Bremsstrahlung from the Collisions of π Mesons with Nucleons *

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1. BREMSSTRAHLUNG from the scattering of mesons by nucleons, arising as a result of the coulomb interaction, was investigated in reference 2; yet, because of the strong interaction of mesons with nucleons, the principal role is played by bremsstrahlung from the nuclear force. Consequently, in the present work the cross section is calculated for the process of radiation of a γ-quantum in the scattering of a pseudoscalar π-meson from a nucleon by the nuclear force, both for pseudoscalar and for pseudovector coupling of the meson to the nucleon. The calculation is carried out to the third order in perturbation theory with the nonrelativistic approximation for the nucleon. Resulting formulas for the cross section of the process $\pi^- + p \to \pi^- + p + \gamma$ are obtained separately for the cases of pseudoscalar and pseudovector coupling.

2. The differential cross section in the coordinate system where the nucleon is originally at rest, for the case of pseudoscalar coupling, has the following form (the quantity $\hbar$ is everywhere designated by h):

$$d\sigma = \sigma_0 \frac{dE_\gamma}{E_\gamma} d\Omega_1 d\Omega_\gamma = \frac{1}{4\pi^2}\left(\frac{\mu}{2m}\right)^2 \frac{e^2}{hc}\left(\frac{h}{\mu c}\right)^2\left(\frac{g^2}{hc}\right)^2 \quad (1)$$

$$\times \frac{k_1}{k_0}\frac{dE_\gamma}{E_\gamma}\left\{\frac{\beta_0^2 - (\vec{\beta}_0 \mathbf{n}_\gamma)^2}{(1-\vec{\beta}_0\mathbf{n}_\gamma)^2} + \frac{\beta_1^2 - (\vec{\beta}_1 \mathbf{n}_\gamma)^2}{(1-\vec{\beta}_1\mathbf{n}_\gamma)^2} - 2\frac{\vec{\beta}_0\vec{\beta}_1 - (\vec{\beta}_0\mathbf{n}_\gamma)(\vec{\beta}_1\mathbf{n}_\gamma)}{(1-\vec{\beta}_0\mathbf{n}_\gamma)(1-\vec{\beta}_1\mathbf{n}_\gamma)}\right\} d\Omega_1 d\Omega_\gamma,$$

where

$$\beta_0 = \frac{v_0}{c} = \frac{chk_0}{E_0};\quad \beta_1 = \frac{v_1}{c} = \frac{chk_1}{E_1};\quad \mathbf{n}_\gamma = \frac{\mathbf{k}_\gamma}{|\mathbf{k}_\gamma|};$$

and where $h\mathbf{k}_0$, E_0; $h\mathbf{k}_1$, E_1 are the momentum and total energy of the meson before and after the collision; $h\mathbf{k}_\gamma, E_\gamma$ are the momentum and energy of the γ-quantum, m the mass of the nucleon and μ the mass of the π-meson.

For the investigation of the angular distribution of the γ-quanta, Eq. (1) can be rewritten in the form

$$d\sigma = \sigma_0 \frac{dE_\gamma}{E_\gamma} d\Omega_1 d\Omega_\gamma = \frac{1}{4\pi^2}\left(\frac{\mu}{2m}\right)^2\left(\frac{g^2}{hc}\right)^2\left(\frac{h}{\mu c}\right)^2\frac{l^2}{hc} \quad (2)$$

$$\times \frac{k_1}{k_0}\frac{dE_\gamma}{E_\gamma}\left(\frac{\vec{\beta}_0}{1-\vec{\beta}_0\mathbf{n}_\gamma} - \frac{\vec{\beta}_1}{1-\vec{\beta}_1\mathbf{n}_\gamma}\right)^2 \sin^2\theta\, d\Omega_1 d\Omega_\gamma,$$

