present in not too small a concentration, is somewhat less (1-3 ev is sufficient) than the ionization potential V_0. In such a case, because of the exponential dependence of the fractional ionization on the ionization potential, x_0 is so small that the validity of the inequality in Eq. (5) is guaranteed.

As a result, Eq. (1) can now be rewritten in the form :

$$\bar{x} = \sum_{i=1}^{k} x_i N_i / N. \qquad (6)$$

Using Eqs. (3) and (6), it is possible to find all the N_i, the concentrations which were to be determined. The temperature, T, and the fractional ionization of one of the components can be found in the usual manner[2].

An experiment has been carried out for the simplest case, $i = 2$, i.e., a three-component arc-vapor*. Magnesium and zinc were used as the test elements. The temperature was determined from the ratio of the intensities of the zinc atomic lines, $\lambda = 3072$ Å and $\lambda = 3076$ Å, the relative (transition) probabilities of which are known[4]. The fractional ionization of the magnesium was found experimentally from the lines Mg I, $\lambda = 2779$ Å and Mg II, $\lambda = 2795$ Å (as in reference 3). Among the Zn and Mg lines, it was possible to find lines with known transition probabilities[4,5] by which the ratio of the concentrations of Zn and Mg atoms could be determined from Eq. (3). We selected the lines Mg I, $\lambda = 2779$ Å and Zn I, $\lambda = 2741$ Å.

All measurements were made by the photographic-photometric method. Zinc was introduced in the test sample in the form of a solution of $ZnSO_4$, magnesium in the form of a solution of $MgCO_3$ and the base element of the test sample was carbon powder. The percentages by weight of the materials in the test sample were: $ZnSO_4$ – 10%; $MgCO_3$ – 3%; C – 87%.

The temperature of the arc-vapor was found to be 6300° K. The measurements of the fractional ionization of the magnesium atoms showed that $x_{Mg} = 0.43$. A calculation using these values of T and x_{Mg} in the Saha formula gives $x_{Zn} = 3.6 \times 10^{-2}$ and $\bar{x} = 4 \times 10^{-3}$. Correspondingly, the computed concentrations of magnesium and zinc were found to be $N_{Mg} = 7.2 \times 10^{15}$ cm^{-3}; $N_{Zn} = 4.3 \times 10^{16}$ cm^{-3}.

Thus, in the test sample, the number of zinc atoms is greater than the number of magnesium atoms by a factor of 1.5; in the discharge-gap, however, it is 6 times greater. It follows that the rate at which zinc atoms from the test sample enter the discharge is 4 times greater than the rate at which magnesium atoms enter. This is in agreement with Rusanov's data on volatile elements[6].

* The experiment was carried out by the student A. E. Kontorovich.

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The Relativistic Mechanics of a Material Point of Variable Mass

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THE work of I. V. Meshcherskov laid the foundations for the non-relativistic mechanics of bodies of variable mass. Meshcherskov's equations are used to determine the motion of rockets, heavenly bodies of variable mass and for the solution of a range of other mechanical problems. These equations, however, which are developed on the basis of Newtonian mechanics, are valid only in the range of velocities small with respect to the velocity of light c.

It is entirely possible that in the future, velocities approaching that of light will be of interest. There exist also radioactive particles which move with velocities close to c. These particles have variable mass.

For these cases it is necessary to apply relativistic mechanics to a material point of variable mass.

For the investigation of a single particle of variable mass we will use four dimensional Minkowski space. Let x_1, x_2, x_3 be the usual

space coordinates. The fourth coordinate is $x_4 = ict$, where c is the velocity of light in vacuum and t is time. The Minkowski space x_i will be denoted by R_4 and the space (x_1, x_2, x_3) by R_3.

Every material particle describes a world line in R_4. The distance between two infinitesimally close events along any world line is given by

$$ds^2 = -dx_i\,dx_i. \tag{1}$$

The four-dimensional velocity of a particle is given by the vector

$$u_i = dx_i/ds. \tag{2}$$

To find its component, we note that according to Eq. (1),

$$ds = c\,dt\sqrt{1-(v^2/c^2)}, \tag{3}$$

where v is the ordinary three-dimensional velocity of the particle in R_3. In this way

$$u_\alpha = v_\alpha\sqrt{1-(v^2/c^2)}, \quad \alpha = 1, 2, 3,$$
$$u_4 = i/\sqrt{1-(v^2/c^2)}.$$

The components of the four-velocity are not independent. We note that for $dx_i^2 = -ds^2$, we have

$$u_i^2 = -1. \tag{4}$$

In a certain coordinate system, which we shall consider at rest, at a given time t, we investigate two material points, one with mass m having velocity u_i with respect to the given system, the other with mass dm having celocity a_i. At the time $t + dt$ these two points form one point (combination of mass) of mass $m + dm$, whose velocity is $u_i + du_i$. Instead of the time t as parameter, we introduce the arc length s, described by the point of mass m.

We now apply the momentum theorem of relativistic mechanics. At the instant s, the momentum of the system is

$$cmu_i + cdma_i,$$

whereas at the instant $s + ds$ it is

$$c(m + dm)(u_i + du_i).$$

The increase in momentum is

$$dp_i = cm\,du_i + cdm\,(u_i - a_i). \tag{5}$$

We introduce the force four-vector, determined by the derivative

$$f_i = \frac{dp_i}{ds} = cm\frac{du_i}{ds} + c\frac{dm}{ds}(u_i - a_i). \tag{6}$$

We shall now write Eq. (6) in R_3. The force vector in R_3 we define by the equation

$$\mathbf{F} = c(f_1, f_2, f_3)\sqrt{1-(v^2/c^2)}.$$

The first components of Eq. (6) give

$$\frac{d}{dt}\frac{m\mathbf{v}}{\sqrt{1-(v^2/c^2)}} - \frac{\mathbf{a}}{\sqrt{1-(a^2/c^2)}}\frac{dm}{dt} = \mathbf{F}. \tag{7}$$

For velocities v and a small with respect to c, Eq. (7) reduces to Mershcherkov's nonrelativistic equation

$$m\frac{d\mathbf{v}}{dt} + \frac{dm}{dt}(\mathbf{v} - \mathbf{a}) = \mathbf{F}. \tag{8}$$

Equation (6) or (7) represents the relativistic generalization of Mershcherkov's basic Eq. (8) for a material point of variable mass.

