third term is 4.4 times smaller than the second, etc. At greater depths, the convergence is much poorer.

From Figure 4 we see that at $\theta = 11.5°$, (just before the twice scattered radiation drops out), the curve for the angular distribution actually rises a little, which can be explained by the methods given above.

The investigation of the angular distribution and energy spectrum of scattered γ - radiation at great depths of penetration in matter will be carried out in another paper.

In conclusion, I must express my profound gratitude to Prof. S. Z. Belen'kii for valuable suggestions, and to Acad. I. E. Tamm and Prof. E. L. Feinberg for supervising the work.

Translated by M. Hamermesh
225

SOVIET PHYSICS JETP VOLUME 2, NUMBER 2 MARCH, 1956

Angular Distribution of Gamma Rays at Great Depths of Penetration in Matter

V. I. OGIEVETSKII

(Submitted to JETP editor June 25, 1954)
J. Exper. Theoret. Phys. USSR 29, 464-472 (October, 1955)

The angular and energy distribution of γ-rays at great depths of penetration in matter is found for the cases of constant and linear dependence of the absorption coefficient on wavelength. The passage of γ-rays through an inhomogeneous medium is examined.

---

1. THE qualitative nature of the angular distribution at great depths of penetration depends strongly on the behavior of the γ-ray absorption coefficient. If the initial energy of the γ-ray is less than that at which the absorption coefficient is a minimum, then on the average, γ-rays scattered through small angles will be more penetrating than those scattered through large angles. With increase in the depth of penetration, the angular distribution will become narrower, or at any rate no wider. The small angle approximation applicable to scattering at energies of the order of several mev remains valid for great depths of penetration.

In the other case, where the absorption coefficient increases with energy, photons scattered through large angles will be more penetrating than those scattered through small angles. As the depth of penetration increases, the angular distribution will be smeared out, and the small angle approximation for each Compton scattering becomes incorrect.

In this respect, the results of reference 1, where the γ-ray energy spectrum at great depths of penetration is calculated using the small angle approximation, arouse some doubt.

In the present article, using the polynomial expansion of Spencer and Fano[2], the energy and angular distribution of γ-rays at great depths of penetration are found for the case of constant absorption coefficient (γ-rays of high energy in light elements, see reference 3), and for an absorption coefficient which increases linearly with wavelength. In these cases the small angle approximation is applicable, as can be seen from the final result.

In the case of constant absorption coefficient, the angular distribution tends to a gaussian one, although the approach to a gaussian distribution takes place significantly slower than indicated in reference 4. At the end of the article these results and those of reference 3 are generalized for an inhomogeneous medium. Below we shall use equations of radiation transport and the notation of the preceeding article[3].

---

[1] U. Fano, Phys. Rev. **76**, 739 (1949); U. Fano, H. Hurwitz, Jr. and L. V. Spencer, Phys. Rev. **77**, 425 (1950)

[2] L. V. Spencer and U. Fano, Phys. Rev. **81**, 464 (1951); J. Research, Nat. Bur. Stand. **46**, 446 (1951)

[3] V. I. Ogievetskii, J. Exper. Theoret. Phys. USSR **29**, 454 (1955); Soviet Phys. **2**, 312 (1956)

[4] L. Foldy, Phys. Rev. **81**, 395, 400 (1951)

2 In the preceeding article[3], the energy and angular distribution function of scattered γ-rays was found in the form of a rapidly converging series in powers of the depth of penetration, multiplied by an exponential which describes the damping of the most penetrating radiation.

The moments of the angular distribution may be obtained in closed form from Eq. (13) of reference 3, as will be shown below, for the cases of constant and linear dependence of the absorption coefficient on wavelength. It also can be shown that at great depths in the constant absorption coefficient case, the angular distribution will be close to the gaussian: $\exp[-\theta^2/2(\lambda-\lambda_0)]$.

It is well known[5] that the values of all moments of a distribution function in principle determine the function (the Stieltjes problem). However, since it is necessary to know every moment in a more or less simple analytic form, then in our case, as well as many others, this method of finding the distribution function from the moments is impractical.

As shown in reference 2, a knowledge of the moments frequently allows finding the distribution function as a rapidly converging series in some well-chosen set of orthogonal polynomials. The method of expansion in polynomials consists of the following[2].

