working from Eq. (3.10), the theory of electric dissociation can be improved.

Since in order to induce dissociation of the deuteron, it is sufficient to give it an impulse of the order of the reciprocal of its dimensions, the nucleus, as it receives the same impulse, does not absorb an appreciable amount of energy and undergoes no reaction.

The detailed results will be published separately.

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The Connection between the Vibrations of the Surface of a Nucleus and Single Nucleon Excitation

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The conditions of validity of a model of single nucleon excitations in a nucleus are investigated by the method of adiabatic approximation. The effect of the relation between single nucleon excitations and the vibrations of the surface of a nucleus on the excited states of the whole nucleus are established.

1. INTRODUCTION

BECAUSE of strong interaction between the nucleons in a nucleus, one can speak in a strictly definedsense only of a state of a nucleus as a whole and not of the states of an individual nucleon. However, such consideration is still impracticable and one has to apply the approximate methods of study of energy states of nuclei.

In the case of an approximate examination of the lowest energy states, one usually proceeds from the notion that the nucleons in a nucleus are in the form of "shells" which can change their shape and dimensions. Individual nucleons are moving in an average field of the surrounding nucleons. This average field is such that the resultant force acting on one nucleon differs from zero mainly on the surface of a nucleus. Since the average field is caused by many nucleons, its change is connected with the collective movement of the nucleons. Because of the small compressibility of nuclear matter, its density can be considered as being constant. In this approximation, one can picture the "collective" movement only as deformation of nucleus surface without change of the volume.

In a series of cases the frequencies of the collective movements are smaller than the frequencies corresponding to the excitations of individual nucleons in a nucleus. Then, by investigation of the energy states of a nucleus, one can apply the adiabatic approximation.

In this article we will consider the limits of applicability of adiabatic approximation to a nucleus and make clear what effect the connection between the single nucleonic excitations and vibrations of the nuclear surface has on the energy states of a whole nucleus.

Particularly, it will be shown that the probability of a single nucleonic transition under the influence of an external excitation decreases because of a connection between single nuclear states and collective vibrations.

2. THE STRUCTURE OF THE ENERGY SPECTRUM OF A NUCLEUS AT SMALL EXCITATIONS

Let r denote the coordinates and the spins of nucleons in a nucleus and R the configuration of the nuclear surface. Assume that for every value of R the characteristic functions $\varphi_n(r, R)$ and the energy of the nucleons $E_n(R)$ are known. Let the index n denote the set of quantum numbers $\{n, j, m_j\}$, which characterizes the state of all nucleons in a nucleus in the case of a single particle approximation. The functions $\varphi_n(r, R)$ satisfy the equation

$$\{H(r, R) - E_n(R)\}\varphi_n(r, R) = 0. \quad (1)$$

To take into account the change of the shape of the surface of a nucleus, we introduce the kinetic

energy operator connected with the change of the nuclear surface:

$$\hat{T}_R \equiv \sum_\mu A_\mu \nabla_\mu^2; \qquad (2)$$

The explicit expressions for A_μ and ∇_μ will be given later.

The complete wave function, which defines the stationary states of a nucleus, has to satisfy the equation

$$\{\hat{T}_R + H(r, R) - E\}\,\Psi(r, R) = 0. \qquad (3)$$

The solution of Eq. (3) can be sought in the form

$$\Psi(r, R) = \sum_n \varphi_n(r, R)\,\Phi(n, R), \qquad (4)$$

where the $\varphi_n(r, R)$ are the solutions of Eq. (1). By substitution of Eq. (4) into Eq. (2) we get, after simple mathematical manipulations,

$$\{\hat{T}_R + E_n(R) - E\}\,\Phi(n, R) = \sum_m D_{nm}\Phi(m, R). \qquad (5)$$

The operator D_{nm} is determined by the expression

$$D_{nm} = \int \varphi_m^* \hat{T}_R \varphi_n\,(dr) + 2\sum_\mu A_\mu \int \varphi_m^* \nabla_\mu \varphi_n\,(dr)\,\nabla_\mu. \qquad (6)$$

The presence of non-diagonal terms on the right-hand side of Eq. (5) indicates that the state $\varphi_n(r, R)\,\Phi(n, R)$ is not stationary. If at some instance this state is fixed, then after some time spontaneous transitions into other states with the same energy will take place.

