Calculation of the Ultimate Thickness of the Emulsion Layer in the Investigation of Nuclear Processes with the Aid of the Photographic Method

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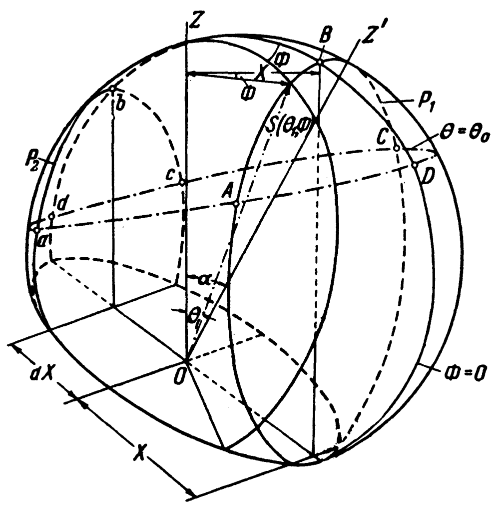
IN a previous study[1] we obtained an analytical expression for the relation of the general number of those secondary particles which were formed during a nuclear reaction in the photoemulsion to the number of those secondary particles whose traces are to be found in their entirety within the emulsion layer. It was assumed thereby that the primary particles fall parallel to the emulsion layer. Below are presented analogous formulas for the more general case where the primary particles fall at a given angle α toward the surface of the emulsion layer.

In the following diagram is shown the reciprocal arrangement of the emulsion layer and its lateral surfaces P_1, P_2 of the surface $S = S(\vartheta)$ representing the geometrical locus of the ends of the traces of secondary particles which flew out at an angle ϑ toward the axis OZ', equal to the direction of incidence of the primary particles and of the cone K with its apex at the point where the secondary particles are formed, and its axis directed along the projection of the OZ'-axis on the plane P_0. The apex angle $2\theta_0$ of the cone as well as the incline α of its axis, are given by the conditions of the experiment.

We designate, as before[1], by σ and σ' the magnitudes which are proportional, respectively, to the full number of the secondary particles diffusing or scattering within cone K, and to the number of secondary particles whose traces, remaining within cone K, terminate outside the emulsion layer. The sought-for magnitude of the correction w is then determined by the formula[1] $w = 1 - \sigma'/\sigma$. If we designate by $f(\vartheta)$ the distribution of the intensity diffusion (scattering) of the secondary particles, then we obtain an expression for σ shown by the following formula:

$$\sigma = \int_0^{2\pi} d\Phi \int_0^{\theta_0} f(\vartheta)\sin\theta\, d\theta, \quad (1)$$

where



Reciprocal arrangement of the surface $S = S(\Theta, \Phi)$ and of the cone K in the emulsion layer.

$$\cos\vartheta = \cos\theta\cos\alpha + \sin\theta\sin\alpha\cos\Phi. \quad (2)$$

We now formulate the expressions for σ' applicable to the following three cases:

I. The thickness d of the emulsion layer is minute and satisfies the condition: $d < \delta = \min S \times (\theta_0, \Phi)\sin\theta_0$,

$$\frac{d}{2}\sigma' = \int_0^{\pi/2} F_1(\theta_0, \Phi)\,d\Phi - \int_{\pi/2}^{\pi} F_1(\theta_0, \Phi)\,d\Phi - \int_0^{\psi_2} F_2(\theta_0, \Phi)\,d\Phi + \int_{\psi_3}^{\pi} F_3(\theta_0, \Phi)\,d\Phi + \int_0^{\psi_2} F_2(\overline{\theta}, \Phi)\,d\Phi - \int_{\psi_3}^{\pi} F_3(\theta', \Phi)\,d\Phi, \quad (3)$$

where the following designations are introduced:

$$F_1(\theta, \Phi) = \cos\Phi\,[M(\theta_0, \Phi) - M(0, \Phi)];$$

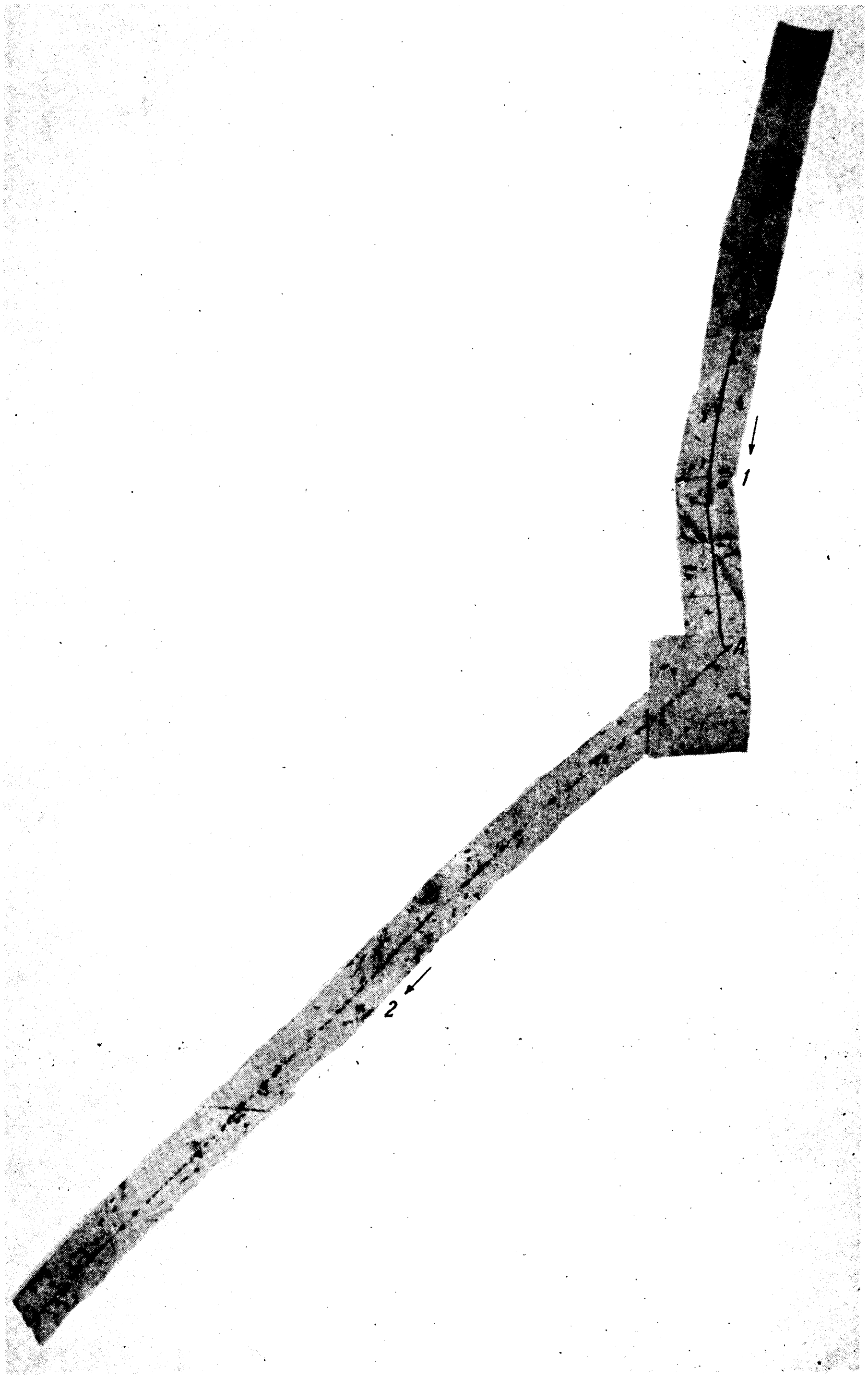
$$F_2(\theta, \Phi) = \{\cos\Phi\cdot M(\theta, \Phi) - d\,\Lambda(\theta, \Phi)\};$$

$$F_3(\theta, \Phi) = \{\cos\Phi\, M(\theta, \Phi) + d\, N(\theta, \Phi)\}.$$

In this case, the magnitudes $\psi_2, \psi_3, \overline{\theta}, \theta'$ are determined, respectively, from the following equations:

$$\cos\psi_2 = \frac{d}{S(\theta_0, \psi_2)\sin\theta_0}; \quad \cos\psi_3 = \frac{-d}{S(\theta_0, \psi_3)\sin\theta_0};$$

$$\sin\overline{\theta} = \frac{d}{S(\overline{\theta}, \Phi)\cos\Phi}; \quad \sin\theta' = \frac{-d}{S(\theta', \Phi)\cos\Phi}.$$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II. The thickness d of the emulsion layer satisfies the condition:

$$d > \Delta = \max S(\theta_0, \Phi) \sin \theta_0.$$

In this case, $\psi_2 = 0$ and $\psi_3 = \pi$ and the expression for σ' has the aspect presented by the next formula:

$$\frac{d}{2} \sigma' = \int_0^{\pi/2} F_1(\theta_0, \Phi)\, d\Phi + \int_{\pi}^{\pi/2} F_1(\theta_0, \Phi)\, d\Phi. \quad (4)$$

III. If the thickness d of the emulsion layer satisfies the condition $\delta < d < \Delta$, then we have either $\psi_2 = 0$; $\psi_3 < \pi$, or $\psi_3 = \pi$ and $\psi_2 > 0$; consequently, the corresponding integrations (quadratures) in Eq. (3) disappear.

