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Determination of the Absolute Concentrations of Atoms in a Multi-Component Arc-Vapor

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THE Prilezhaeva method[1] makes it possible to determine the absolute concentrations of atoms in the positive column of a direct-current arc[2,3]. It is generally believed that absolute concentrations can be measured by this method only in the case where the arc-vapor contains two components (a compound which is difficult to ionize --- air and carbon --- and an easily ionized one --- the vapor of the element being investigated). However, we have also applied the Prilezhaeva method in the case of a more complex arc-vapor.

Let an arc-vapor contain k elements which are to be analyzed and whose concentrations are N_1, N_2, . . . N_k respectively. The general idea of the concentration of atoms is based on the fact that the degree of ionization of a mixture of different gases at a given temperature depends on the composition of the gas. By determining the fractional ionization of the gas experimentally, it is possible to determine the concentrations of the base components of the mixture.

The resultant (average) fractional ionization of the gas as a whole (i.e., the mixture), $\overline{x}$, is related to the concentrations of the components in the following way:

$$\overline{x} = \sum_{i=0}^{k} x_i N_i \Big/ \sum_{i=0}^{k} N_i, \tag{1}$$

where x_i is the fractional ionization of the ith component. Consequently, in order to determine the N_i, it is necessary to know the average fractional ionization of the gas and also the fractional ionizations of all the components. To determine these quantities, it suffices to measure experimentally the temperature of the gas and the fractional ionization of only one of the components. Then the Saha equation can be written for each component:

$$\lg \frac{x_i}{1-x_i} \frac{\overline{x}}{1+\overline{x}} = 2.5 \lg T - \frac{5040}{T} V_i - 6.5, \tag{2}$$

where T is the temperature and V_i is the ionization potential of each component in electron volts. From Eq. (2) as written for that component whose fractional ionization is measured, it is possible to determine $\overline{x}$, and then from this same equation as written for all the remaining components, it is possible to determine all the x_i. However, knowing $\overline{x}$ and the x_i is still not sufficient for finding the N_i from Eq. (1). It is necessary to have additional relations between the concentrations, N_i. Such relations can be obtained if we measure $k-1$ ratios of the intensities of pairs of spectral lines (regardless of whether they are atomic or ionic) of the elements being investigated. For example, we take the ratio of two atomic lines of the ith and nth components to be:

$$\frac{I_i}{I_n} = \frac{N_i}{N_n} \frac{A_i g_i g_{n0}}{A_n g_{i0} g_n} e^{-(E_i - E_n)/kT} \frac{\nu_i}{\nu_n}, \tag{3}$$

where A is the transition probability, g is the statistical weight and E is the excitation potential of the line.

The ratio N_i/N_n can be determined from Eq. (3). Here, however, a complication arises from the fact that it is necessary to know the transition probabilities of the lines which are being used. In order to determine k quantities, the N_i, from $k-1$ relations in the form of Eq. (3) and Eq. (1), it is still necessary that N_0, the last unknown quantity in Eq. (1), be eliminated. In the denominator, N_0, enters together with the N_i in the form of a sum. Since the pressure p (atmospheric) and the temperature T are known, this sum can be determined very simply from a consideration of gas kinetics by the formula:

$$N = \sum_{i=0}^{k} N_i = \frac{9.7 \cdot 10^{18}}{T} p, \tag{4}$$

where p is the pressure in mm Hg.

The numerator term containing N_0 can usually be neglected, i.e., in the majority of cases:

$$\sum_{i=1}^{k} x_i N_i \gg x_0 N_0. \tag{5}$$

This is always justified when the ionization potential, V_i, of even one of the components, which is present in not too small a concentration, is somewhat less (1-3 ev is sufficient) than the ionization potential V_0. In such a case, because of the exponential dependence of the fractional ionization on the ionization potential, x_0 is so small that the validity of the inequality in Eq. (5) is guaranteed.

As a result, Eq. (1) can now be rewritten in the form :

$$\bar{x} = \sum_{i=1}^{k} x_i N_i / N. \tag{6}$$

Using Eqs. (3) and (6), it is possible to find all the N_i, the concentrations which were to be determined. The temperature, T, and the fractional ionization of one of the components can be found in the usual manner[2].

An experiment has been carried out for the simplest case, $i = 2$, i.e., a three-component arc-vapor*. Magnesium and zinc were used as the test elements. The temperature was determined from the ratio of the intensities of the zinc atomic lines, $\lambda = 3072$ Å and $\lambda = 3076$ Å, the relative (transition) probabilities of which are known[4]. The fractional ionization of the magnesium was found experimentally from the lines Mg I, $\lambda = 2779$ Å and Mg II, $\lambda = 2795$ Å (as in reference 3). Among the Zn and Mg lines, it was possible to find lines with known transition probabilities[4,5] by which the ratio of the concentrations of Zn and Mg atoms could be determined from Eq. (3). We selected the lines Mg I, $\lambda = 2779$ Å and Zn I, $\lambda = 2741$ Å.

All measurements were made by the photographic-photometric method. Zinc was introduced in the test sample in the form of a solution of $ZnSO_4$, magnesium in the form of a solution of $MgCO_3$ and the base element of the test sample was carbon powder. The percentages by weight of the materials in the test sample were: $ZnSO_4$ – 10%; $MgCO_3$ – 3%; C – 87%.

The temperature of the arc-vapor was found to be 6300° K. The measurements of the fractional ionization of the magnesium atoms showed that $x_{Mg} = 0.43$. A calculation using these values of T and x_{Mg} in the Saha formula gives $x_{Zn} = 3.6 \times 10^{-2}$ and $\bar{x} = 4 \times 10^{-3}$. Correspondingly, the computed concentrations of magnesium and zinc were found to be $N_{Mg} = 7.2 \times 10^{15}$ cm^{-3}; $N_{Zn} = 4.3 \times 10^{16}$ cm^{-3}.

Thus, in the test sample, the number of zinc atoms is greater than the number of magnesium atoms by a factor of 1.5; in the discharge-gap, however, it is 6 times greater. It follows that the rate at which zinc atoms from the test sample enter the discharge is 4 times greater than the rate at which magnesium atoms enter. This is in agreement with Rusanov's data on volatile elements[6].

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* The experiment was carried out by the student A. E. Kontorovich.

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The Relativistic Mechanics of a Material Point of Variable Mass

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THE work of I. V. Meshcherskov laid the foundations for the non-relativistic mechanics of bodies of variable mass. Meshcherskov's equations are used to determine the motion of rockets, heavenly bodies of variable mass and for the solution of a range of other mechanical problems. These equations, however, which are developed on the basis of Newtonian mechanics, are valid only in the range of velocities small with respect to the velocity of light c.

It is entirely possible that in the future, velocities approaching that of light will be of interest. There exist also radioactive particles which move with velocities close to c. These particles have variable mass.

For these cases it is necessary to apply relativistic mechanics to a material point of variable mass.

For the investigation of a single particle of variable mass we will use four dimensional Minkowski space. Let x_1, x_2, x_3 be the usual