If the velocity of the added mass is relativistic, but that of mass m is nonrelativistic, then Eq. (7) assumes the form

$$\frac{d}{dt}(m\mathbf{v}) - \frac{\mathbf{a}}{\sqrt{1-(a^2/c^2)}}\frac{dm}{dt} = \mathbf{F}. \tag{9}$$

If the velocity of the added mass vanishes ($a = 0$), then Eq. (7) assumes the form

$$\frac{d}{dt}\frac{m\mathbf{v}}{\sqrt{1-(v^2/c^2)}} = \mathbf{F}. \tag{10}$$

For $a = v$ (the relative velocity of the combining masses is zero), Eq. (7) takes the form

$$m\frac{d}{dt}\frac{\mathbf{v}}{\sqrt{1-(v^2/c^2)}} = \mathbf{F}. \tag{11}$$

Let us form the scalar product

$$f_iu_i = cm\frac{du_i}{ds}u_i + c\frac{dm}{ds}(u_iu_i - a_iu_i).$$

Since $u_iu_i = -1$, and therefore $(du_i/ds)u_i = 0$, then

$$f_iu_i = -c\frac{dm}{ds}(1 + a_iu_i). \tag{12}$$

In R_3, Eq. (12) has the following form

$$\mathbf{Fv} = \frac{d}{dt}\frac{mc^2}{\sqrt{1-(v^2/c^2)}} - \frac{dm}{dt}\left(c^2\sqrt{1-(v^2/c^2)} + \frac{\mathbf{av}}{\sqrt{1-(a^2/c^2)}}\right), \tag{13}$$

One must bear in mind that in all these equations m depends on time. It can depend on time either explicitly, or implicitly by means of quantities determining the position or motion of the point. If, for example, $m = f(x_\alpha, \dot{x}_\alpha, t)$, then

$$\frac{dm}{dt} = \frac{\partial m}{\partial x_\alpha}\dot{x}_\alpha + \frac{\partial m}{\partial \dot{x}_\alpha}\ddot{x}_\alpha + \frac{\partial m}{\partial t} \quad (\alpha = 1, 2, 3).$$

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Hyperfine Structure of Paramagnetic Resonance in Copper Tutton Salts at Intermediate Fields

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UP to the present time the hyperfine structure of paramagnetic resonance in compounds of elements of the iron group has been studied at high frequencies, usually of the order of 10^{10} cps. In this frequency region, strong magnetic fields are required to satisfy the conditions for resonance in elements of the iron group. Such fields give rise to splittings at least an order of magnitude greater than the splittings which result from the interactions of electrons with the moments of the nucleus.

At much lower frequencies, say in the decimeter or meter region, conditions obtain which correspond to the intermediate- or weak-field Zeeman effect[1-3]. Low-frequency studies can be useful not only in checking the general theory of paramagnetic resonance absorption in crystals at low and intermediate fields[4], but also in obtaining more precise determinations of the hyperfine structure constant inasmuch as under these conditions the energy splittings produced by the nuclear moments become equal to or greater than the splitting produced by the dc magnetic field.

We have examined the paramagnetic resonance in a $CuK_2(SO_4)_2 \times 6H_20$ single-crystal diluted in an isomorphous zinc salt in the ratio 1:200 at a frequency of 526.74×10^6 cps at liquid air temperature. The paramagnetic resonance was observed using the grid-current method[5] in conjunction with modulation of the dc magnetic field. The free radical α, α-diphenyl β-picryl hydrazyl was used to calibrate the magnetic field. The accuracy in the determination of the resonance values of the field was limited by the width of the absorption line and consequently was not better than 2%.

It is known that the unit cell of a Tutton-salt cyrstal contains two copper ions[6] and that the tetragonal symmetry axes of the electric fields around these ions form an angle of 96°.

We investigated the paramagnetic resonance spectrum in the case for which the dc magnetic field was oriented along one of the tetragonal symmetry axes of the electric field. Six absorption lines were found. The resonance values in oersteds of the dc magnetic field are shown in the Table.

An interpretation of the paramagnetic resonance spectrum at strong fields of the salt which is the subject of this paper was carried out in reference 7 with the use of the following "spin" Hamiltonian:

$$\mathcal{H} = g_\parallel \beta H_z S_z + g_\perp \beta (H_x S_x + H_y S_y) + A S_z I_z + B (S_x I_x + S_y I_y) + Q\left[I_z^2 - \frac{1}{3} I(I+1)\right], \quad (1)$$

where $g_\parallel$ and $g_\perp$ are, respectively, the spectroscopic splitting factors parallel and perpendicular to the tetragonal symmetry axis of the electric field; A, B and Q are hyperfine structure constants of which the first two depend on the magnetic moment of the nucleus and the last depends on the nuclear electric quadrupole moment.

For the frequency which we used, the applied field corresponds to that of the intermediate field

Resonance values of the dc magnetic field in oersteds

Experimental	36	96	143	168	216	250
Calculated	36.11	95.85	140	168.5	221	252.5