where θ is the angle between the vector $\left(\frac{\vec{\beta}_0}{1-\vec{\beta}_0\mathbf{n}_\gamma} - \frac{\vec{\beta}_1}{1-\vec{\beta}_1\mathbf{n}_\gamma}\right)$ and $\mathbf{k}_\gamma$. In the nonrelativistic approximation for the meson this formula has the form

$$d\sigma = \sigma_0 \frac{dE_\gamma}{E_\gamma} d\Omega_1 d\Omega_\gamma = \frac{1}{4\pi^2}\left(\frac{\mu}{2m}\right)^2\left(\frac{g^2}{hc}\right)^2\frac{e^2}{hc}\left(\frac{h}{\mu c}\right)^2\frac{k_1}{k_0}(\vec{\beta}_0 - \vec{\beta}_1)^2 \times \sin^2\theta\frac{dE_\gamma}{E_\gamma}d\Omega_1 d\Omega_\gamma. \quad (2')$$

The angular distribution of the radiation is similar to that of a dipole vibrating along an axis which coincides with the vector $\mathbf{v}_0 - \mathbf{v}_1$.

The angular distribution of the γ-quanta, disregarding the angle of the scattered π-meson, is given by the formula

$$\frac{dE_\gamma}{E_\gamma} d\Omega_\gamma \int \sigma_0 d\Omega_1 = \frac{1}{\pi}\left(\frac{\mu}{2m}\right)^2\frac{e^2}{hc}\left(\frac{g^2}{hc}\right)^2\left(\frac{h}{\mu c}\right)^2 \quad (3)$$

$$\times \frac{k_1}{k_0}\frac{dE_\gamma}{E_\gamma} d\Omega_\gamma\left\{\frac{\beta_0^2\sin^2\vartheta}{(1-\beta_0\cos\vartheta)^2} + \frac{1}{\beta_1}\ln\frac{1+\beta_1}{1-\beta_1} - 2\right\},$$

where ϑ is the angle between the vectors $\mathbf{k}_0$ and $\mathbf{k}_1$. In nonrelativistic approximation for the meson this formula takes the form

$$\frac{dE_\gamma}{E_\gamma} d\Omega_\gamma \int \sigma_0 d\Omega_1 = \frac{1}{\pi}\left(\frac{\mu}{2m}\right)^2\frac{e^2}{hc}\left(\frac{g^2}{hc}\right)^2 \quad (3')$$

$$\times \left(\frac{h}{\mu c}\right)^2\frac{k_1}{k_0}\frac{dE_\gamma}{E_\gamma}\left\{\beta_0^2\sin^2\vartheta + \frac{2}{3}\beta_1^2\right\}.$$

In this case of nonrelativistic energy for the meson, the majority of the γ-quanta emerge perpendicular to the direction of the incident mesons and with increasing energy up to this maximum deviation from the direction of the incident meson. The energy distribution of the γ-quanta has the characteristic formula

$$\frac{dE_\gamma}{E_\gamma}\int \sigma_0 d\Omega_1 d\Omega_\gamma \tag{4}$$

$$= 4\left(\frac{\mu}{2m}\right)^2\left(\frac{g^2}{hc}\right)^2\frac{e^2}{hc}\left(\frac{h}{\mu c}\right)^2\frac{k_1}{k_0}\frac{dE_\gamma}{E_\gamma}$$

$$\times\left\{\frac{1}{\beta_0}\ln\frac{1+\beta_0}{1-\beta_0}+\frac{1}{\beta_1}\ln\frac{1+\beta_1}{1-\beta_1}-4\right\}.$$

In the nonrelativistic approximation for the meson this expression takes the simpler form

$$\frac{dE_\gamma}{E_\gamma}\int \sigma_0 d\Omega_1 d\Omega_\gamma \tag{4'}$$

$$= \frac{8}{3}\left(\frac{\mu}{2m}\right)^2\left(\frac{h}{\mu c}\right)^2\frac{e^2}{hc}\left(\frac{g^2}{hc}\right)^2\frac{dE_\gamma}{E_\gamma}(\beta_0^2+\beta_1^2).$$

