which corresponds to the total energy E_n, we obtain:

$$\psi_n(\mathbf{x}_1, \mathbf{x}_2, t) = \left\{\Lambda^{(1)}(t) \exp\left[-\frac{i}{\hbar}\left(H_1 - \frac{E_n}{2}\right)t\right] + \right. \\ \left. + \Lambda^{(2)}(-t) \exp\left[\frac{i}{\hbar}\left(H_2 - \frac{E_n}{2}\right)t\right]\right\} \chi_n(\mathbf{x}_1, \mathbf{x}_2), \quad (11)$$

where

$$\chi_n(\mathbf{x}_1, \mathbf{x}_2) = -\frac{1}{(2\pi)^6} \quad (12)$$

$$\times \int \frac{\exp\{i\mathbf{k}_1(\mathbf{x}_1 - \mathbf{x}_1') + i\mathbf{k}_2(\mathbf{x}_2 - \mathbf{x}_2')\}\, d^3k_1\, d^3k_2}{E_n - H_1 - H_2}$$

$$\times \varphi_n(\mathbf{x}_1', \mathbf{x}_2') \times K(\mathbf{x}_1', \mathbf{x}_2')\, d^3x_1'\, d^3x_2',$$

$$\varphi_n(\mathbf{x}_1, \mathbf{x}_2) = \psi_n(\mathbf{x}_1, \mathbf{x}_2, t)\,|_{t=0}.$$

Equations (11) and (12) establish the connection between $\psi_n(\mathbf{x}_1, \mathbf{x}_2, t)$ and $\phi_n(\mathbf{x}_1, \mathbf{x}_2)$, and they indicate the form of the unknown operator $\Theta_n(t)$, $\psi_n(\mathbf{x}_1, \mathbf{x}_2, t) = \Theta_n(t)\,\phi_n(\mathbf{x}_1, \mathbf{x}_2)$, which describes the causal development of the coupled system in relative time $t = t_1 - t_2$. If we multiply (12) by $\exp\{-\frac{i}{\hbar} E_n t\}$, then the general wave function can be written in the form:

$$\psi_n(t, T) = \exp\left\{-\frac{i}{\hbar} E_n T\right\} \psi_n(t)$$

$$= \begin{cases} \Lambda_+^{(1)} \exp\left\{-\frac{i}{\hbar} H_1 t_1\right\} \exp\left\{-\frac{i}{\hbar}(E_n - H_1)\, t_2\right\} \\ -\Lambda_-^{(2)} \exp\left\{-\frac{i}{\hbar} H_2 t_2\right\} \exp\left\{-\frac{i}{\hbar}(E_n - H_2)\, t_1\right\} \chi_n, \end{cases} \quad t > 0;$$

$$= \begin{cases} \Lambda_+^{(2)} \exp\left\{-\frac{i}{\hbar} H_2 t_2\right\} \exp\left\{-\frac{i}{\hbar}(E_n - H_2)\, t_1\right\} \\ -\Lambda_-^{(1)} \exp\left\{-\frac{i}{\hbar} H_1 t_1\right\} \exp\left\{-\frac{i}{\hbar}(E_n - H_1)\, t_2\right\} \chi_n, \end{cases} \quad t < 0.$$

We can say that the wave function corresponds either to the propagation of the first particle into the future in the form of a free wave with positive frequency $\Lambda_+^{(1)}$ (the frequency $H_1 = |H_1|$) and of the second particle into the past with a much more complicated sort of "coupling" (the frequency is $E_n - H_1$), or to the propagation of the second particle into the past in the form of a free wave with a negative frequency (the frequency : $-H_2 = |H_2|$) and of the first one into the future with a much more complicated sort of "coupling" (the frequency: $E_n - H_2$)**.

This result is a generalization of the result which was obtained by Salpeter and Bethe for the nonrelativistic case, and it takes into account a new possibility which is connected with the propagation of particles with negative frequencies.

The operator of the causal development in time $\Theta(t)$ may be successfully applied to the integration over relative time of the matrix elements which occur in the theory of excitation and, in particular, for finding the effective excitation energy in the theory of two bodies (see reference 6).

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* Instead of $K(\mathbf{x}_1, \mathbf{x}_2)$ one can also take the phenomenological potential.

**The future and the past of each particle is counted from the moment of interaction.

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The "Equilibrium" Energy Spectrum of Cascades of Photons

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In the present work the "equilibrium" spectrum of photons generated in cascading electromagnetic processes is calculated, taking into account not only radiation damping and the creation of pairs, but also ionization losses and the Compton-effect.