Let the distribution function $f(x)$ be defined in the interval $(0, \infty)$. For an arbitrary function $\varphi(x)$, such that $\int\limits_0^\infty \varphi(x)\, x^n\, dx$ exists for all n, it is possible to construct a set of polynomials $P_0(x)$, $P(x)$, . . . , orthonormal on the interval $(0, \infty)$ with weight $\varphi(x)$:

$$\int\limits_0^\infty \varphi(x)\, P_n(x)\, P_m(x)\, dx = \delta_{mn}. \qquad (1)$$

Let us examine the expansion of $f(x)$ in the following form

$$f(x) = \varphi(x) \sum_{n=0}^{\infty} b_n P_n(x). \qquad (2)$$

From the values of the first m moments $a_m = \int\limits_0^\infty f(x)\, x^m\, dx$, using the orthogonality of the polynomial set $P(x)$, it is possible to find the first m coefficients b_m of expansion (2).

It often happens that certain characteristics of the distribution function are known, or even that there is an approximate expression for $f(x)$.

[5] Titchmarsh, *Theory of Fourier Integrals*

Physical considerations may sometimes determine the behavior of $f(x)$ for very large x, or, as in our case, the distribution function depends both on depth of penetration and on the energy, so that one may suppose that at great depths of penetration the angular distribution will be gaussian.

Let us choose a weighting function $\varphi(x)$ which reflects the properties of $f(x)$, and can thus be used as a first approximation to $f(x)$. Then the first and second terms of the polynomial expansion (2) can be considered corrections; the nth correction changes the nth moment, and does not affect the preceeding approximations. If the weight function has been well chosen, then the expansion (2) converges rapidly. Conversely, the rapidity of convergence of expansion (2) allows evaluation of the quality of the first approximation $\varphi(x)$.

The method of polynomial expansion has considerable generality, and can also be used in other branches of physics, for example, in the calculation of the spatial distribution of (cosmic-ray) shower particles.

3. In accordance with the above let us calculate the moments of the angular distribution. In the case of linear dependence of the absorption coefficient on wavelength $\tau(\lambda) = \tau_0 + \tau_1(\lambda-\lambda_0)$ the kinetic radiation transport equation in the variables s and l [reference 3, Eq. (13)] has the solution:

$$F_l(x, s) = \exp[-\tau_0 x] \exp\left\{a \int\limits_0^x \frac{\exp\left[-\frac{l^2}{2(s-\tau_1 y)}\right]}{s-\tau_1 y}\, dy\right\}, \qquad (3)$$

where the desired energy and angular distribution of the γ radiation are related to $F_l(x, s)$ by [Eq. (15), reference 3]

$$\Gamma(x, \lambda, \theta) = \left(\frac{\lambda_0}{\lambda}\right)^{1.8} \frac{1}{2\pi i} \int\limits_{\delta-i\infty}^{\delta+i\infty} e^{(\lambda-\lambda_0)s}\, ds\, \frac{1}{2\pi} \int\limits_0^\infty F_l(x, s)\, J_0(l\theta)\, l\, dl, \qquad (4a)$$

$$F_l(x, s) = \int\limits_{\lambda_0}^\infty d\lambda \left(\frac{\lambda}{\lambda_0}\right)^{1.8} \exp[-(\lambda-\lambda_0)s]\, ds\, 2\pi \times \int\limits_0^\infty \Gamma(x, \lambda, \theta)\, J_0(l\theta)\, \theta\, d\theta. \qquad (4b)$$

Direct Laplace and Hankel transformation (on variables s and l) without expansion in powers of x is very difficult[3]. However, the moments can be determined. Since

$$J_0(l\theta) = \sum_{m=0}^{\infty} \frac{(-1)^m (l\theta)^{2m}}{2^{2m}(m!)^2}, \quad (5)$$

then

$$\overline{\theta^{2n}}(x,\lambda) = 2\pi \int_0^{\infty} \Gamma(x,\lambda,\theta)\,\theta^{2n}\,\theta d\theta \quad (6)$$

$$= \left(\frac{\lambda_0}{\lambda}\right)^{1,8} \frac{1}{2\pi i} \int_{\delta-i\infty}^{\delta+i\infty} \exp[(\lambda-\lambda_0)s]\,ds$$

$$\times \left[(-1)^n 2^{2n} n! \left(\frac{\partial}{\partial (l^2)}\right)^n F_l(x,s)\right]_{l=0}$$

To make this convincing, the operator in the square brackets of Eq. (6) can be applied to Eq. (4b). A somewhat more difficult method to determine the moments in the case of a Hankel transform is used in reference 6, p. 167.

