TABLE

Substance	Density ρ in gm/cm^3	Frequency ν in 10^6 cps	Absorption Coefficient in cm^{-1}	Ultrasonic Velocity v in m/sec	Measurements of Other Investigations	
					a in cm^{-1}	v in m/sec
Armco Iron	7.85	0.66	0.0024			
		6	0.024			
		10	0.038	5920		
Plexiglass	1.19	0.66	0.11		0.14 [5]	
		1.4	0.21		0.22 [5]	2660 [4]
		6	0.40			2640 [5]
		10	0.55	2680		2662 [6]

absorption has not been tested experimentally. Therefore the investigation of the propagation of ultrasonic waves in different solids over a wide range of frequencies and temperatures presents considerable scientific and practical interest.

In this letter we report the results of measurements by the pulse method of the absorption and propagation velocity of ultrasonic waves over a frequency range from 0.66 to 10 mc in two substances: Armco iron and plexiglass. The experimental arrangements and techniques of measurement will be reported separately.

The results of the measurement of the absorption coefficient of ultrasonic waves a, and the propagation velocity ν in the two materials are listed in the table.

From the values obtained in the frequency range 0.6 - 10 mc, it follows that the ultrasonic absorption coefficient in Armco iron is directly proportional to the ultrasonic frequency. Investigations[1] of ultrasonic absorption in magnesium over a wide frequency range also gave a linear frequency dependence.

The data for plexiglass indicate that the absorption increases proportional to $\sqrt{\nu}$, which is in agreement with the results of other authors[2,3]. The measured velocity of the ultrasonic waves in plexiglass at $\nu = 10$ mc is also in agreement with the values in the literature[4-6]. There were no available data on ultrasonic absorption in Armco iron.

Translated by R. T. Beyer
63

[1] W. Roth, J. Appl. Phys. 19, 901 (1948)

[2] I. G. Mikhailov, *The Propagation of Ultrasound in Liquids*, Moscow, 1949

[3] W. Mason, *Piezoelectric Crystals and Their Applications in Ultrasonics*

[4] N. F. Otpushchennikov, J. Exper. Theoret. Phys. USSR 22, 436 (1952)

[5] D. S. Hughes, W. L. Pandrom and R. L. Mims, Phys. Rev. 73, 1552 (1949)

[6] I. G. Mikhailov, Doklady Akad. Nauk SSSR 59, 1555 (1948)

On the Causal Development of a Coupled System in Relative Time

V. N. TSYTOVICH
Moscow State University

(Submitted to JETP editor August 21, 1954)
J. Exper. Theoret. Phys. USSR 28, 372-374
(March, 1955)

IN the relativistic covariant equation which describes the coupled motion of two interacting particles[1-3], one ascribes to each particle its own time: to the first particle t_1 and to the second t_2. The question in what manner are these times t_1 and t_2 related to each other is of interest. If the external fields are stationary, we can introduce a general time $T = (t_1 + t_2)/2$ and a relative time $t = t_1 - t_2$; for simplicity we assume that the masses of the particles are equal (this corresponds, for example, to the case of positronium). We are interested in how the wave function of different times ($t \neq 0$) can be found by means of the wave function of the same times ($t = 0$), in other words in the development of a coupled system in relative time.

We recall, for example, that the development of the wave function in time of a freely moving particle is described by the operator $e^{-(i/\hbar)Ht}$

$$\psi(t) = e^{-(i/\hbar)Ht}\,\psi(0), \tag{1}$$

where H is the Hamiltonian, independent of time. If we know (for example by means of some measurements performed on the system) the wave function at the initial moment of time, then Eq. (1) determines its value for all future moments of time. It is known that in the theory of a vacuum there occur causal functions of propagation which describe a causal development in time, different from (1). According to Feynman[4]

$$\psi(\mathbf{x}_1, t_1) = \int G(\mathbf{x}_1, t_1, \mathbf{x}_1', 0)\,\psi(\mathbf{x}_1', 0)\,d^3x_1', \qquad (2)$$

