{0,1}}{\delta t}, \tag{53}$$

$$F_{0,2} = -\frac{1}{12v^2\Omega}\frac{\partial}{\partial v}\left\{v^3\left(\frac{\Gamma_{\perp}^2}{A} + \frac{\Gamma_{\parallel}^2 l}{\omega^2 l^2 + v^2}\right)\mu\left(\frac{9}{8} + \frac{\mu^2}{16}\right)\frac{\partial G_{0,1}}{\partial v}\right\} - \frac{\mu}{16\Omega}\frac{\delta G_{0,1}}{\delta t}, \tag{54}$$

$$F_{0,3} = -\frac{1}{18v^2\Omega}\frac{\partial}{\partial v}\left\{v^3\left(\frac{\Gamma_{\perp}^2}{A} + \frac{\Gamma_{\parallel}^2 l}{\omega^2 l^2 + v^2}\right)\frac{3\mu^2}{8}\frac{\partial G_{0,1}}{\partial v}\right\}, \tag{55}$$

$$F_{0,4} = \frac{\mu}{16} F_{0,3}. \tag{56}$$

Here the operator $\delta/\delta t$ is determined by Eq. (5). We shall also write the functions of $G_{0,3}$ and $G_{0,4}$ which, for the first non-vanishing approximation, are of the order of $(\delta\nu/\Omega)^3\,\Phi_{0,0}$

$$G_{0,3} = \frac{1}{3\Omega}\frac{1}{6v^2}\frac{\partial}{\partial v}\left\{v^3\left(\frac{\Gamma_{\perp}^2}{A} + \frac{\Gamma_{\parallel}^2 l}{\omega^2 l^2 + v^2}\right)\left(\frac{\mu^2}{4}\frac{\partial F_{0,1}}{\partial v} + \mu\frac{\partial F_{0,2}}{\partial v} + \left(1 + \frac{\mu^2}{2}\right)\frac{\partial F_{0,3}}{\partial v} + \frac{\mu^2}{4}\frac{\partial F_{0,4}}{\partial v}\right)\right\} + \frac{1}{3\Omega}\frac{\delta F_{0,3}}{\delta t}, \tag{57}$$

$$\frac{\Gamma_{\parallel}^2 l}{\omega^2 l^2 + v^2}\Big)\Big[\left(1 + \frac{\mu^2}{2}\right)\frac{\partial F_{0,4}}{\partial v} + \mu\frac{\partial F_{0,3}}{\partial v} + \Gamma_{\parallel}^2\frac{\mu}{2}\xi_{1,\pm 1}\Big]\Big\} = \frac{\delta\Phi_{2,\pm 2}}{\delta t}. \tag{58}$$

$$+ \frac{\mu^2}{4}\frac{\partial F_{0,2}}{\partial v}\Big]\Big\} + \frac{1}{4\Omega}\frac{\delta F_{0,4}}{\delta t}.$$

The functions $\phi_{n,k}$, $\psi_{n,k}$, $\xi_{n,k}$ can easily be found from Eqs. (38) - (40) by substituting in them the functions obtained for $\Phi_{n,k}$. The functions $F_{0,1}$, $G_{0,1}$, $F_{0,2}$, $G_{0,2}$, etc., are normalized at 0, as seen from the corresponding formulas. Consequently, it is necessary to normalize only the function $\Phi_{0,0}$.

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