When the non-diagonal terms on the right-hand side of Eq. (5) are small, (we will evaluate these terms later) then in the zeroth approximation we will get the system of equations

$$\{T_R + E_n(R) - E_\nu\}\,\Phi_\nu(n, R) = 0, \qquad (7)$$

where $E_n(R)$ plays the part of the potential energy of surface deformation. Equation (7) characterizes the vibrations of the nuclear surface about the equilibrium positions determined (for each state of n-particle model) from the condition of minimum of $E_n(R)$. Let n_0 denote the complete set of quantum numbers $\{n, j, m_j\}$ for which the nucleons have the lowest of all possible energy states satisfying the Pauli principle. The energy of this state $E_{n_0}(R)$ will be a function of configuration R. The equilibrium configuration of the nucleus will be determined by the condition $\min E_{n_0}(R)$. We expand $E_n(R)$ in powers of relative departure $\alpha = (R - R_0)/R_0$ from a spherical shape of the nucleus R_0:

$$E_n(R) = E_n(R_0) + V_n^{(1)}\alpha + V_n^{(2)}\alpha^2 + \ldots \qquad (8)$$

The term $V_n^{(2)}\alpha^2$ can be considered as the potential energy of a surface deformation and can be expressed in terms of "macroscopic" parameters, for instance, the surface tension γ. Let

$$V_n^{(2)}\alpha^2 = {}^1/_2 \sum_{\lambda\mu} C_\lambda^{(n)} |\alpha_{\lambda\mu}|^2, \qquad (9)$$

where $\alpha_{\lambda\mu}$ is determined by the expansion in spherical functions of the relative displacement

$$\alpha = \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\vartheta, \phi),$$

where

$$\alpha_{\lambda\mu} = (-)^\mu \alpha^*_{\lambda,-\mu}, \qquad Y^*_{\lambda\mu} = (-)^\mu Y_{\lambda,-\mu}.$$

The value $\lambda = 1$ corresponds to a displacement of the whole nucleus and not to its deformation. Therefore, in Eq. (9), $\lambda > 1$. The sequence of numbers λ is bounded above by the value $\lambda \approx 6$ because of the finite number of particles in the nucleus. The coefficients $C_\lambda^{(n)}$ depend on quantum states of nucleons. If the distribution of charge is homogeneous in the volume of the spherical nucleus with radius R_0, the $C_\lambda^{(n)}$ are given by [1]

$$C_\lambda^{(n)} = (\lambda - 1)(\lambda + 2) R_0^2 \gamma - \frac{3(\lambda - 1)}{2\pi(2\lambda + 1)} \frac{Z^2 e^2}{R_0}.$$

To find the expression for the operator of the kinetic energy $\hat{T}_R$, we first consider the classical expression for kinetic energy of surface deformation of a nucleus. The kinetic energy of surface deformation of a nucleus can be expressed in terms of the impulses $\pi_{\lambda\mu} = B_\lambda \dot{\alpha}^*_{\lambda\mu}$ by the expression

$$T_R = \frac{1}{2} \sum_{\lambda\mu} \frac{|\pi_{\lambda\mu}|^2}{B_\lambda}.$$

For an incompressible nucleus with density ρ,

$$B_\lambda = \rho\,(R_0^5/\lambda).$$

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[1] A. Bohr, Dan. Mat. Fys. Medd. **26**, 14 (1952); **26**, 16 (1953)

By letting

$$\omega_\lambda^{(n)} = \left(\frac{C_\lambda^{(n)}}{B_\lambda}\right)^{1/2},$$

$$\alpha_{\lambda\mu} = \sqrt{\frac{\hbar}{2B_\lambda\omega_\lambda}}\,(b_{\lambda\mu} + (-)^\mu b^*_{\lambda,-\mu}),$$

$$\pi_{\lambda\mu} = i\sqrt{\frac{\hbar B_\lambda\omega_\lambda}{2}}\,(b^*_{\lambda\mu} + (-)^\mu b_{\lambda,-\mu}),$$

we get

$$T_R + V_n^{(2)}\alpha^2 \equiv \frac{\hbar}{2}\sum_{\lambda\mu}\omega_\lambda\,(b_{\lambda\mu}b^*_{\lambda\mu} + b^*_{\lambda\mu}b_{\lambda\mu}).$$

