values of α_K for the dipole magnetic and quadrupole electric transitions, which have been found with the use of extrapolation in reference 6 for $Z = 23$.

The comparison of the experimental and theoretical values of α_K leads to the conclusion that electric quadrupole character of transition should be ascribed to $V^{51*} \to V^{51}$. It is possible that magnetic dipole emission is available in a very small quantity. In accordance with the theory of nuclear shells with a strong spin-orbital bond, the state $f_{7/2}$ corresponds to the principal state of V^{51}. In the case when the nuclear shells are successively filled up, the first excited level will have state $f_{5/2}$, hole-level will have the state $S_{1/2}$ or $d_{3/2}$.

The conclusion about the quadrupole character of an emission may find its confirmation in the fact that the nucleus of V^{51}, after getting into an excited state, fills up its nuclear levels successively, which is in agreement with an inference of reference 11. The transition $V^{51*} \to V^{51}$ belongs to the type $f_{5/2} \to f_{7/2}$.

The correlation of the decay probabilities of Cr^{51} into ground or excited state of V^{51} [8,9] is in agreement with the state $f_{7/2}$ of a nucleus of Cr^{51}, which was predicted by the theory of nuclear shells.

Translated by M. Hadsinskyj
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[11] L. K. Peker, L. A. Sliv and L. V. Zolotavin, Doklady Akad. Nauk SSSR **88**, 781 (1953); L. K. Peker and L. A. Sliv, Izv. Akad. Nauk SSSR, Ser. Fiz. **17**, 411 (1953)

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Some Observations on Possible Formulations of the Theory of Extended Particles

V. S. BARASHENKOV

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Some of the features of possible formulations of a theory of non-localized fields are considered. In particular, it is shown that if the operators of the non-localized fields are considered to be non-diagonal matrices in coordinate space, then non-interacting non-localized fields cannot be equivalent to an aggregate of fields of local type. Finally some considerations are presented concerning the comparison of results of a theory of extended particles with experiment.

1. The search for a possibility of eliminating the fundamental difficulties of field theory, associated with the presence of divergent expressions in the apparatus of the present theory, constitutes one of the central parts of contemporary physical literature, devoted to the study of the properties and interaction of elementary particles. The importance of the problem is due to the close connection of these difficulties with the most profound problems of the structure of matter: the mass and structure of elementary particles, the coupling of these particles, nuclear forces, etc.

One may feel[1] that the elimination of many of the difficulties of present physical theory, among which are the difficulties with divergences connected with the incorrect application, for the description of phenomena taking place in small space-time regions, of concepts and principles which are in accord with experiment only over large regions of space-time*. From this point of view, some ideas which have been recently thoroughly discussed are of great interest; these are the hypothesis of non-local fields[2-4], in which the errors in determination of field, ΔA, and coordinate, Δx, are connected by the relation $\Delta A\ \Delta x \sim \lambda_0 A$, and the closely connected hypothesis of non-local interaction[5], i.e., the hypothesis that the interaction is "smeared" over a small space-time domain. Mathematically these ideas are formulated by the introduction into

[1] D. I. Blokhintsev, Uch. Zap., Moscow State University, Phys. **3**, 77, 101 (1945); M. A. Markov, J. Exper. Theoret. Phys. USSR **8**, 124 (1938)

* On the other hand, it is hard to deny that a definite part of our difficulties is due to incorrect application in various cases of one or another mathematical method. In particular, it may be that taking account of higher approximations of perturbation theory will bring the essential corrective measure.

[2] M. A. Markov, J. Exper. Theoret. Phys. USSR **10**, 1311 (1940); **21**, 11 (1951)

[3] H. Yukawa, Phys. Rev. **77**, 219 (1950); **80**, 1047 (1950)

[4] H. Yukawa, Phys. Rev. **91**, 415, 416 (1953)

[5] C. Bloch, Kgl. Danske Videnskab. Selskab., Mat.-Fys. Medd. **27**, No. 8 (1952)

the theory of cut-off form factors, which can be interpreted as an attempt to introduce into the theory a space-time extension of the particles.

The hypotheses of non-local field and non-local interaction have no specific quantum character, and are applicable to the classical theory of fields[6].