The total cross section is obtained by integrating over a very small energy interval δ_E about the kinetic energy of the incident meson $T = E_0 - \mu c^2$. For the case $E_0 \gg \mu c^2$ the total cross section takes the form:

$$\sigma = \int\frac{dE_\gamma}{E_\gamma}\sigma_0 d\Omega_1 d\Omega_\gamma \tag{5}$$

$$= 16\left(\frac{\mu}{2m}\right)^2\left(\frac{g^2}{hc}\right)^2\frac{e^2}{hc}\left(\frac{h}{\mu c}\right)^2\ln\frac{2E_0}{\mu c^2}\ln\frac{E_0}{\delta_E};$$

in the case of nonrelativistic energy for the meson

$$\sigma = \int\frac{dE_\gamma}{E_\gamma}\sigma_0 d\Omega_1 d\Omega_\gamma \tag{5'}$$

$$= 8\left(\frac{\mu}{2m}\right)^2\left(\frac{g^2}{hc}\right)^2\frac{e^2}{hc}\left(\frac{h}{\mu c}\right)^2\frac{2T_0}{\mu c^2}\left[\ln\frac{2T_0}{\delta_E}-\frac{5}{3}\right],$$

where $T_0 = (hk_0)^2/2\mu$, that is, the energy which the meson under consideration would have nonrelativistically. The total cross section increases as the first power of this kinetic energy, and at extreme relativistic energies the total cross section increases as the square of the logarithm of this energy.

3. In the case of pseudovector coupling the differential cross section takes the following form

$$d\sigma = \sigma_0\frac{dE_\gamma}{\mu c^2}d\Omega_1 d\Omega_\gamma \tag{6}$$

$$= \frac{1}{4\pi^2}\frac{e^2}{hc}\left(\frac{f^2}{hc}\right)^2\left(\frac{h}{\mu c}\right)^2\frac{k_1}{k_0}\frac{E_\gamma dE_\gamma}{(\mu c^2)^2}d\Omega_1 d\Omega_\gamma$$

$$\times\{(\vec\beta_0-\vec\beta_1)^2+4(\vec\beta_0\vec\beta_1-(\vec\beta_0\mathbf{n}_\gamma)(\vec\beta_1\mathbf{n}_\gamma))$$

$$+2\frac{\beta_1^2-(\vec\beta_1\mathbf{n}_\gamma)^2}{1-\vec\beta_1\mathbf{n}_\gamma}\left(\vec\beta_0\mathbf{n}_\gamma+\frac{E_0^2}{2mc^2E_\gamma}\right)$$

$$+2\frac{\beta_0^2-(\vec\beta_0\mathbf{n}_\gamma)}{1-\vec\beta_0\mathbf{n}_\gamma}\left(\vec\beta_1\mathbf{n}_\gamma-\frac{E_1^2}{2mc^2E_\gamma}\right)$$

$$+\frac{\beta_0^2-(\vec\beta_0\mathbf{n}_\gamma)^2}{(1-\vec\beta_0\mathbf{n}_\gamma)^2}\left(\beta_1^2\left(\frac{k_0}{k_\gamma}\right)^2+\beta_1^2-2\beta_1\frac{E_0}{E_\gamma}\right.$$

$$\left.+\frac{E_1^4}{E_\gamma^2(2mc^2)^2}+\frac{(ch)^2(\mathbf{k}_1\mathbf{k}_0-\mathbf{k}_\gamma)E_1}{mc^2E_\gamma}\right)$$

$$+\frac{\beta_1^2-(\vec\beta_1\mathbf{n}_\gamma)^2}{(1-\vec\beta_1\mathbf{n}_\gamma)^2}\left(\beta_0^2\left(\frac{k_1}{k_\gamma}\right)^2+\beta_0^2+2\beta_0^2\frac{E_1}{E_\gamma}\right.$$

