The potential Ω is defined by the following relation[2]

$$\Omega = \mp kTV \int \frac{4\pi^2 p dp}{(2\pi\hbar)^3} \ln\left(1 \pm \exp\left\{\frac{\mu - \varepsilon}{kT}\right\}\right). \tag{8}$$

Substituting Eq. (7) for the energy in Eq. (8), and carrying out the substitution of variables $p \to p\, T^{1/n}$, we obtain

$$\Omega = -pV = VT^{1+3/n} f(\mu/T), \tag{9}$$

where f is some function of a single argument. We take advantage now of a thermodynamic identity for Ω and compute the entropy σ. Inasmuch as Ω is a homogeneous function of μ and T of order $1 + 3/n$, we have

$$\sigma = \frac{(\partial\Omega/\partial T)_{V,\mu}}{(\partial\Omega/\partial\mu)_{T,V}} = \varphi\left(\frac{\mu}{T}\right). \tag{10}$$

Thus, for adiabatic processes (σ = const) the relation μ/T is a constant quantity[2], i.e.,

$$\frac{\partial}{\partial\rho}\left(\frac{\mu}{T}\right)_\sigma = 0. \tag{11}$$

Furthermore, from the relation $N = -(\partial\Omega/\partial\mu)_{T,V}$ it follows that for adiabatic processes, $VT^{3/n}$ = const, and, consequently,

$$\left(\frac{\partial T}{\partial\rho}\right)_\sigma = \frac{n}{3}\frac{T}{\rho}. \tag{12}$$

By considering Eqs. (11) and (12), we can convince ourselves that the following expression holds:

$$\rho\frac{\partial}{\partial\rho}\left(\frac{\mu}{T}\right)_\sigma + \frac{1}{T}\left(\frac{\varepsilon}{T}\left(\frac{\partial T}{\partial\rho}\right)_\sigma - \frac{1}{3}\frac{\partial\varepsilon}{\partial p}p\right) = 0. \tag{13}$$

Thus, in the case under consideration, ($\epsilon = ap^n$) the second viscosity vanishes. Thus, for example, the second viscosity is equal to zero in a photon gas ($\epsilon = cp$), and also in a monatomic gas in the ultra-relativistic case. Evidently, the second viscosity will vanish in the liquid isotope of helium with mass 3 (He^3), which represents a set of Fermi particles. It is easy to see that condition (7) is necessary in order that the second viscosity equal zero. Actually, according to Eq. (6), if the second viscosity vanishes, then it is necessary that for all values of momenta, the following expression vanishes:

$$\varepsilon\left(\frac{\partial T}{\partial\rho}\right)_\sigma\frac{\rho}{T} - \frac{1}{3}p\frac{\partial\varepsilon}{\partial p} = 0, \tag{14}$$

or also,

$$\frac{1}{3}\frac{\partial\ln\varepsilon}{\partial\ln p} = \left(\frac{\partial\ln T}{\partial\ln\rho}\right)_\sigma. \tag{15}$$

Consequently, the energy is proportional to a power of the momentum, for which the power n is given by

$$n = 3(\partial\ln T/\partial\ln\rho)_\sigma. \tag{16}$$

[1] L. D. Lanadu and E. M. Lifshitz, *The Mechanics of Continuous Media*, Moscow, 1944

[2] L. D. Landau and E. M. Lifshitz, *Statistical Physics*, 1951

Translated by R. T. Beyer
185

The Surface Energy Associated with a Tangential Velocity Discontinuity in Helium II

V. L. GINZBURG

P. N. Lebedev Institute of Physics, Academy of Sciences, USSR

(Submitted to JETP editor April 21, 1955)