It is essential to point out that all schemes of this sort so far proposed lead to serious difficulties connected with the breakdown of relativistic invariance[7,8]. One can try[8] to eliminate these difficulties by introducing into the theory "dynamically deformable" form factors, which correspond to the consideration of "soft" particles in the sense that the velocity of propagation of a signal for such a particle does not exceed the velocity of light. However, in our opinion, it appears extremely reasonable to investigate, within the realm of a theory with "rigid" particles, the possibility of constructing a theory in which the deviations from relativistic invariance are localized within small space-time domains. It was shown earlier[9] that such a "small space-time region" ($\lesssim \lambda_0$), in which the signal velocity can exceed that of light, remains small ($\lesssim \lambda_0$) in any coordinate system. In this sense we may say that the theory is relativistically invariant, and consider it to be the limiting case of a theory with dynamically deformable form factor.

It is also essential that the apparatus of the theory automatically guarantee the limitation of measurement in small regions ($\sim \lambda_0$); the physically observable quantities in such a theory must be quantities corresponding to a large ($> \lambda_0$) space-time interval. In this respect, the theory of non-localized fields, in which field and coordinate satisfy the relation $\Delta A \, \Delta x \sim \lambda_0 A$, has an advantage over the theory of non-local interaction.

2. An interesting variant of the theory of non-localized fields was presented and discussed in detail by Yukawa[3]. In contrast to the theory of localized fields, in which the field is regarded as a local point function $A = A(x)$, the field in Yukawa's non-localized field theory is considered to be a non-diagonal matrix in coordinate space:

$$A = (x'|A|x'') \qquad \text{with} \qquad [A, x] \neq 0.$$

The complete system of equations describing the non-interacting, non-localized field $(x'|A|x'')$, in the form in which it was given in reference 3 is:

$$\begin{aligned} &[p[p, A]] + m_0^2 A = 0, \\ &[x[x, A]] - \lambda_0^2 A = 0, \\ &[p[x, A]] = 0. \end{aligned} \tag{1}$$

Here p is the momentum operator: $(x'|p|x'') = -i \frac{\partial}{\partial x'} \partial(x' - x'')$; x is the coordinate operator: $(x'|x|x'') = x'\delta(x' - x'')$.

Thus this variant of the theory differs from the theory of localized fields because of the change in the laws of motion for the free fields; the second quantization is carried out in accordance with the altered equations of motion[3].

Subsequently, several authors[10,11] have come to the conclusion that the theory of a free non-localized field is merely one of the possible ways of describing an aggregate of localized fields each describing particles with mass m_0 and different spins. In the light of the original idea concerning the essentially non-diagonal character of the field operator $(x'|A|x'')$, such a conclusion seems very surprising and merits more careful scrutiny.

It is easy to see that the matrix elements of the commutator of the coordinate operator and an arbitrary matrix quantity $(x'|T|x'')$ can, in accordance with the rules for matrix multiplication, be written in the form:

$$\begin{aligned} (x'|[x, T]|x'') &= \int x'\delta(x' - t)(t|T|x'')\,dt \\ &- \int (x'|T|t)\, t\,\delta(t - x'')\,dt = (x' - x'')(x'|T|x''). \end{aligned} \tag{2}$$

Similarly, we obtain for the matrix elements of the commutator of the momentum operator and a matrix T the expression:

$$\begin{aligned} &(x'|[p, T]|x'') \\ &\quad = i \int (x'|T|t) \frac{\partial}{\partial t} \delta(t - x'')\,dt \\ &\quad - i \int \frac{\partial}{\partial x'} \delta(x' - t)(t|T|x'')\,dt \\ &\quad = -i \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''}\right)(x'|T|x''). \end{aligned} \tag{3}$$

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[6] D. I. Blokhintsev, J. Exper. Theoret. Phys. USSR **16**, 480 (1946); **18**, 566 (1948); H. Yukawa, Progr. Theor. Phys. 2, 209 (1947)

[7] M. A. Markov, J. Exper. Theoret. Phys. USSR **16**, 790 (1946)

[8] M. A. Markov, J. Exper. Theoret. Phys. USSR **25**, 527 (1953)

[9] D. I. Blokhintsev, J. Phys. USSR **10**, 167 (1946); J. Exper. Theoret. Phys. USSR **22**, 254 (1952)

[10] O. Hara and H. Shimazu, Progr. Theor. Phys. **7**, 255 (1952)