$$\left.+\frac{E_0^4}{E_\gamma^2(2mc^2)^2}+2\frac{(ch)^2(\mathbf{k}_0,\mathbf{k}_1+\mathbf{k}_\gamma)}{2mc^2E_\gamma^2}E_0\right)$$

$$-2\frac{\vec\beta_0\vec\beta_1-(\vec\beta_0\mathbf{n}_\gamma)(\vec\beta_1\mathbf{n}_\gamma)}{(1-\vec\beta_0\mathbf{n}_\gamma)(1-\vec\beta_1\mathbf{n}_\gamma)}\left[\beta_0^2\beta_1^2\frac{E_0E_1}{E_\gamma^2}\right.$$

$$-\beta_1^2\frac{E_1}{E_\gamma}+\beta_0^2\frac{E_0}{E_\gamma}+\frac{E_0^2E_1^2}{E_\gamma^2(2mc^2)^2}+\frac{E_1^2}{2mc^2E_\gamma}-\frac{E_0^2}{2mc^2E_\gamma}$$

$$+(\vec\beta_1\mathbf{n}_\gamma)\left(1-\frac{E_0^2+E_1^2}{2mc^2E_\gamma}\right)+(\vec\beta_0\mathbf{n}_\gamma)\left(-1+\frac{E_0^2+E_1^2}{2mc^2E_\gamma}\right)$$

$$\left.\left.+(\vec\beta_0\vec\beta_1)\left(1+\frac{E_0^3+E_1^3}{2mc^2E_\gamma^2}\right)\right]\right\}.$$

With further analysis this formula, carried out in the range of energies which satisfy the inequality $1.1\,\mu c^2 < E_0 \ll mc^2$, takes the form

$$d\sigma = \sigma_0\frac{dE_\gamma}{\mu c^2}d\Omega_1 d\Omega_\gamma \tag{6'}$$

$$= \frac{1}{4\pi^2}\left(\frac{f^2}{hc}\right)^2\frac{e^2}{hc}\left(\frac{h}{\mu c}\right)^2\frac{E_\gamma dE_\gamma}{(\mu c^2)^2}d\Omega_1 d\Omega_\gamma$$

$$\times\left\{(\vec\beta_0-\vec\beta_1)^2+\frac{4\beta_0^2\quad\beta_1^2}{(\beta_0^2-\beta_1^2)^2}[(\beta_0^2-(\vec\beta_1\mathbf{n}_\gamma)^2)\right.$$

$$+(\beta_1^2-(\vec\beta_1\mathbf{n}_\gamma)^2)-2(\vec\beta_0\vec\beta_1-(\vec\beta_0\mathbf{n}_\gamma)(\vec\beta_1\mathbf{n}_\gamma))]$$

$$\left.+\frac{4}{\beta_0^2-\beta_1^2}[\beta_1^2(\vec\beta_0\mathbf{n}_\gamma)^2-\beta_0^2(\vec\beta_1\mathbf{n}_\gamma)^2]\right\}.$$

The angular distribution of the γ-quanta, disregarding the angle of the scattered π-meson, has the characteristic form

$$\frac{dE_\gamma}{\mu c^2}d\Omega_\gamma\int\sigma_0 d\Omega_1 \tag{7}$$

$$= \frac{1}{\pi}\frac{e^2}{hc}\left(\frac{f^2}{hc}\right)^2\left(\frac{h}{\mu c}\right)^2\frac{k_1}{k_0}\frac{dE_\gamma E_\gamma}{(\mu c^2)^2}d\Omega_\gamma$$

$$\times\left\{\beta_0^2+\beta_1^2+4\frac{\beta_0^2\beta_1^2}{(\beta_0^2-\beta_1^2)^2}\left[\frac{\beta_0^2\sin^2\vartheta_0}{(1-\beta_0\cos\vartheta_0)^2}+\frac{2}{3}\beta_1^2\right]. + 4\frac{\beta_0^2\beta_1^2}{\beta_0^2-\beta_1^2}\left(\cos^2\vartheta_0-\frac{1}{3}\right)\right\}.$$