Substituting Eq. (3) into Eq. (6), and applying a Laplace transform, we obtain for the first few moments:

$$\overline{\theta^0}(x,\lambda) = e^{-\tau_0 x}\Big\{\delta(\lambda-\lambda_0) \quad (7a)$$

$$+ ax\left(\frac{\lambda_0}{\lambda}\right)^{1,8} {}_1F_1\left(1-\frac{a}{\tau};2;-\tau_1 x(\lambda-\lambda_0)\right)\Big\},$$

$$\overline{\theta^2}(x,\lambda) = 2ax(\lambda-\lambda_0)\left(\frac{\lambda_0}{\lambda}\right)^{1,8} e^{-\tau_0 x} \quad (7b)$$

$$\times\ {}_1F_1\left(1-\frac{a}{\tau_1};2;-\tau_1 x(\lambda-\lambda_0)\right),$$

$$\overline{\theta^4}(x,\lambda) = 8ax(\lambda-\lambda_0)^2\left(\frac{\lambda_0}{\lambda}\right)^{1,8} e^{-\tau_0 x} \quad (7c)$$

$$\times\Big\{{}_1F_1\left(1-\frac{a}{\tau_1};2;-\tau_1 x(\lambda-\lambda_0)\right)$$

$$+\frac{1}{2\tau_1(\lambda-\lambda_0)x}\Big[{}_1F_1\left(2-\frac{a}{\tau_1};2;-\tau_1 x(\lambda-\lambda_0)\right)$$

$$-{}_1F_1\left(-\frac{a}{\tau_1};2;-\tau_1 x(\lambda-\lambda_0)\right)\Big]\Big\},$$

$$\overline{\theta^6}(x,\lambda) = 48ax(\lambda-\lambda_0)^3\left(\frac{\lambda_0}{\lambda}\right)^{1,8} e^{-\tau_0 x} \quad (7d)$$

$$\times\Big\{\frac{1}{3!}\,{}_1F_1\left(1-\frac{a}{\tau_1};4;-\tau_1 x(\lambda-\lambda_0)\right)$$

$$+\frac{(3a-\tau_1)x(\lambda-\lambda_0)}{4!}$$

$$\times\ {}_1F_1\left(2-\frac{a}{\tau_1};5;-\tau_1(\lambda-\lambda_0)x\right)$$

$$+\frac{(\lambda-\lambda_0)^2 x^2 (2\tau_1^2 - 9a\tau_1 + 6a^2)}{6!}$$

$$\times\ {}_1F_1\left(3-\frac{a}{\tau_1};6;-\tau_1(\lambda-\lambda_0)x\right)\Big\},$$

where ${}_1F_1(\alpha;\beta;z)$ is a degenerate hypergeometric function.

For the transition to the case of constant absorption coefficient $\tau(\lambda) = \tau_0$, substitute $\tau_1 = 0$ into (3). Then

$$F_l(x,s) = \exp[-\tau_0 x]\exp\left\{\frac{ax}{s}\exp\left(-\frac{l^2}{2s}\right)\right\}. \quad (8)$$

Using Eq. (6) for the moments of the angular distribution we obtain:

$$\overline{\theta^0}(x,\lambda) = e^{-\tau_0 x}\left\{\delta(\lambda-\lambda_0) + \left(\frac{\lambda_0}{\lambda}\right)^{1.8} 2\frac{ax}{\rho} I_1(\rho)\right\}, \quad (9a)$$

$$\overline{\theta^2}(x,\lambda) = 4\left(\frac{\lambda_0}{\lambda}\right)^{1.8} e^{-\tau_0 x}\frac{ax(\lambda-\lambda_0)}{\rho} I_1(\rho), \quad (9b)$$

$$\overline{\theta^4}(x,\lambda) = 16 e^{-\tau_0 x}\left(\frac{\lambda_0}{\lambda}\right)^{1.8}\frac{ax(\lambda-\lambda_0)^2}{\rho} \quad (9c)$$

$$\times\left\{I_1(\rho) - 2\frac{I_0(\rho)}{\rho} + 4\frac{I_1(\rho)}{\rho^2}\right\},$$

$$\overline{\theta^6}(x,\lambda) = 96\left(\frac{\lambda_0}{\lambda}\right)^{1.8} e^{-\tau_0 x}\frac{ax(\lambda-\lambda_0)^3}{\rho} \quad (9d)$$

$$\times\left\{I_1(\rho) - \frac{6}{\rho} I_1(\rho) + \frac{28}{\rho^2} I_1(\rho) - \frac{64}{\rho^3} I_0(\rho) + \frac{128}{\rho^4} I_1(\rho)\right\},$$

where $I_\nu(\rho)$ is a ν-th order Bessel function of an imaginary argument, and as in reference 3, $\rho = 2\sqrt{ax(\lambda-\lambda_0)}$.