J. Exper. Theoret. Phys. USSR 29, 244-246 (August, 1955)

ONE of the most essential problems of the theory of superfluidity is, as we have already had occasion to point out[1,2], the question concerning the character of the tangential discontinuity in the velocity v_s of the superfluid component of helium, at the boundary between the fluid and a wall. The existence of such a discontinuity follows from the fact that the helium atoms adhere to the wall (a solid body), while at the same time, from the macroscopic viewpoint, i.e., in the immediate vicinity of the wall, the tangential component of v_s at the wall is not equal to zero[3]. The well-known proof[3] for the possibility of superfluidity consists in this case of the establishment of the conditions for stability of the discontinuity at the wall. The discontinuity and the superfluid flow are completely stable, provided the flow velocity $v < v_c$:

$$v_c = [\varepsilon(p)/p]_{\min}, \tag{1}$$

where $\epsilon(p)$ and p are the energy and the momentum of the "excitations" which may appear in the liquid (both quantities are measured in the coordinate system associated with the liquid). Within the framework of the microscopic representation, the thickness of the discontinuity is clearly not equal to zero, but is in order of magnitude equivalent to the atomic distance $a \sim N^{1/3} \simeq 3.5 \times 10^{-8}$ ($N = 2.2 \times 10^{22}$ is the concentration of atoms in liquid helium). Analogous discontinuities, in accordance with references 4 and 5, may also exist within the bulk helium II, in which case the situation is even simpler, since the question of the possible influence of the wall material upon the character of the discontinuity does not arise*. A certain surface energy σ must be associated with the velocity discontinuity, since this discontinuity represents a local disturbance of the superflow, requiring the expenditure of energy[5]. As it seems to us, such a conclusion is even more natural from the quantum viewpoint, inasmuch as the Ψ-function for the He atoms changes at the discontinuity within a distance $\sim a$; whence the average additional kinetic energy associated with unit surface area of the discontinuity**,

$$\sigma \sim \frac{\hbar^2 N}{2m_{\mathrm{He}}a^2}a \sim \frac{\hbar^2}{2m_{\mathrm{He}}a^4} \sim 5\times 10^{-2}\ \frac{\mathrm{erg}}{\mathrm{cm}^2} \qquad (2)$$

The expression (2) for σ was derived in reference 4 on dimensional grounds; in reference 5 there was obtained from similar considerations the formula

$$\sigma \sim \rho_s (kT_\lambda U^4/\rho)^{1/3}, \qquad (3)$$

in which ρ_s and ρ are, respectively, the density of the superfluid component of the helium and the total density of the helium, $T_\lambda = 2.19^0$, and U is the second sound velocity. Setting $\rho_s \sim \rho \sim 0.15$ and $U = 2\times 10^3$ cm/sec, we obtain $\sigma \sim 5\times 10^{-2}$; i.e., the approximations (2) and (3) are essentially in agreement, as was to be expected. The nature of the approximations is such that even a value of $\sigma \sim 5\times 10^{-3}$ is compatible with them [this value is obtained from (2) when the thickness of the transition layer $\sim 10a \sim 3\times 10^{-7}$ cm]. Calculation of the surface energy σ at the velocity discontinuities within the bulk helium II is essential to an understanding of the peculiarities arising for velocities greater than the critical velocity v_c, and, in particular, for rotation of helium in a beaker[4,5]. As was pointed out long ago by Landau, the formula $v_c \sim \sqrt{\sigma/\rho_s d}$, where d is the width of the slit (or capillary) through which helium II flows, may be derived on dimensional grounds.