[11] M. Fierz, Phys. Rev. **78**, 184 (1950); Helv. Phys. Acta **23**, 412 (1950)

If we choose the matrix T in the form $(x'|T|x'') = (x'|[x, A]|x'')$ or $(x'|T|x'') = (x'|[p, A]|x'')$, then the system of equations (1) can be rewritten in the form:

$$(x'|F_1|x'') \equiv \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''}\right)_\mu \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''}\right)^\mu (x'|A|x'') - m_0^2 (x'|A|x'') = 0, \quad (4)$$

$$(x'|F_2|x'') \equiv (x'-x'')_\mu (x'-x'')^\mu (x'|A|x'') - \lambda_0^2 (x'|A|x'') = 0,$$

$$(x'|F_3|x'') \equiv (x'-x'')_\mu \left(\frac{\partial}{\partial x'} + \frac{\partial}{\partial x''}\right)^\mu (x'|A|x'') = 0.$$

In addition we introduce, in place of the matrix operators F_1, F_2, F_3, new operators E_1, E_2, E_3, whose matrix elements we define as follows:

$$(X'|E_i|X'') = (X'+X''|F|X'-X'') = (x'|F_i|x''). \quad (5)$$

If the matrix elements of the operators F_i and E_i are regarded as functions of the variables x', x'' and X', X'', then Eq. (5) corresponds to a change from the variables x', x'' to new variables $X' = \frac{1}{2}(x'+x'')$ and $X'' = \frac{1}{2}(x'-x'')$. Each matrix element of E_i coincides with one of the matrix elements of F_i; in other words, the matrix E_i is obtained from the matrix F_i by rearranging elements in accordance with the rule (5).

Thus,

$$(X'|E_1|X'') \equiv \left(\frac{\partial}{\partial X'_\mu}\frac{\partial}{\partial X'^\mu} - m_0^2\right)(X'|\bar{A}|X'') = 0, \quad (6)$$

$$(X'|E_2|X'') \equiv (X''_\mu X''^\mu - \lambda_0^2)(X'|\bar{A}|X'') = 0,$$

$$(X'|E_3|X'') \equiv X''_\mu \frac{\partial}{\partial X'_\mu}(X'|\bar{A}|X'') = 0,$$

where the operator $(X'|\bar{A}|X'')$ is defined in accordance with Eq. (5).

It is essential to remark that X', X'', just as x', x'', are completely equivalent eigenvalues of the coordinate operator.

The matrix operators E_i can be represented in the form:

$$E_1 \equiv (K - m_0^2)\bar{A} = 0, \quad (7)$$
$$E_2 \equiv \bar{A}(L - \lambda_0^2) = 0,$$
$$E_3 \equiv M\bar{A}N = 0,$$

where

$$(X'|K|X'') = \frac{\partial}{\partial X'_\mu}\frac{\partial}{\partial X'^\mu}\delta(X'-X''), \quad (8)$$
$$(X'|L|X'') = X'_\mu X'^\mu \delta(X'-X''),$$
$$(X'|M|X'') = \frac{\partial}{\partial X'}\delta(X'-X''),$$
$$(X'|N|X'') = X'\delta(X'-X'').$$

The construction, by means of a canonical transformation, of relations describing a non-localized, non-interacting field from the corresponding relations describing a local field ($\lambda_0 = 0$), is based on the possibility of presenting the system of equations (6) in the form:

$$(K - m_0^2)\bar{A} = (L - \lambda_0^2)\bar{A} = MN\bar{A} = 0,$$

where the operators K, L, M, N commute with one another[10]. However, such a form for the equation system (6) is possible only if X' and X'' are considered to be independent variables and, in accord with this, the field operator is taken as a function $\bar{A} = \bar{A}(X'; X'')$ in the space of X' and X''.

The analysis of Yukawa's theory presented in reference 11 is also based, to a significant extent, on a consideration of the field operator $\bar{A}$ as a function $A(X', X'')$ in the space of X' and X''. But such a formulation of the problem differs essentially from the initially proposed idea of regarding the field A as a non-diagonal operator in the space of the coordinate x.