where G is the Green's function which satisfies the equation for the operator of the field with δ-function on the right side. If G can be computed by means of the theory of residues, under the assumption that the mass of the particle has an infinitesimally small negative imaginary part, we obtain

$$\psi(t_1) = \Lambda(t_1)\,e^{-(i/\hbar)Ht_1}\,\psi(0), \qquad (3)$$

where $\Lambda(t)$ is a generalization of the projection operators:

$$\Lambda_+ = \frac{|H|+H}{2|H|}, \quad \Lambda_- = \frac{|H|-H}{2|H|}; \qquad (4)$$

$$\Lambda(t_1) = \frac{H+(t_1/|t_1|)|H|}{2|H|} = \begin{cases} \Lambda_+; & t_1 > 0, \\ -\Lambda_-; & t_1 < 0, \end{cases}$$

with $|H| = \sqrt{H^2}$.

The effect of $\Lambda(t_1)$ on the wave function $\psi(0)$ with $t > 0$ is to cut off all negative energies and with $t_1 < 0$, all positive, and also to change the sign of the wave function:

$$\Lambda(t)|_{t>0}\,\psi(0) = \Lambda(t)|_{t>0}\sum_n c_n\psi_n = \sum_{E_n>0} c_n\psi_n;$$

$$\Lambda(t)|_{t<0}\,\psi(0) = \Lambda(t)|_{t<0}\sum_n c_n\psi_n = -\sum_{E_n<0} c_n\psi_n. \qquad (5)$$

Thus the positive frequencies are propagated in the future and the negative ones in the past[5].

In the theory of two bodies the operator of evolution in relative time $t = t_1 - t_2$ can be found in any case if the interaction is instantaneous, for in such a case the moment $t_2 = t_1$; $t = 0$ is singled out by the interaction. In this case an equation of the type used in references 1 - 3 has the form

$$\{F_1F_2 + i\hbar K(\mathbf{x}_1, \mathbf{x}_2)\,\delta(t)\}\,\psi_n(\mathbf{x}_1, \mathbf{x}_2, t, T) = 0, \qquad (6)$$

where $K(\mathbf{x}_1, \mathbf{x}_2)\delta(t)$ is the operator ot the instantaneous interaction *

$$F_1 = \frac{E_n}{2} - \frac{\hbar}{i}\frac{\partial}{\partial t} - H_1; \quad F_2 = \frac{E_n}{2} + \frac{\hbar}{i}\frac{\partial}{\partial t} - H_2. \qquad (7)$$

Insofar as $\delta(t)$ enters the second term of (6) one can write

$$\psi(\mathbf{x}_1, \mathbf{x}_2, t, T) = -\frac{1}{F_1F_2}\,i\hbar K(\mathbf{x}_1, \mathbf{x}_2)\,\delta(t)\,\psi(\mathbf{x}_1, \mathbf{x}_2, 0, T), \qquad (8)$$

or

$$\psi(\mathbf{x}_1, \mathbf{x}_2, t, T) = \int G(\mathbf{x}_1, \mathbf{x}_2, t_1, t_2, \mathbf{x}_1', \mathbf{x}_2', t_1', t_2') \qquad (9)$$

$$\times\; i\hbar\,\delta(t')\,\psi(\mathbf{x}_1', \mathbf{x}_2', 0, T)\,d^4x_1'\,d^4x_2'.$$

Further,

$$G(\mathbf{x}_1, \mathbf{x}_2, t_1, t_2, \mathbf{x}_1', \mathbf{x}_2', t_1', t_2') =$$

$$\frac{1}{\hbar^2}\,G^{(1)}(\mathbf{x}_1, t_1, \mathbf{x}_1', t_1')\,G^{(2)}(\mathbf{x}_2, t_2, \mathbf{x}_2', t_2'),$$

where

$$G^{(1)} = \Lambda^{(1)}(t_1 - t_1')\exp\left\{-\frac{i}{\hbar}H_1(t_1 - t_1')\right\}\delta(\mathbf{x}_1 - \mathbf{x}_1');$$

$$G^{(2)} = \Lambda^{(2)}(t_2 - t_2')\exp\left\{-\frac{i}{\hbar}H_2(t_2 - t_2')\right\}\delta(\mathbf{x}_2 - \mathbf{x}_2').$$