The "equilibrium" spectrum of photons $\Theta(E)$ is determined by the following method:

$$\Theta(E) = \int_0^\infty \Theta(E, t)\, dt,$$

where $\Theta(E, t)dE$ is the average number of photons in the energy interval $(E, E + dE)$ at a depth t.

The approximate expression for $\Theta(E)$ occurs in Belenko's book[1] [Sec. 17, Eq. (17.8)]. For the calculation of this magnitude for the probability of the Compton-effect W_k, [reference 1, Eq. (2.20)] the following approximation was used:

$$W_k(E', E)\,dE = g\,dE/EE'.$$

Here $W_k(E', E)$ is the probability that a photon of energy E', in its passage through a layer of unit thickness in the cascade, experiences Compton scattering by the electron, after which it will have the energy E; g is a certain constant, equal, for air, to 1.32 MeV; for carbon, 1.53 MeV; for aluminum, 0.839 MeV; for iron, 18.4 MeV; for copper, 0.404 MeV; and for lead 0.175 MeV.

In the work reported in reference 2, a more exact expression was obtained for the "equilibrium" spectrum of photons $\Theta(E)$. Precision in this case is connected with the fact, that for the probability of the Compton-effect the following more exact expression was taken:

$$W_k(E', E)\,dE = \frac{g\,dE}{EE'}\left[1 + \left(\frac{E}{E'}\right)^2\right] = \frac{g\,dE}{E'E}\,[1 + \delta(E'E)],$$

in which it was assumed that $\delta(E'E)$ is small compared to unity. The correction $\kappa(E)$ to the approximate "equilibrium" spectrum of photons $\Gamma(E)$, given in reference 1 [Sec. 17, Eq. (17.8)], was calculated only for air in reference 2.

In the Table there are presented the results of the calculation of the equilibrium photon spectrum $\Theta(E) = \Gamma(E) + \kappa(E)$ for hydrogen, aluminum, iron, copper and lead. The calculation was carried out according to formulas obtained in reference 2. From these data it is evident that, as was to be expected, the corrections are large for heavy elements in the energy region of the order of the critical energy or lower.

The following values of critical energy were used in the present work: for hydrogen 120, for aluminum 37.2, for iron 18.4, for copper 22.4, for lead 6.4 mev. The results of the calculations are given in the Table.

From a comparison of the spectra for a given energy for hydrogen, aluminum, iron, copper and lead, it is easy to see that $\Theta(E)$ increases with increasing Z. Copper is an "exception" (compare with Fe). However, this result is connected with the choice of an inaccurate value of the critical energy given in the review of Rossi and Greisen[3], which was made on the basis of calculations for copper.

It should be noted that the correction can be regarded as small up to energies $\sim 0.1\beta$ or up to $Z = 30$, where the corrections for $\Gamma(E)$ amount to less than 10%. However, for Pb, the correction at 0.5 β is already 10.7 / 14.4 or 74.5%, and it is impossible to speak of the complete applicability of the method used in reference 2.

Comparison of the spectrum obtained in the present work for Pb with the spectrum obtained by Richards and Nordheim[4] indicates that, in contrast to the spectrum for air (for the case $E_m = \infty$, see reference 2), where the discrepancy of the spectra has been observed only in the region $E = 1$ because of the doubtful approximation carried out in reference 4 at this point, for lead the divergence is obtained for all energies $E < 2$. This comparison shows that the method set forth by Rossi and Greisen cannot give the correct result in the calculation of the photon spectrum of heavy elements at energies $E \sim 1.5\beta$ and lower.

In conclusion I express my gratitude to S. Z. Belen'kii for supervision of this research, and also to L. Ia. Zhil'tsov who carried out the principal calculations.

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The Gravitational Self Energy of Particles in the Classical Field Theories

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In several classical field theories[1,2] the electron is assigned an exclusively field self-energy. This energy can be calculated by a common formula[3], which allows the electron's own magnetism to be taken into account:

$$\mathcal{E}_0 = \int \varepsilon_0(\mathbf{E}, \mathbf{H}) \sqrt{{}^1/_4(\mathbf{H}^2 - \mathbf{E}^2)^2 + (\mathbf{E}, \mathbf{H})^2}\,(d\mathbf{x}), \qquad (1)$$

where **E**, **H** are the vector field intensities, ϵ_0 is the dielectric permeability of the vacuum. The result (without considering the magnetic moment) turns out to be $\mathcal{E}_0 = (1.2361\ \ldots)(e/x_0)$ in the Born-Infeld theory[1] and $\mathcal{E}_0 = e/2x_0$ in the theory of Bopp-Podolsky[2] where x_0 is the classical radius of the electron.

The question of the form of the linear gravitational field of the electron with a field mass is usually not considered, although it is a matter of