To go over to quantum mechanics, we will consider that $b_{\lambda\mu}$ are operators satisfying the permutational relationship $[b_{\lambda\mu},\ b^*_{\lambda'\mu'}] = \delta_{\lambda\lambda'}\delta_{\mu\mu'}$. To simplify the future notation, we will write only one index μ instead of both of them (λ and μ). If new real variables ξ_μ are introduced according to the expressions

$$b_\mu = 2^{-1/2}\left(\xi_\mu + \frac{\partial}{\partial\xi_\mu}\right), \qquad b^*_\mu = 2^{-1/2}\left(\xi_\mu - \frac{\partial}{\partial\xi_\mu}\right),$$

we will get

$$\hat{T}_R + V_n^{(2)}\alpha^2 \equiv \frac{\hbar}{2}\sum_\mu\omega_\mu^{(n)}\left\{\xi_\mu^2 - \frac{\partial^2}{\partial\xi_\mu^2}\right\}.$$

The term $V_n^{(1)}\alpha$ in Eq. (8) characterizes the difference between the equilibrium shape of a nucleus in the nth state and the spherical shape. By letting

$$V_n^{(1)}\alpha = -\sum_\mu\hbar\omega_\mu^{(n)}\zeta_\mu^{(n)}\xi_\mu, \tag{11}$$

the surface deformation can be expressed in terms of displacements $\zeta_\mu^{(n)}$ from equilibrium values of variables ξ_μ. Now one can write

$$\hat{T}_R + E_n(R) - E_n(R_0) \tag{12}$$

$$= \frac{\hbar}{2}\sum_\mu\omega_\mu^{(n)}\left\{(\xi_\mu - \zeta_\mu^{(n)})^2 - \frac{\partial^2}{\partial\xi_\mu^2}\right\} - \frac{\hbar}{2}\sum_\mu\omega_\mu^{(n)}\zeta_\mu^2.$$

By substitution of Eq. (8) into Eq. (7) and taking into consideration Eq. (12), we get the equation which determines the states of vibration of the nuclear surface when the single nucleonic states are characterized by the set of quantum numbers n

$$\left\{\frac{\hbar}{2}\sum_\mu\omega_\mu^{(n)}\left[(\xi_\mu - \zeta_\mu^{(n)})^2 - \frac{\partial^2}{\partial\xi_\mu^2}\right] - \varepsilon_\nu\right\} \tag{13}$$

$$\times\ \Phi_\nu(n; \ldots\xi_\mu\ldots) = 0,$$

where

$$\varepsilon_\nu = E_\nu - E_n(R_0) + \frac{\hbar}{2}\sum_\mu\omega_\mu^{(n)}[\zeta_\mu^{(n)}]^2. \tag{14}$$

Let ν denote the set of quantum numbers which characterize the vibrational state of a nuclear surface. Equation (13) has the solution

$$\Phi_\nu(n; \ldots\xi_\mu\ldots) = \prod_\mu\psi_{\nu_\mu}(\xi - \zeta_\mu^{(n)}), \tag{15}$$

where $\psi_{\nu\mu}(x)$ is the wave function of a harmonic oscillator. The energy of vibrations corresponding to a certain set of quantum numbers $\nu \equiv (\ldots\nu_\mu\ldots)$, where $\nu_\mu = 0, 1, 2 \ldots$ is equal to

$$\varepsilon_\nu^{(n)} = \hbar\sum_\mu\omega_\mu^{(n)}(\nu_\mu + 1/2).$$

The total energy of a nucleus is determined by the quantum states of the nucleons (in a single particle approximation) and by the quantum states of the vibration of a nuclear surface.

$$E_{n\nu} = E_n(R_0) - \frac{\hbar}{2}\sum_\mu\omega_\mu^{(n)}[\zeta^{(n)}]^2 \tag{16}$$

$$+ \hbar\sum_\mu\omega_\mu(\nu_\mu + 1/2).$$

The wave function of such a state is equal to

$$\Psi_{n\nu} = \varphi_n(r, R)\,\Phi_\nu(n; \ldots\xi_\mu\ldots). \tag{15a}$$

In general, the series of states with different n and ν can have approximately equal energies $E_{n\nu} \approx E_{n'\nu'} \approx E_{n''\nu''} \approx \ldots$. The states (16) are not stationary. For investigation of the spontaneous transitions of a system from the state $n\nu$ into the state $n'\nu'$ (with approximately the same energy) it is necessary to consider the time equation

$$i\hbar\frac{\partial\Psi}{\partial t} = \{H(r, R) + \hat{T}_R\}\,\Psi. \tag{17}$$