The indices (1) and (2) refer to the first and second particles respectively. Inserting these values into Eq. (9), integrating over t', and using $\delta(t')$, we obtain

$$\psi(\mathbf{x}_1, \mathbf{x}_2, t, T) = \frac{1}{(2\pi)^6\hbar^2}\int \Lambda^{(1)}(t_1 - T')\,\Lambda^{(2)}(t_2 - T')\times$$

$$\times \exp\left\{-\frac{i}{\hbar}H_1(t_1 - T')\right\}\exp\left\{-\frac{i}{\hbar}H_2(t_2 - T')\right\}\times$$

$$\times \exp\{\mathbf{k}_1(\mathbf{x}_1 - \mathbf{x}_1') + i\mathbf{k}_2(\mathbf{x}_2 - \mathbf{x}_2')\} \qquad (10)$$

$$\times\; i\hbar K(\mathbf{x}_1', \mathbf{x}_2')\,\psi(\mathbf{x}_1', \mathbf{x}_2', 0, T').$$

We give the integration of T' for the two cases: $t_1 > t_2$ and $t_1 < t_2$, by dividing the range of integration into the subintervals , for example, for $t_1 > t_2$ $[-\infty, t_2]$, $[t_2, t_1]$ and $[t_1, \infty]$. In this way we take it into account that the insertion of $T = \pm\infty$ in the resulting equation yields zero, due to the presence of the factors $\Lambda^{(1)}(T_1 - T)$ and $\Lambda^{(2)}(t_2 - T)$ and $m = m - i\delta$ for $\delta \to 0$. Then for the staionary state, which corresponds to the total energy E_n, we obtain:

$$\psi_n(\mathbf{x}_1, \mathbf{x}_2, t) = \left\{\Lambda^{(1)}(t) \exp\left[-\frac{i}{\hbar}\left(H_1 - \frac{E_n}{2}\right)t\right] + \right.$$
$$\left. + \Lambda^{(2)}(-t) \exp\left[\frac{i}{\hbar}\left(H_2 - \frac{E_n}{2}\right)t\right]\right\} \chi_n(\mathbf{x}_1, \mathbf{x}_2), \quad (11)$$

where

$$\chi_n(\mathbf{x}_1, \mathbf{x}_2) = -\frac{1}{(2\pi)^6} \quad (12)$$
$$\times \int \frac{\exp\{i\mathbf{k}_1(\mathbf{x}_1 - \mathbf{x}_1') + i\mathbf{k}_2(\mathbf{x}_2 - \mathbf{x}_2')\}\, d^3k_1\, d^3k_2}{E_n - H_1 - H_2}$$
$$\times \varphi_n(\mathbf{x}_1', \mathbf{x}_2') \times K(\mathbf{x}_1', \mathbf{x}_2')\, d^3x_1'\, d^3x_2',$$

$$\varphi_n(\mathbf{x}_1, \mathbf{x}_2) = \psi_n(\mathbf{x}_1, \mathbf{x}_2, t)\,|_{t=0}.$$