If we consider Eq. (2), it becomes clear that the formula $v_c \sim (\hbar/m_{\mathrm{He}})\sqrt{1/ad}$ derived in reference 4 agrees in essence with the preceeding. It is not, perhaps, superfluous to point out that an analogous result is obtained by application of the fundamental criterion (1). We assume, actually, that within helium II there may form a region of volume V and surface area S, isolated from the remainder of the liquid. Then $p = Mu$ and $\epsilon(p) = \frac{1}{2}Mu^2 + \sigma S$, where $M = \rho_s V$ is the corresponding mass and u is the velocity of motion of the region in the coordinate system associated with the liquid. In this case, in accordance with Eq. (1)

$$v_c = \left[\frac{\frac{Mu^2}{2} + \sigma S}{Mu}\right]_{\min} = \sqrt{\frac{2\sigma}{\rho_s}\left(\frac{S}{V}\right)_{\min}} \qquad (4)$$

$$\sim \sqrt{\frac{\sigma}{\rho_s d}} \sim \frac{\hbar}{m_{\mathrm{He}}}\sqrt{\frac{1}{ad}} \sim \frac{0.1 \text{ to } 1}{\sqrt{d}}\ \frac{\mathrm{cm}}{\mathrm{sec}}$$

where d is the width of the slit or capillary, so that $(S/V)_{\min} \sim 1/d$. Here $u_{\min} \sim v_c$; i.e., the region formed (playing the role of the "elementary excitation" of reference 3) is at rest relative to the capillary walls. The minimal energy $\epsilon_{\min} = \frac{1}{2}Mu^2_{\min} + \sigma S = 2\sigma S$. The relation (4), within the limits of the very low accuracy achieved, agrees with experiment[7]; in any event, it agrees better than the relation $v_c \sim \hbar/m_{\mathrm{He}}d$ (cf. reference 2).

In view of all that has just been said, it now appears to us that the most natural explanation for the properties of the critical processes in helium is to be sought, neither through consideration of the quantum character of the excitations[2,7] nor through investigation of the surface excitations[2], but rather in the results of a study of the possibilities for formation of discontinuities[4,5].

The principle reason for the present letter, however, is the wish to emphasize another circumstance --- the necessity for calculating the surface energy σ' associated with the velocity discontinuity in the vicinity of the boundary between the helium II and a solid wall. According to all of the data, as has been said, such a discontinuity must necessarily exist, and therefore, the existence of a surface energy $\sigma' \sim \sigma$ is difficult to doubt. In order that the discontinuity should not leave the wall (which, apparently, is the case for $v < v_c$), it is necessary for the inequality $\sigma' < \sigma$ to be fulfilled (σ being the surface energy for a discontinuity within the bulk helium II)***. The value of σ' may to a certain extent depend upon the material of the wall, which is of interest from the standpoint of the possibility, considered below, of determining the influence of the surface energy σ' upon the flow of helium II. This influence should, first of all, manifest itself by the existence of a certain minimum energy $\sigma' S$ required for the setting in motion of a solid body of surface area S in helium II. It is obvious that a similar expenditure of energy should occur as well in the establishment of flow through slits and capillaries. Here, clearly, we are concerned with an effect analogous to that seen in the presence of so-called dry friction between solid surfaces. The situation is more complicated in the case of non-steady motion, since the disappearance of the surface energy σ' (the same applies as well to σ) cannot follow instantaneously upon the stopping of the body, and, apparently, metastable "discontinuities" can exist corresponding to zero velocity of relative motion between the wall and the helium II. The problem of the mechanism and the relaxation time τ, and also of the nature of such "discontinuities" (these appear to be strata in which the Ψ-function is perturbed relative to the corresponding lowest state), remains unclear Thus, as regards the influence of the energy σ' upon nonsteady flow, it is difficult to make any but purely qualitative statements. For example, in determining the viscosity of helium II from the damping of the oscillations of a disk, this damping may for $v < v_c$ be explained partially by the formation of discontinuities at the surface of the disk. Here, in the quasistationary case, an energy $4\sigma' S$ must be expended on the formation of discontinuities during the period θ of the oscillations, S being the surface area of the disk. Actually, however, in the experiments[8,9] the damping corresponding to $\sigma \sim 10^{-2}$ is 4 to 5 orders of magnitude smaller than the indicated quasistationary value, which demonstrates that the inequality $\tau \gg \theta \sim 10$ sec is fulfilled. Nevertheless, it is not impossible that the contribution to the damping associated with the formation of discontinuities is substantial, and that with it is to be connected the disparity between the results of the experiments[8,9] with oscillating disks and the measurements of the viscosity from the moment developed in rotating two coaxial cylinders relative to one another[10] (in the latter case the process is stationary, and the damping must be due solely to the viscosity); in reference 10 lower values are obtained for the damping at a low temperature, than in references 8 and 9, which accords with what has been said. The effect of the discontinuities may also be responsible for the peculiarities in the damping of a disk at large amplitudes[9]. In virtue of what has been stated, it seems to us that discontinuites in the flow velocity of helium II in the vicinity of walls deserve close attention.