We will seek the solution of this equation in the form $\Psi = \sum_{n\nu} a_{n\nu}(t)\,\Psi_{n\nu}(r, R)$. We then get

$$i\hbar\dot{a}_{n\nu} = E_{n\nu}a_{n\nu} + \sum_{n'\nu'}a_{n'\nu'}F_{n\nu}^{n'\nu'}, \tag{18}$$

where

$$F_{n\nu}^{n'\nu'} = \int\Phi^*_{n\nu}D_{nn'}\Phi_{n'\nu'}\,dR. \tag{18a}$$

Assume that at $t = 0$, the nucleus is in the state $n\nu$, i.e., $a_{n'\nu'} = \delta_{nn'}\delta_{\nu\nu'}$. Then the system of

equations (18) can be written as follows:

$$i\dot{a}_{n\nu} = \omega_{n\nu}a_{n\nu} + \frac{1}{\hbar}\sum_{n'\nu'} a_{n'\nu'}F_{n\nu}^{n'\nu'},$$

$$i\dot{a}_{n'\nu'} = \omega_{n'\nu'}a_{n'\nu'} + \frac{1}{\hbar}a_{n\nu}F_{n'\nu'}^{n\nu},$$

where $\omega_{n\nu} = E_{n\nu}/\hbar$. By means of the Laplace's transform

$$A_{n\nu} = \int_0^\infty e^{-(\eta - i\gamma)t}a_{n\nu}(t)\,dt,$$

this system of equations can be transformed into the algebraic system

$$A_{n\nu}(\omega_{n\nu} - \gamma - i\eta) = -i - \frac{1}{\hbar}\sum_{n'\nu'} A_{n'\nu'}F_{n\nu}^{n'\nu'},$$

$$A_{n'\nu'}(\omega_{n'\nu'} - \gamma - i\eta) = -\frac{1}{\hbar}\sum_{n\nu} A_{n\nu}F_{n'\nu'}^{n\nu}.$$

By substitution of the second equation into the first one we get

$$A_{n\nu} = \frac{i}{\omega_{n\nu} - \gamma - i[\eta + {}^1/_2\,T(\gamma)]},$$

where

$${}^1/_2\,T(\gamma) = \frac{1}{i\hbar}\sum_{n\nu}|F_{n\nu}^{n'\nu'}|^2/(\omega_{n'\nu'} - \gamma - i\eta).$$

By means of an inverse Laplace transform we find

$$a_{n\nu} = \exp\{-i\omega_{n\nu}t - {}^1/_2T(\gamma_0)\,t\},$$

where

$$\gamma = \omega_{n\nu} - i\left(\eta + \frac{T(\gamma_0)}{2}\right).$$

Hence the probability of a nucleus to be in the state $n\nu$ varies according to the law $|a_{n\nu}|^2 = e^{-\Gamma t}$, where Γ can be found from the equation

$$1 = \frac{1}{\hbar^2}\sum_{n'\nu'}|F_{n\nu}^{n'\nu'}|^2/[(\omega_{n'\nu'} - \omega_{n\nu})^2 + {}^1/_4\Gamma^2]. \quad (19)$$

Thus all energy levels at $n \neq n_0$ have the width

$$\Delta E_{n\nu} = \hbar\Gamma, \quad (20)$$

where $1/\Gamma$ is the average lifetime of the state $n\nu$ with respect to a spontaneous transition into some other states.

The adiabatic approximation is valid when the inequality $\hbar\,\Gamma < |E_{n\nu} - E_{n'\nu'}|$ holds. In this case, the terms on the right-hand side of Eq. (19) have sharp maxima for $\omega_{n\nu} = \omega_{n'\nu'}$. Since we assume that the energy of an individual quantum of the surface vibrations is smaller than the excitation energy of a single nucleonic state, the equality $\omega_{n\nu} = \omega_{n'\nu'}$ is possible only when the vibrative state ν' differs from ν by many quanta of vibrations of a nuclear surface.