Equations (11) and (12) establish the connection between $\psi_n(\mathbf{x}_1, \mathbf{x}_2, t)$ and $\phi_n(\mathbf{x}_1, \mathbf{x}_2)$, and they indicate the form of the unknown operator $\Theta_n(t)$, $\psi_n(\mathbf{x}_1, \mathbf{x}_2, t) = \Theta_n(t)\, \phi_n(\mathbf{x}_1, \mathbf{x}_2)$, which describes the causal development of the coupled system in relative time $t = t_1 - t_2$. If we multiply (12) by $\exp\{-\frac{i}{\hbar} E_n t\}$, then the general wave function can be written in the form:

$$\psi_n(t, T) = \exp\left\{-\frac{i}{\hbar} E_n T\right\} \psi_n(t)$$

$$= \begin{matrix} \Lambda_+^{(1)} \exp\left\{-\frac{i}{\hbar} H_1 t_1\right\} \exp\left\{-\frac{i}{\hbar}(E_n - H_1)\, t_2\right\} \\ -\Lambda_-^{(2)} \exp\left\{-\frac{i}{\hbar} H_2 t_2\right\} \exp\left\{-\frac{i}{\hbar}(E_n - H_2)\, t_1\right\} \chi_n, \quad t > 0; \end{matrix}$$

$$= \begin{matrix} \Lambda_+^{(2)} \exp\left\{-\frac{i}{\hbar} H_2 t_2\right\} \exp\left\{-\frac{i}{\hbar}(E_n - H_2)\, t_1\right\} \\ -\Lambda_-^{(1)} \exp\left\{-\frac{i}{\hbar} H_1 t_1\right\} \exp\left\{-\frac{i}{\hbar}(E_n - H_1)\, t_2\right\} \chi_n, \quad t < 0. \end{matrix}$$

We can say that the wave function corresponds either to the propagation of the first particle into the future in the form of a free wave with positive frequency $\Lambda_+^{(1)}$ (the frequency $H_1 = |H_1|$) and of the second particle into the past with a much more complicated sort of "coupling" (the frequency is $E_n - H_1$), or to the propagation of the second particle into the past in the form of a free wave with a negative frequency (the frequency : $-H_2 = |H_2|$) and of the first one into the future with a much more complicated sort of "coupling" (the frequency: $E_n - H_2$)**.

This result is a generalization of the result which was obtained by Salpeter and Bethe for the nonrelativistic case, and it takes into account a new possibility which is connected with the propagation of particles with negative frequencies.

The operator of the causal development in time $\Theta(t)$ may be successfully applied to the integration over relative time of the matrix elements which occur in the theory of excitation and, in particular, for finding the effective excitation energy in the theory of two bodies (see reference 6).

* Instead of $K(\mathbf{x}_1, \mathbf{x}_2)$ one can also take the phenomenological potential.

**The future and the past of each particle is counted from the moment of interaction.

[1] E. E. Salpeter and H. A. Bethe, Phys. Rev. **84**, 1232 (1951)

[2] J. Schwinger, Proc. Natl. Acad. **7**, 432, 445 (1951)

[3] A. D. Galanin, J. Exper. Theoret. Phys. USSR **23**,488 (1952)

[4] R. P. Feynman, Phys. Rev. **76**, 749 (1949)

[5] E. Stuckelberg, Helv. Phys. Acta. **15**, 23 (1942)

[6] V. N. Tsytovich, J. Exper. Theoret. Phys. USSR **28**, 113 (1955)

Translated by S. I. Gaposchkin
64

The "Equilibrium" Energy Spectrum of Cascades of Photons

P. S. Isaev
P. N. Lebedev Institute of Physics, Academy of Sciences, USSR
(Submitted to JETP editor June 24, 1955)
J. Exper. Theoret. Phys. USSR **28**, 374-376 (March, 1955)

In the present work the "equilibrium" spectrum of photons generated in cascading electromagnetic processes is calculated, taking into account not only radiation damping and the creation of pairs, but also ionization losses and the Compton-effect.

The "equilibrium" spectrum of photons $\Theta(E)$ is determined by the following method:

$$\Theta(E) = \int_0^\infty \Theta(E, t)\, dt,$$

where $\Theta(E, t)\,dE$ is the average number of photons in the energy interval $(E, E + dE)$ at a depth t.

The approximate expression for $\Theta(E)$ occurs in Belenko's book[1] [Sec. 17, Eq. (17.8)]. For the calculation of this magnitude for the probability of the Compton-effect W_k, [reference 1, Eq. (2.20)]