Clearly, the initial energy of the circuit, W, is equal to the total dissipated heat energy.

$$W = \int_0^\infty \frac{1}{2} R \mid I \mid^2 e^{-2\delta t} dt = \frac{R}{4\delta} \mid I \mid^2.$$

Substituting Eq. (1) into the above relation, and noting that $\frac{L}{C}|I|^2 = |V|^2$, we obtain:

$$W = \frac{1}{4}\left[\mid I \mid^2 \frac{d}{d\omega}(\omega L) + \mid V \mid^2 \frac{d}{d\omega}(\omega C)\right] \tag{2}$$

$$= \frac{1}{4} L_0 \mid I \mid^2 \frac{d}{d\omega}(\omega\mu) + \frac{1}{4} C_0 \mid V \mid^2 \frac{d}{d\omega}(\omega\varepsilon).$$

However, $L_0 |I|^2 = \frac{1}{4\pi} |H|^2 \tau_m$, $C_0 |V|^2 = \frac{1}{4\pi} |E|^2 \tau_e$, where H is the amplitude of the magnetic vector, τ_m is the volume of the solenoid, E is the amplitude of the electric vector, and τ_e is the volume of the capacitor. Hence, Eq. (2) can be written as follows:

$$W = \frac{\mid H \mid^2}{16\pi} \frac{d}{d\omega}(\omega\mu)\tau_m + \frac{\mid E \mid^2}{16\pi} \frac{d}{d\omega}(\omega\varepsilon)\tau_e. \tag{3}$$

From this we can conclude immediately, that the average electric and magnetic energy densities of a sinusoidal field in a dispersive medium are given by:

$$w_e = \frac{\mid E \mid^2}{16\pi} \frac{d}{d\omega}(\varepsilon\omega); \quad w_m = \frac{\mid H \mid^2}{16\pi} \frac{d}{d\omega}(\mu\omega). \tag{4}$$

These relations were obtained in references 1 and 2 by means of the Fourier integral method.

[1] S. M. Rytov and F. S. Iudkevich, J. Exper. Theoret. Phys. USSR 10, 887 (1940)

[2] S. M. Rytov, J. Exper. Theoret. Phys. USSR 17, 930 (1947)

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The Second Viscosity of Monatomic Gases

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IT is well-known that monatomic gases, which obey Boltzmann statistics and which possess an energy spectrum of the form

$$\varepsilon = p^2/2m, \tag{1}$$

do not have a second viscosity[1]. We show here that this result also takes place if the monatomic gas* obeys quantum statistics (Fermi or Bose), provided that the energy is a power function of the momentum of the particle.

We write the kinetic equation for the distribution function n of the gas under consideration:

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \mathbf{r}}\frac{\partial H}{\partial \mathbf{p}} - \frac{\partial n}{\partial \mathbf{p}}\frac{\partial H}{\partial \mathbf{r}} = I(n). \tag{2}$$

Here $H(\mathbf{p}, \mathbf{r})$ is the Hamiltonian of the particle, $I(n)$ is the collision integral. Let there be macroscopic motion of the gas with velocity $\mathbf{u}$, for which $\operatorname{div} \mathbf{u} \neq 0$. In this case the unexcited distribution function is equal to

$$n = \frac{1}{\exp\{(\varepsilon - \mu - \mathbf{pu})/T\} + 1} \tag{3}$$

(the sign is negative in the case of Bose statistics, positive for Fermi statistics, $\epsilon(p)$ is the energy of the particle in the quiescent gas, μ is the chemical potential. In the absence of an external field the equality $H = \epsilon(p)$ holds. We substitute the n of Eq. (3) in the left side of the kinetic equation; for simplicity we shall consider the case here for which $\mathbf{u} = 0$ (but $\operatorname{div} \mathbf{u} \neq 0$). Moreover, inasmuch as we are interested only in the second viscosity, i.e., in terms in the momentum flux which are porportional to $\operatorname{div} \mathbf{u}$, it is reasonable that temperature gradients can be considered absent. For such a case we obtain for the left side of the kinetic equation

$$n'\left\{-\frac{\partial}{\partial t}\frac{\mu}{T} - \frac{\varepsilon}{T^2}\frac{\partial T}{\partial t} - \frac{1}{T}\frac{\partial \varepsilon}{\partial \mathbf{p}}\nabla(\mathbf{p}, \mathbf{u})\right\} = I(n). \tag{4}$$

We now consider the equation of continuity for the entropy σ and the density ρ at $\mathbf{u} = 0$, which have the following forms:

$$\frac{\partial \rho}{\partial t} + \rho \operatorname{div} \mathbf{u} = 0, \quad \frac{\partial \sigma}{\partial t} = 0 \tag{5}$$

(adiabatic condition).

In this case, Eq. (4) transforms to

$$n'\left\{\left[\rho\frac{\partial}{\partial \rho}\left(\frac{\mu}{T}\right)_\sigma + \frac{\varepsilon}{T^2}\left(\frac{\partial T}{\partial \rho}\right)_\sigma \rho - \frac{1}{3T} p \frac{\partial \varepsilon}{\partial p}\right] \operatorname{div} \mathbf{u} - \frac{1}{2T}\left(\frac{\partial \varepsilon}{\partial p_i} p_k - \frac{1}{3}\delta_{ik}\frac{\partial \varepsilon}{\partial p} p\right) \times \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial u_l}{\partial x_l}\right)\right\} = I(n). \tag{6}$$

The second viscosity is absent in this case if the expression in square brackets on the left side of Eq. (6) vanishes. We show that this takes place if the energy ϵ is proportional to some power of the momentum

$$\varepsilon = ap^n. \tag{7}$$

* By a monatomic gas we understand a set of particles each of which is characterized by three degrees of freedom (the three component vector momentum p).