3. THE CALCULATION OF THE PROBABILITY OF A NONRADIATIVE TRANSITION OF ENERGY OF A SINGLE NUCLEONIC EXCITATION INTO THE VIBRATION ENERGY OF A NUCLEAR SURFACE

The operators $D_{nn'}$, which are functions of R, appear in the matrix element $F_{n\nu}^{n'\nu'}$. To simplify the evaluation of $F_{n\nu}^{n'\nu'}$ we will neglect the less essential first integral $D_{nn'}$, which contains the second derivatives of φ_n with respect to ξ_μ. Also, in remaining terms we will replace R by the equilibrium value R_0. Then

$$D_{nn'} = -\hbar\sum_\mu \omega_\mu^{n_0}L_{nn'}(\mu)\frac{\partial}{\partial x_\mu}, \quad (21)$$

where

$$L_{nn'}(\mu) = \left\{\int \varphi_{n'}^*\frac{\partial\varphi_n}{\partial\xi_\mu}(dr)\right\}_{R=R_0} \quad (21a)$$

By the substitution of Eq. (21) into Eq. (18a), and by taking Eq. (15) into consideration, and also the equality

$$\frac{\partial}{\partial\xi_\mu}\Psi_{\nu\mu} = \sqrt{\frac{\nu_\mu}{2}}\Psi_{\nu_\mu - 1} - \sqrt{\frac{\nu_\mu + 1}{2}}\Psi_{\nu_\mu + 1},$$

we will get

$$F_{n\nu}^{n'\nu'} \approx -\hbar\sum_\mu \omega_\mu^{n_0}L_{nn'}(\mu) \quad (22)$$

$$\times\left\{\sqrt{\frac{\nu_\mu}{2}}M_{\nu_\mu\nu_\mu - 1}^{nn'}\prod_{\mu'\neq\mu}M_{\nu_{\mu'}\nu'_{\mu'}}^{nn'}\right.$$

$$\left. - \sqrt{\frac{\nu_\mu + 1}{2}}M_{\nu_\mu\nu_\mu + 1}^{nn'}\sum_{\mu'\neq\mu}M_{\nu_{\mu'}\nu'_{\mu'}}^{nn'}\right\},$$

where

$$M_{\nu_\mu\nu'_\mu}^{nn'} = \int\varphi_{\nu_\mu}(\xi_\mu - \zeta_\mu^{(n)})\,\varphi_{\nu'_\mu}(\xi_\mu - \zeta_\mu^{(n')})\,d\xi_\mu. \quad (22a)$$

With the accuracy to the terms of order $(\zeta_\mu^n - \zeta_\mu^{n'})^2$ the matrix elements $M_{\nu_\mu\nu'_\mu}^{nn'}$ are equal to zero when $\nu'_\mu \neq \nu_\mu$, $\nu_\mu \pm 1$. If $\nu'_\mu = \nu_\mu$, $\nu_\mu \pm 1$, we have

$$M_{\nu_\mu\nu_\mu}^{nn'} = 1 - {}^1/_2(\nu_\mu + {}^1/_2)(\zeta_\mu^n - \zeta_\mu^{n'})^2; \quad (22b)$$

$$M_{\nu_\mu\nu_\mu + 1}^{nn'} = \sqrt{\frac{\nu_\mu + 1}{2}}(\zeta_\mu^n - \zeta_\mu^{n'});$$

$$M^{nn'}_{\nu_\mu \nu_\mu - 1} = - \sqrt{\frac{\nu_\mu}{2}} (\zeta^n_\mu - \zeta^{n'}_\mu).$$

Now let us consider the excited states n with only one nucleon on the first excited level, and such that there are no quanta of the surface vibrations ($\nu = 0$). Then in the summation (19) over $n'\nu'$ there will remain only the terms with $F^{n_0 \nu}_{n 0}$. Moerover, considering Eq. (22b) and retaining in Eq. (19) only terms of order not higher than $(\zeta^n_\mu - \zeta^{n_0}_\mu)^2$, we get $F^{n_0 \nu'}_{n,0} = \hbar/2 \sum_\mu \omega^{n_0}_\mu L_{nn_0}(\mu) \times (\zeta^n_\mu - \zeta^{n_0}_\mu)^2$ and the summation (19) will be expressed as follows:

$$1 = \frac{1}{2} \sum_\nu \frac{\left| \sum_\mu \omega^{n_0}_\mu L_{nn_0}(\mu) (\zeta^n_\mu - \zeta^{n_0}_\mu) \right|^2}{(\omega_{n_0 \nu} - \omega_{n,0})^2 + {}^1/_4 \Gamma^2}. \qquad (23)$$