The moments (9) for the case of a constant absorption coefficient can also be obtained from the moments (7) by the limiting process $\tau_1 \to 0$. The zero moments are energy spectra, and were calculated in several ways in reference 3.

---

[6] S. Z. Belen'kii, *Avalanche Processes in Cosmic Rays*, Gov. Tech. Publishers, USSR, 1948

We note that the mean square of the angular deviation is:

$$\langle\theta^2\rangle = \frac{\overline{\theta^2}(x, \lambda)}{\overline{\theta^0}(x, \lambda)} = 2(\lambda - \lambda_0) \tag{10}$$

and does not depend on the depth of penetration. This is also correct for an arbitrary absorption coefficient in the small angle approximation.

The moments have now been found, and to construct the distribution function we must choose a weighting function.

4. The angular and energy distribution functions for γ-radiation scattered two or more times, for the case of constant absorption coefficient, was found in the form [reference 3, Eq. (22)]:

$$\Gamma(x, \lambda, \theta) = \frac{1}{2\pi}\left(\frac{\lambda_0}{\lambda}\right)^{1,8} \frac{e^{-\tau_0 x}}{(\lambda-\lambda_0)^2} \sum_{n=2}^{\infty} b_n,$$

where

$$b_n = \frac{(n-1)}{(n!)^2}\left(\frac{\rho}{2}\right)^{2n} \times \left(1 - \frac{\theta^2}{2n(\lambda-\lambda_0)}\right) u \left(1 - \frac{\theta^2}{2n(\lambda-\lambda_0)}\right). \tag{11}$$

At large depths of penetration ($\rho \gg 1$) the main contributions to expansion (11) come from terms with large n, and b_n may then be approximated as:

$$\tilde{b}_n = \frac{n-1}{(n!)^2}\left(\frac{\rho}{2}\right)^{2n} \exp\left\{-\frac{\theta^2}{2(\lambda-\lambda_0)}\right\}. \tag{12}$$

Recalling that the Bessel function of an imaginary argument has the expansion[7]:

$$I_0(\rho) = \sum_{n=0}^{\infty} \frac{1}{(n!)^2}\left(\frac{\rho}{2}\right)^{2n}; \; I_1(\rho) = \frac{dI_0(\rho)}{d\rho},$$

the distribution function (11) at great depths of penetration may be approximated by

$$\Gamma(x, \lambda, \theta) \sim \frac{1}{2\pi}\left(\frac{\lambda_0}{\lambda}\right)^{1,8} \frac{e^{-\tau_0 x}}{(\lambda-\lambda_0)^2} \times \left[\frac{\rho}{2} I_1(\rho) - I_0(\rho) + 1\right] \exp\left[-\frac{\theta^2}{2(\lambda-\lambda_0)}\right]. \tag{13}$$

The substitution of $\tilde{b}_n$ for b_n is also possible for $\theta^2 / 2(\lambda - \lambda_0) \ll 1$, i.e., for angles considerably smaller than the mean square. The similarity of the angular distribution of multiply scattered radiation to a gaussian distribution was pointed out by Foldy[4], in whose paper the angular distribution is approximated in a different but equally valid way. From Eq. (13), or from the corresponding approximation of reference 4, it follows only that the angular distribution tends to a gaussian, while an investigation of this trend, as done by Foldy, is impossible if only because the approximate substitution is not single valued [in Eq. (13), for example, we immediately obtained a gaussian exponential]. As will be shown below, the gaussian distribution is attained considerably more slowly than claimed by Foldy.