The author is obliged to Academician L. D. Landau and to Professor E. M. Lifshitz for their discussion of this problem.

* At the same time, the hypothesis of the possible existence of velocity discontinuities within the bulk helium II becomes especially likely when it is considered that such discontinuities are known to exist in the vicinity of the wall.

** We note that in the theory of superconductivity[6] the surface energy σ_{ns} at the boundary between the superconducting and normal phases can be successfully evaluated from similar considerations. Thus, assuming that the thickness of the transition layer between the phases δ_0/κ (cf reference 6), we obtain

$$\sigma_{ns} \sim \frac{\hbar^2 n_s (\delta_0/\varkappa)}{2m(\delta_0/\varkappa)^2} \sim \frac{H_{km}^2 \delta_0}{2\pi\varkappa},$$

which agrees with more exact calculations[6] (for the symbols, reference 6, noting that $n_s = mc^2/4\pi e^2 \delta_0^2$ and $\kappa^2 = (2e^2/\hbar^2 c^2) H_{km}^2 \delta_0^4$).

*** If σ' and σ depend on v_s [for example, if $\sigma' = \sigma'(v_s \to 0) + bv_s^2$], then, in principle, it is possible for critical processes to develop, in conjunction with the fact that for $v \geq v_c$, $\sigma' \geq \sigma$. We note further that if we set $\sigma = \propto u^2$ (cf. reference 4), then $v_c = 0$. This circumstance, together with a number of others, indicates that (provided that all of the ideas under consideration are correct) the surface energies σ and σ' tend to a limit different from zero as $v_s \to 0$.

[1] V. L. Ginzburg, J. Exper. Theoret. Phys. USSR **14**, 135 (1944)

[2] V. L. Ginzburg, Dokl. Akad. Nauk SSSR **69**, 161 (1949)

[3] L. D. Landau, J. Phys. USSR **5**, 71 (1941)

[4] N. F. Mott, Phil. Mag. **40**, 61 (1949)

[5] L. D. Landau and E. M. Lifshitz, Dokl. Akad. Nauk SSSR **100**, 669 (1955)

[6] V. L. Ginzburg and L. D. Landau, J. Exper. Theoret. Phys. USSR **20**, 1064 (1950)

[7] J. G. Daunt and R. S. Smith, Revs. Mod. Phys. **26**, 172 (1954)

[8] E. L. Andronikashvili, J. Exper. Theoret. Phys. USSR **18**, 429 (1948)

[9] A. C. Hollis-Hallett, Proc. Roy. Soc. (London) A210, 404 (1952)

[10] A. C. Hollis-Hallett, Proc. Camb. Phil. Soc. **49**, 717 (1953)

Translated by S. D. Elliott
186

The Effective Density of Rotating Liquid Helium II

I. M. LIFSHITZ AND M. I. KAGANOV
Physico-Technical Institute, Academy of Sciences, Ukrainian SSR, Kharkov

(Submitted to JETP editor March 31, 1955)
J. Exper. Theoret. Phys. USSR **29**, 257-258
(August, 1955)

LANDAU and Lifshitz have shown[1] that in the rotation of a vessel containing He II, the normal part of the helium rotates as a whole, while