In performing the summation over ν, one should keep in mind that ν_μ can have the values 0, 1; further,

$$\omega_{n_0 \nu} - \omega_{n,0} = \Omega_{nn_0} - \sum_\mu \omega^{n_0}_\mu \nu_\mu, \qquad (23a)$$

where

$$\Omega_{nn_0} = \frac{E_n - E_{n_0}}{\hbar} + \sum_\mu \Big\{ \omega^{n_0}_\mu (\zeta^{n_0}_\mu)^2 - \omega^n_\mu (\zeta^n_\mu)^2 + \frac{1}{2} (\omega^n_\mu - \omega^{n_0}_\mu) \Big\}. \qquad (23b)$$

The first term in Eq. (23b) corresponds to the change of energy due to the transition of nucleons from the state n into n_0, under the assumption of a spherical shape for the nucleus. The terms in square brackets account for the change of energy connected with the deformation of a nuclear surface, and the last term accounts for the change of energy of the zeroth vibrations, which happens because of changes in frequencies of vibrations.

Considering the sharp maximum of Eq. (23) at $\Omega_{nn_0} - \sum_\mu \omega^{n_0}_\mu \nu_\mu = 0$, we can write Eq. (23) as follows:

$$1 = \frac{1}{2} \left| \overline{\sum_{\mu''}} \omega^{n_0}_{\mu''} L_{nn_0}(\mu) (\zeta^n_{\mu''} - \zeta^{n_0}_{\mu''}) \right|^2 \times \sum_\nu \frac{1}{\left(\Omega_{nn_0} - \sum_\mu \omega^{n_0}_\mu \nu_\mu \right)^2 + {}^1/_4 \Gamma^2},$$

where $\overline{\sum_{\mu''}}$ means the average value of the sum over those quantum numbers μ'' for which $\nu_{\mu''} \approx 1$ and $\sum_{\mu''} \omega^{n_0}_{\mu''} \nu_{\mu''} = \Omega_{nn_0}$. Let $\sum_\mu \omega^{n_0}_\mu \nu_\mu \equiv \Omega$; then,

$$\sum_\nu \frac{1}{\left(\Omega_{nn_0} - \sum_\mu \omega^{n_0}_\mu \nu_\mu \right)^2 + {}^1/_4 \Gamma^2} \approx \int \frac{\rho(\Omega)\, d\Omega}{(\Omega_{nn_0} - \Omega)^2 + {}^1/_4 \Gamma^2} = \frac{2\pi}{\Gamma} \rho(\Omega_{nn_0}),$$

where $\rho(\Omega)$ is the number of states of surface vibrations per unit of frequency interval. According to Bethe[2]

$$\rho(\Omega) \approx {}^4/_3 R_0^2 \omega^{1/3} (\rho/\gamma)^{2/3}.$$

Hence

$$\Gamma = \pi \rho(\Omega) \left| \overline{\sum_{\mu''}} \omega^{n_0}_{\mu''} L_{nn_0}(\mu) (\zeta^n_{\mu''} - \zeta^{n_0}_{\mu''}) \right|^2, \qquad (24)$$

where $L_{nn_0}(\mu)$ is determined in Eq. (21a).

For magnitude evaluation of $L_{nn_0}(\mu)$ the change of the function due to deformation of a nuclear surface has to be calculated. Assuming that the potential field for the nucleons in a nucleus can be pictured by a spherical rectangular well of radius R_0 and depth D, the operator of excitation at small deformation of nuclear surface can be represented by

$$W = - D R_0 \sum_p \delta(r_p - R_0) \times \sum_{\lambda\mu} \sqrt{\hbar / B_\lambda \omega_\lambda}\, \xi_{\lambda\mu} (\mathbf{Y}_{\lambda\mu} + \mathbf{Y}^*_{\lambda\mu}).$$

The summation $\sum_p$ extends over all nucleons of a nucleus.