5. Thus, to find the angular and energy distributions in the case of constant absorption coefficient at great depths of penetration, it is natural to set the weight function equal to

$$\theta \exp[-\theta^2/2(\lambda - \lambda_0)].$$

The corresponding set of orthonormal polynomials are those of Chebishev-Laguerre $L_n\left[\frac{\theta^2}{2(\lambda-\lambda_0)}\right]$

$$L_n(\alpha) = \frac{1}{n!} e^{\alpha} \frac{d^n}{d\alpha^n}(e^{-\alpha}\alpha^n). \tag{14}$$

With the help of the polynomial expansion method, using the moments (9), we obtain the γ-radiation distribution function in the form:

$$\Gamma(x, \lambda, \theta) = \frac{4a^2x^2}{\pi}\left(\frac{\lambda_0}{\lambda}\right)^{1.8} \times \exp[-\tau_0 x] \exp\left[-\frac{\theta^2}{2(\lambda-\lambda_0)}\right] \times \left\{\frac{1}{\rho^3} I_1(\rho) - \frac{2}{\rho^4}\left[I_0(\rho) - \frac{2}{\rho} I_1(\rho)\right] L_2\left(\frac{\theta^3}{2(\lambda-\lambda_0)}\right) - \frac{16}{\rho^5}\left[I_1(\rho) - \frac{4}{\rho} I_0(\rho) + \frac{8}{\rho^2} I_1(\rho)\right] \times L_3\left(\frac{\theta^2}{2(\lambda-\lambda_0)}\right) + \ldots\right\}. \tag{15}$$

The term which describes unscattered radiation in (15) is omitted. The ratios of successive coefficients of Chebishev-Laguerre polynomials fall off as $1/\rho$, i.e., Eq. (15) is valid for $\rho \gg 1$. This is quite natural, since it is at great depths that the chosen weighting function (a gaussian exponent) closely describes the angular distribution. Expression (15) is valid for all materials in which, at the given energy, the absorption coefficient can be considered constant (the variable ρ depends on the constant a which varies with material).

Figure 1 shows normalized angular distributions of multiply scattered γ-radiation with initial

---

[7] H Watson, *Theory of Bessel Functions*

energy 17.34 mev at depths of $\rho = 4.8$ and 16 at an energy 9.18 mev. The curve for $\rho = 4$ is not certain, since the expansion (15) converges poorly for this ρ.

Figure 1 clearly shows the trend to a gaussian distribution as the depth of penetration increases.

For $\rho = 4$, $\Gamma(x, \lambda, \theta)$ increase with θ for small θ. This is also, although to a lesser extent, true for $\rho = 8$. It is explained by the influence of single scattered radiation at these depths.

For $\rho = 16$ the angular distribution is already very close to gaussian.

Figure 2 shows the evolution of the angular distribution of multiply scattered γ-radiation with increasing depth of penetration. The curves for $\rho = 1$, 4 and 6 were calculated by the method used in reference 3, the curve for $\rho = 16$ was calculated by the method of polynomial expansion. A gaussian distribution is shown for comparison. Thus, for the case of constant absorption coefficient we have accounted for all depths.

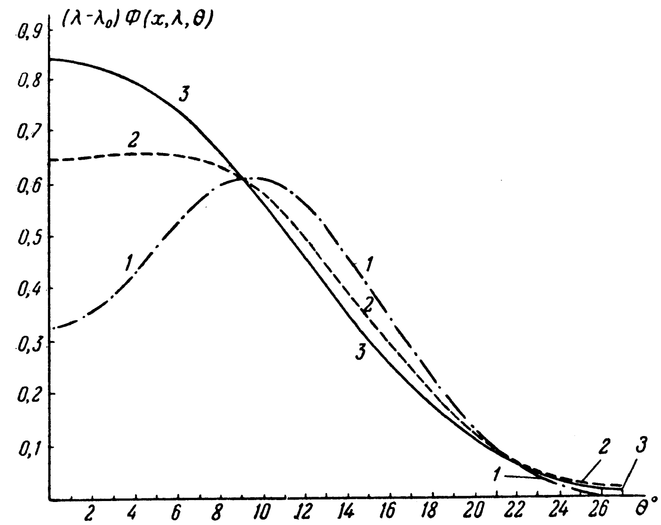

FIG. 1. Normalized angular distributions of multiply scattered γ-radiation of energy 9.18 mev. Initial energy 17.34 mev. *1.* $\rho = 4$; *2.* $\rho = 8$; *3.* $\rho = 16$.