The potential Ω is defined by the following relation[2]

$$\Omega = \mp kTV \int \frac{4\pi^2 p dp}{(2\pi\hbar)^3} \ln\left(1 \pm \exp\left\{\frac{\mu - \varepsilon}{kT}\right\}\right). \quad (8)$$

Substituting Eq. (7) for the energy in Eq. (8), and carrying out the substitution of variables $p \to p\, T^{1/n}$, we obtain

$$\Omega = -pV = VT^{1+3/n} f(\mu/T), \quad (9)$$

where f is some function of a single argument. We take advantage now of a thermodynamic identity for Ω and compute the entropy σ. Inasmuch as Ω is a homogeneous function of μ and T of order $1 + 3/n$, we have

$$\sigma = \frac{(\partial\Omega/\partial T)_{V,\mu}}{(\partial\Omega/\partial\mu)_{T,V}} = \varphi\left(\frac{\mu}{T}\right). \quad (10)$$

Thus, for adiabatic processes (σ = const) the relation μ/T is a constant quantity[2], i.e.,

$$\frac{\partial}{\partial\rho}\left(\frac{\mu}{T}\right)_\sigma = 0. \quad (11)$$

Furthermore, from the relation $N = -(\partial\Omega/\partial\mu)_{T,V}$ it follows that for adiabatic processes, $VT^{3/n}$ = const, and, consequently,

$$\left(\frac{\partial T}{\partial\rho}\right)_\sigma = \frac{n}{3}\frac{T}{\rho}. \quad (12)$$

By considering Eqs. (11) and (12), we can convince ourselves that the following expression holds:

$$\rho\frac{\partial}{\partial\rho}\left(\frac{\mu}{T}\right)_\sigma + \frac{1}{T}\left(\frac{\varepsilon}{T}\left(\frac{\partial T}{\partial\rho}\right)_\sigma - \frac{1}{3}\frac{\partial\varepsilon}{\partial p}p\right) = 0. \quad (13)$$

Thus, in the case under consideration, ($\epsilon = ap^n$) the second viscosity vanishes. Thus, for example, the second viscosity is equal to zero in a photon gas ($\epsilon = cp$), and also in a monatomic gas in the ultra-relativistic case. Evidently, the second viscosity will vanish in the liquid isotope of helium with mass 3 (He^3), which represents a set of Fermi particles. It is easy to see that condition (7) is necessary in order that the second viscosity equal zero. Actually, according to Eq. (6), if the second viscosity vanishes, then it is necessary that for all values of momenta, the following expression vanishes:

$$\varepsilon\left(\frac{\partial T}{\partial\rho}\right)_\sigma\frac{\rho}{T} - \frac{1}{3}p\frac{\partial\varepsilon}{\partial p} = 0, \quad (14)$$

or also,

$$\frac{1}{3}\frac{\partial\ln\varepsilon}{\partial\ln p} = \left(\frac{\partial\ln T}{\partial\ln\rho}\right)_\sigma. \quad (15)$$

Consequently, the energy is proportional to a power of the momentum, for which the power n is given by

$$n = 3(\partial\ln T/\partial\ln\rho)_\sigma. \quad (16)$$

[1] L. D. Lanadu and E. M. Lifshitz, *The Mechanics of Continuous Media*, Moscow, 1944

[2] L. D. Landau and E. M. Lifshitz, *Statistical Physics*, 1951

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The Surface Energy Associated with a Tangential Velocity Discontinuity in Helium II

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ONE of the most essential problems of the theory of superfluidity is, as we have already had occasion to point out[1,2], the question concerning the character of the tangential discontinuity in the velocity v_s of the superfluid component of helium, at the boundary between the fluid and a wall. The existence of such a discontinuity follows from the fact that the helium atoms adhere to the wall (a solid body), while at the same time, from the macroscopic viewpoint, i.e., in the immediate vicinity of the wall, the tangential component of v_s at the wall is not equal to zero[3]. The well-known proof [3] for the possibility of superfluidity consists in this case of the establishment of the conditions for stability of the discontinuity at the wall. The discontinuity and the superfluid flow are completely stable, provided the flow velocity $v < v_c$:

$$v_c = [\varepsilon(p)/p]_{\min}, \quad (1)$$

where $\epsilon(p)$ and p are the energy and the momentum of the "excitations" which may appear in the liquid (both quantities are measured in the coordinate system associated with the liquid). Within the framework of the microscopic representation, the thickness of the discontinuity is clearly not equal to zero, but is in order of magnitude equivalent to the atomic distance $a \sim N^{1/3} \simeq 3.5 \times 10^{-8}$ ($N = 2.2 \times 10^{22}$ is the concentration of atoms in liquid helium). Analogous discontinuities, in accordance with references 4 and 5, may also exist within the bulk helium II, in which case the situation is even simpler, since the question of the possible influence of the wall material upon the character of the discontinuity does not arise*. A certain surface energy σ must be associated