In first approximation of the theory of excitations

$$\varphi_{n_0}(R) = \varphi_{n_0}(R_0) + \sum_m {}' (W_{mn_0}/\hbar\Omega_{n_0 m}) \varphi_m(R_0), \qquad (25)$$

where

$$\Omega_{n_0 m} = (E_{n_0} - E_m)/\hbar, \qquad W_{mn_0} = \int \varphi^*_m W \varphi_{n_0}\, d\tau.$$

Substitution of Eq. (25) into Eq. (21a) gives

$$L_{nn_0} = - \sum_{p,\lambda,\mu} \frac{D R_0^3 \Re_n(R_0) \Re_{n_0}(R_0)}{\hbar \Omega_{n_0 n}} \times \sqrt{\frac{\hbar}{B_\lambda \omega_\lambda}} \int \mathbf{Y}^*_{n_0} (\mathbf{Y}_{\lambda\mu} + \mathbf{Y}^*_{\lambda\mu})\, \mathbf{Y}_n\, d\Omega.$$

[2] H. Bethe, Rev. Mod. Phys. 3, 362 (1948)

For simplicity we will consider the nuclei containing only one nucleon outside of the complete shells*. Then the lowest excited states of the nucleus are determined according to the shell model by the transition of this nucleon (in the future we will call it an external nucleon) into some other quantum states. Consequently, the quantum states n_0 and n will differ only by the state of the external nucleon. As it is shown in references 3, the summation

$$\sum_p \int Y^*_{lm} Y_{\lambda\mu} Y_{lm}\, d\Omega = 0,$$

when it includes all nucleons contained in complete shells. Therefore, only external nucleons will contribute to $L_{nn_0}(\mu)$

$$L_{nn_0}(\mu) = -\frac{DR_0^3 \Re_n(R_0)\,\Re_{n_0}(R_0)}{\hbar\Omega_{n_0 n}} \qquad (26)$$

$$\times \sum_{\lambda\mu} \sqrt{\frac{\hbar}{B_\lambda \omega_\lambda}} \int Y^*_{l_0 m_0} (Y_{\lambda\mu} + Y^*_{\lambda\mu})\, Y_{\lambda\mu}\, d\Omega.$$

The factor $DR_0^3\, \Re_n(R_0)\, \Re_{n_0}(R_0)$ can be computed under certain assumptions on the potential well in which the nucleons are moving. These computations are very approximate. They were done in works mentioned in reference 3.

It was shown that this factor does not depend much on the state of a nucleon and is approximately equal to 40 mev (at $D = 28.3$ mev).

Let g denote the integral $\int Y_{l_0 m_0} Y_{\lambda\mu} Y_{lm}\, d\Omega$. This integral is zero when the selection rules $|\lambda - l_0| \leq l \leq |\lambda + l_0|$ and $\lambda + l + l_0 = 2n$ (where n is an integer) do not hold. For the transition $f \rightleftarrows p$ the average value of second power of g^2 according to the computation of Reifman[4] is $g^2 \leq 0.1$.

Hence, the following approximation can be made:

$$|L_{n_0 n}|^2 \approx \frac{0.4\hbar}{B\omega}\left(\frac{40}{E_{\text{mev}}}\right)^2,$$

where E_{mev} is the energy of the single nucleonic transition expressed in mev and ω is the frequency of surface vibrations.

By substitution of these results in Eq. (24) we get

$$\Gamma = \frac{0.4\pi\rho(\Omega)\hbar}{B}\left(\frac{40}{E_{\text{MeV}}}\right)^2 \left| \sum_\mu \omega_\mu^{1/2} (\zeta_\mu^n - \zeta_\mu^{n_0}) \right|^2.$$

Thus, the probability of a nonradiative transfer of energy of a single nucleonic excitation to the collective degrees of freedom of a nucleus is the greater, the greater is the square of the difference $(\zeta_\mu^n - \zeta_\mu^{n_0})$ characterizing the change of equilibrium configuration of a nucleus during the transition of a nucleon from an excited into the ground state. Such nonradiative transitions must widen the levels which correspond to the excitation of separate nucleons in a nucleus. The total width will be equal to the sum of widths of the nonradiative transition processes, considered above, γ-emission, conversion, etc.