In all the enumerated formulas, the functions $M(\theta, \Phi)$ and $N(\theta, \Phi)$ denote, respectively, the following equations:

$$M(\theta, \Phi) = \int f(\vartheta)\, S(\vartheta) \sin^2 \theta\, d\theta;$$

$$N(\theta, \Phi) = \int f(\vartheta) \sin \theta\, d\theta.$$

If we approximate $f(\vartheta)$ and $S(\vartheta)$ with polynomials with regard to $\cos \vartheta$[2,3], then the calculation of the first four integrations (quadratures) in Eq. (3) can be performed easily and exactly. The last two integrations (quadratures), however, can be calculated by means of any formula of mechanical quadratures.

In this way formula (3) [accordingly, Eq. (4) in the case II], together with formula (1), fully solve the problem of finding the correction $w = 1 - (\sigma'/\sigma)$.

From the cited formulas it is easy to obtain the correction w for the case which was examined earlier[1]. For this purpose we posit $\alpha = 0$, then $\vartheta \equiv \theta$ and $S(\vartheta) \equiv S(\theta)$. The expressions for σ' assume, therefore, the following aspect. In case I:

$$\frac{d}{4} \sigma' = M(\theta_0) - M(0) + d \psi_2 N(\theta_0) - M(\theta_0) \sin \psi_2 + \int_0^{\psi_2} F(\bar{\theta})\, d\Phi. \quad (5)$$

Accordingly, in case II, we obtain:

$$\sigma' = \frac{4}{d} [M(\theta_0) - M(0)]. \quad (6)$$

Formulas (5) and (6) are simpler for practical calculations than the formulas cited earlier[1].

Let us remark that the choice of the layer thickness d and of the angle θ_0 permits us to reduce the majority of practical problems to case II. If, at the same time, $f(\vartheta)$ and $S(\vartheta)$ are given in the form of polynomials with regard to $\cos \vartheta$, then the computation of the correction w according to formulas (1) and (4) does not present any difficulties whatever.

I wish to express here my appreciation to Dr. Westmeier for his formulation of the problem and to A. Benet for a certain preliminary analysis.

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[2] L. E. Darlington et al, Phys. Rev. 90, 1049 (1953)

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An Elementary Derivation of the Formula for the Electromagnetic Energy in a Dispersive Medium

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LET us suppose that a substance of dielectric and magnetic susceptibility $\epsilon(\omega)$ and $\mu(\omega)$, respectively, fills a parallel plate condenser of capacity $C(\omega) = \epsilon(\omega) C_0$ and a thin solenoid of inductance $L(\omega) = \mu(\omega) L_0$, thus forming an oscillatory circuit. The free space values C_0 and L_0 are so chosen that the frequency ω is the natural frequency of the system given by: $\omega^2 = [L(\omega) C(\omega)]^{-1}$. Let us suppose that at $t < 0$ undamped oscillations take place in the circuit. The current through the solenoid, $Ie^{i\omega t}$, and the potential across the condenser, $Ve^{i\omega t}$, due to these oscillations satisfy the well-known condition $V = -i \sqrt{L/C}\, I$.

At $t = 0$ let us insert into the circuit a vanishingly small resistance R; then, at $t > 0$, the oscillations will have a complex frequency $\tilde{\omega}$, determined by the relation:

$$\tilde{\omega} L(\tilde{\omega}) - 1/\tilde{\omega} C(\tilde{\omega}) = iR.$$

It is easy to see that for the case of $R \to 0$, the solution of the above equation is $\tilde{\omega} = \omega + i\delta$, where δ and R are related by the equation:

$$\frac{R}{\delta} = \frac{d}{d\omega}(\omega L) + \frac{1}{\omega^2 C^2} \frac{d}{d\omega}(\omega C) = \frac{d}{d\omega}(\omega L) + \frac{L}{C} \frac{d}{d\omega}(\omega C). \quad (1)$$