If we regard the operators of the non-localized field as non-diagonal matrices in the coordinate space, $A = (x'|A|x'')$, a different physical interpretation of the quantities X' and X'', which are completely equivalent in the matrix sense, is in our opinion incorrect. This last remark applies in particular to a new, recently proposed[4] variant of the theory, where the operators of the non-localized field in the equations of motion are regarded from two essentially different points of view: first, as matrices in coordinate space, $A = (x'|A|x'')$, where x' and x'' are equivalent sets of eigenvalues of the coordinate operator x; second, as functions $A = A(X', X'')$, where the independent variables X' and X'' have different physical meanings ascribed to them.

It is also very important to remark that in Yukawa's theory[3] the deviations from relativistic invariance are not localized within small space-time regions[12].

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[12] Y. Ono and M. Sugawara, Progr. Theor. Phys. **6**, 182 (1951)

3. Recently completed experiments on the scattering of π-mesons by protons[13] show that the total cross sections in the energy range studied ($\leq$ 1.45 bev) have two well-defined maxima. One might attempt to explain such an energy dependence of the cross section in terms of the presence of isobaric states (in analogy with the classical Franck-Hertz experiments on scattering of electrons by atoms), or in terms of phenomena analogous to the Ramsauer effect[14].

Both cases require us to take account of the structure of the particles taking part in the reaction. The presence of cut-off form factors in the expressions describing the reaction can lead[8] to a rapid decrease in the magnitude of the cross section with increasing energy of the particles taking part in the reaction. But this is the case only for form factors which decrease rapidly with increasing energy, e.g., for form factors of the type $e^{-\lambda k_0 p}$ [*]. Apparently, to describe the structure of the particles it is necessary to introduce form factors which oscillate for energies $E < E^* \times (\lambda_0)$ and drop rapidly for $E > E^*(\lambda_0)$ [**].

To explain the difference in energy dependence of the cross section for scattering of π^+- and π^--mesons, it may turn out to be necessary to make such additional assumptions.

[*] It is interesting to observe that the rapid decrease in cross section with increasing energy does not occur in a theory with non-local interaction, described by a form function $F(x', x'')$; but in such a theory the divergence difficulty is still not eliminated.

[**] Other interesting possibilities in this respect are the "sign-variable form-factors", i.e., form factors like the cosine factor considered earlier (or sin kx/kx, etc.). However, in this case the expressions for the interaction energy become non-Hermitian.

[13] R. L. Cool, L. Madansky and O. Piccioni, Bull. Am. Phys. Soc. **28**, No. 6, 14 (1953)

[14] N. Mott and H. Massey, *Theory of Atomic Collisions*, 2'nd Ed., Oxford, 1949, chapt. X

All of these questions require further study. We should remark that at present the fundamental obstacle to the study of the effect of possible extension of particles is not so much the insufficient amount of experimental data as our inexact knowledge of the limits of applicability of the various methods used in the present theory of fields. In particular, it remains unclarified how the value of a total cross section, in a theory with form-factor, is affected by the inclusion of higher approximations of the perturbation theory.

In addition, so long as the magnitude of the constant λ_0 in such a theory is not understood, one will always be able to choose it to be so small that, within the energy range reached in experiments, the dimenisons of the particle either do not as yet show themselves, or have an insignificant effect***.

In conclusion, it is a pleasant duty to express my thanks to Prof. D. I. Blokhintsev for guidance, valuable advice and interesting discussions, and also to Prof. M. A. Markov for interesting discussions concerning general questions of form-factor theories.

Note added in proof: In a recently published paper [Progr. Theor. Phys. **10**, 533 (1953)], Hayashi calls attention to the fact that in a theory with form-function, combinations of the familiar $\theta^{\pm}(x_i - x_j)$ functions which link the vertices in closed parts of the Feynman diagram have no well-defined value [e.g., Eqs. (42) and (47) of Hayashi's paper]. Consequently, the singular functions Δ, $\Delta_{\pm}$ etc., in the expression for the S-matrix cannot be combined into a product of causal Δ^c-functions, which leads to a breakdown of causality in macroscopic ($\gg \lambda$) domains. The relative contribution of causal effects is proportional to a power of λ and goes to zero for $\lambda \to 0$.

Translated by M. Hamermesh
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*** The arbitrariness which exists at present in the choice of type of form factor and in the magnitude of the constant λ_0 is one of the basic defects of the theory of non-localized fields and non-localized interaction.