For the evaluation of magnitude of Γ we assume that the transition takes place from a spherically symmetrical state n_0 (or into spherically symmetrical state), and that the interaction exists only with the surface vibrations $\lambda = 2$; then,

$$\Gamma \approx 0.4\pi\rho(\Omega)\left(\frac{40}{E_{\text{MeV}}}\right)^2 \omega^2 \beta,$$

where ω is the frequency of vibrations of the nuclear surface at $\lambda = 2$. The value

$$\beta = \sum_\mu \frac{\hbar}{B\omega}(\zeta_\mu^n)^2 = \sum_\mu |\alpha_\mu|^2$$

determines the ellipsoidal deformation of a nuclear surface. According to reference 1, it can be calculated from the value of a quadruple moment of a nucleus Q_0 (with respect to the axis of symmetry of a nucleus) by using the formula $\beta = (5\pi)^{1/2} Q_0 / 3ZR_0^2$ or from the moment of inertia of a nucleus $J = 3B\beta^2$. Further, from Eq. (24a) it follows that $\rho(\Omega) \approx 2.6 \times 10^{-30}\, \omega^{1/3} A^{2/3}$. For example, by letting $E = 1.5$ mev, $\omega = 8 \times 10^{19}$; and $A = 216$; $\beta = 0.1$, we will get $\Gamma \approx 2 \times 10^{20}$ which corresponds to the width of a level ≈ 0.1 mev. Knowing the magnitude of Γ, one can determine the mean free path of a nucleon in a nucleus $\Lambda = v/\Gamma$, where v is the velocity of a nucleon. Since Γ differs from zero only for the excited states of a nucleus, $\Lambda \neq \infty$ only for the excited states of a nucleus.

In some works[5] it was shown that the experimental data[6] about the scattering of neutrons with

* The results obtained below apply only to nuclei which lack one nucleus to fill the outer shell.

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the energy less than 3 mev can be phenomenologically explained as a scattering in a complex potential, the imaginary part of which is responsible for the formation of an excited compound nucleus. This verifies the notion that during the first stages of nuclear reactions the average field of a nucleus acts on the incident nucleon. In this case the mean free path, which is connected with the probability of formation of an intermediate nucleus, can be determined in an analogous way.

Of course, one should keep in mind that the possibility of a separation of single nucleonic excitations is determined by conditions, which make the inequality $\hbar\Gamma < \Delta$ valid. Here Δ is the distance between the neighboring single particle levels. If this inequality is violated, which apparently must be the case at high excitation energies of a nucleus, the separation of single particle excitations from a total excitation of a nucleus is entirely impossible.

4. EFFECT OF THE DEFORMATION OF A NUCLEAR SURFACE ON THE PROBABILITY OF SINGLE NUCLEONIC EXCITATIONS

In the adiabatic approximation the state of a nucleus is characterized by the energy levels $E_{n\nu}$ (16) and wave functions

$$\psi_{n\nu} = \varphi_n(r, R) \prod_\mu \psi_{\nu_\mu}(\xi_\mu - \zeta_\mu^n).$$

We assume that under the influence of an excitation, the operator $A(r)$ of which operates only on the coordinates of the nucleons, the transition from the state $n_0\nu$ into the state $n\nu$ takes place without change of the quantum state $\nu = (\ldots 0_\mu \ldots)$ of the vibration of a nuclear surface. The probability of the transition within one second will be equal to

$$W_{nn_0} = \frac{2\pi}{\hbar} \left| \int \varphi_n^*(r) A(r) \varphi_{n_0}(r) dr \right|^2 \times \left\{ 1 - {}^1/_4 \sum_\mu (\zeta_\mu^n - \zeta_\mu^{n_0})^2 \right\}.$$

The ratio of the probability of a transition $n_0 \to n$, upon consideration of possible deformation of a nucleus, to the probability of the transition computed for the case of a rigid nuclear shell will be equal to

$$P = 1 - {}^1/_4 \sum_\mu (\zeta_\mu^n - \zeta_\mu^{n_0})^2;$$

The summation is performed over all possible types of vibrations of a nuclear surface. The last expression can be simplified when one of the states, between which the transition takes place, corresponds to the spherically symmetrical nucleus; then $P = 1 - 1/4 \sum_\mu \zeta_\mu^2$. By assuming, for simplicity, that the greatest interaction is accomplished with the surface vibrations $\lambda = 2$, we get $P = 1 - (B\omega\beta/4\hbar)$. When $A = 216$, $\Omega = 8 \times 10^{19}$, $\beta = 0.3$, we get $P \approx 0.8$.

Translated by G. Filipovich
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