The energy distribution of the γ-quanta is obtained after integrating over angles :

$$\frac{dE_\gamma}{\mu c^2}\int\sigma_0 d\Omega_\gamma\, d\Omega_1 \qquad (8)$$

$$=4\frac{e^2}{hc}\left(\frac{f^2}{hc}\right)^2\left(\frac{h}{\mu c}\right)^2\frac{k_1}{k_0}\frac{E_\gamma dE_\gamma}{(\mu c^2)^2}$$

$$\times\left\{\beta_0^2+\beta_1^2+\frac{2}{3}\beta_0^2\beta_1^2\frac{1}{E_\gamma^2}\left[(chk_1)^2+(chk_0)^2\right]\right\}.$$

The total cross section for the case $\mu c^2 \ll E_0 \ll mc^2$ takes the form

$$\sigma=\int\sigma_0 d\Omega_\gamma d\Omega_1\frac{dE_\gamma}{\mu c^2} \qquad (9)$$

$$=2\frac{e^2}{hc}\left(\frac{f^2}{hc}\right)^2\left(\frac{h}{\mu c}\right)^2\frac{4}{3}\left(\frac{E_0}{\mu c^2}\right)^2\ln\frac{2E_0}{\mu c^2}\ln\frac{E_0}{\delta_E}.$$

Note that the cross section for the process $\pi^+ + p \to \pi^+ + p + \gamma$ differs somewhat from the cross section for the process $\pi^- + p \to \pi^- + p + \gamma$ in the case of pseudovector coupling.

4. The cross section obtained by us, $d\sigma_T$, for bremsstrahlung from a nonrelativistic energy meson and with $E_\gamma \ll E_0$, together with the cross section for elastic scattering of π-mesons by nucleons derived in reference 5, gives the most general relationship

$$d\sigma_T=\frac{2}{3\pi}\frac{e^2}{hc}\frac{(v_0-v_1)^2}{c^2}\frac{dE_\gamma}{E_\gamma}d\sigma_{\pi\text{-el.}} \qquad (10)$$

An analogous relation has been obtained earlier in references 3-5.

The total cross section for the process of bremsstrahlung from the scattering of π-mesons by nucleons for both types of coupling of the meson to the nucleon has the order of magnitue of 10^{-28} cm^2 for $E_0 \sim 2\mu c^2$, for $\frac{f^2}{hc}=\frac{\mu}{2m}\frac{g^2}{hc}\approx\frac{1}{6}$ and for the energy interval $\delta_E = 1$ mev.

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The Role of Isobaric States of Nucleons in Meson Creation

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As is known[1], there is experimental evidence that the isobaric states of the nucleons play an important role in the process of meson production in nucleon-nucleon collisions. A series of authors have taken the point of view that the production of pions proceeds entirely through an isobaric state[2-4]. However, it has not been excluded that direct production plays a significant role in such processes.

Let us examine the experiments on production of charged pions in the reaction $Be^9 + p$ [2]. In these experiments the ratio $\rho = \pi^+/\pi^-$ (π^+ and π^- indicating the number of π^+ and π^- mesons formed) is equal to 6 at an energy of 1 bev, and to 1.8 at 2.3 bev. In the paper of Peaslee[2] there is a discussion of the possibility of explaining such a dependence of ρ on energy by assuming that the production of pions proceeds only through an isobaric state. At an energy of 2.3 bev, in order to calculate ρ, one must evaluate matrix elements or else make definite assumptions about them[2]. At 1 bev, however, there is not enough energy to produce real excitation of two isobaric states and one must therefore assume that the production of pions goes only by the excitation of one isobaric state. Then, assuming only the hypothesis of charge independence, it is easy to calculate ρ. The result obtained by Peaslee[2] for ρ is equal to 6, agreeing well with experiment. However, in this work an error was made, as pointed out by the author himself in a following paper[3]. The corrected value