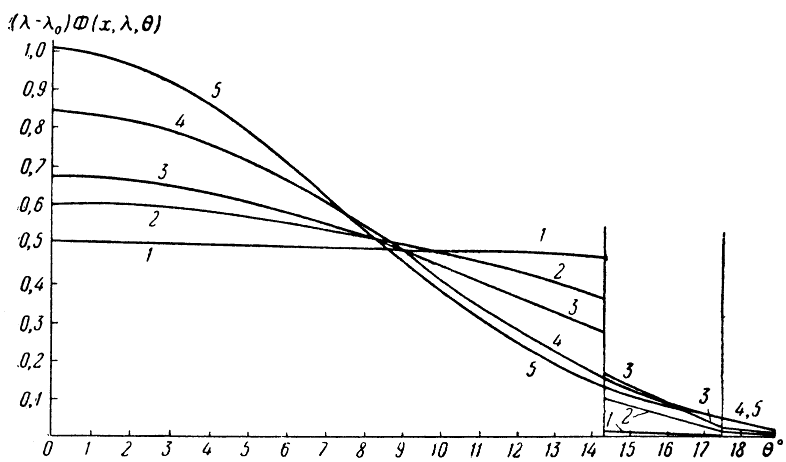

FIG. 2. Evolution of the angular distribution of γ-rays with increase in depth of penetration. Normalized angular distributions of 9 mev radiation scattered two or more times. Initial energy 12.5 mev. *1.* $\rho = 1$; *2.* $\rho = 4$; *3.* $\rho = 6$; *4.* $\rho = 16$; *5.* $\exp[-\theta^2/2(\lambda - \lambda_0)]$.

6. Let us now consider the case of a linear absorption coefficient $\tau(\lambda) = \tau_0 + \tau_1 (\lambda - \lambda_0)$. Here, as was discussed in Sec. 1, the angular distribution does not tend to a gaussian with increase in the depth of penetration. But if τ_1 is positive, then the angular distribution is already gaussian, and if $\tau_1 \ll 1$, the distribution will not differ strongly from a gaussian. This gives the basis for the selection of the same weighting function as for the case of constant absorption coefficient: $\theta \exp[-\theta^2/2(\lambda - \lambda_0)]$. The final result will allow us to judge the success of this choice.

The moment expression (7) is very clumsy; since we are interested in great depths, we shall use an asymptotic expansion of the moments in our calculation. This is obtained from the known asymptotic expansion of the degenerate hypergeometric function[8]

$$ {}_1F_1(\alpha; \gamma; z) \sim \frac{\Gamma(\gamma)}{\Gamma(\gamma-\alpha)} (-z)^{\alpha} \times {}_2F_0(\alpha; \alpha-\gamma+1; -z) + \frac{\Gamma(\gamma)}{\Gamma(\alpha)} e^z z^{\alpha-\gamma} {}_2F_0(1-\alpha; \gamma-\alpha; z), \qquad (16)$$

where $\Gamma(\gamma)$ is the gamma function, and

$$ {}_2F_0(\alpha;\gamma;z) = 1 + \frac{\alpha\gamma}{1!\,z} + \frac{\alpha(\alpha+1)\gamma(\gamma+1)}{2!\,z^2} + \dots $$

has asymptotic meaning (the series diverges for any z as long as α or γ are not negative integers).

In this case we obtain the energy and angular distributions at great depths in the form:

$$ \Gamma(x, \lambda, \theta) = \left(\frac{\lambda_0}{\lambda}\right)^{1.8} \frac{ax\exp[-\tau_0 x]}{2\pi(\lambda-\lambda_0)} \frac{v^{a/\tau_1 - 1}}{\Gamma\left(1+\frac{a}{\tau_1}\right)} \qquad (17)$$

$$ \times \left\{1 + \frac{a(a-\tau_1)}{\tau_1^2 v} + \frac{a(a^3 - 4a^2\tau_1 + 5a\tau_1^2 - 2\tau_1^3)}{2\tau_1^4 v^2} - \left[\frac{\tau_1}{2(a+\tau_1)} + \frac{a\tau_1}{2\tau_1^2 v} + \frac{a\tau_1(a^2 - a\tau_1 - 2\tau_1^2)}{4\tau_1^4 v^2}\right] \times L_2\left(\frac{\theta^2}{2(\lambda-\lambda_0)}\right) \right.$$

$$ \left. - \left[\frac{4\tau_1^2}{3(a+\tau_1)(a+2\tau_1)} + \frac{4a\tau_1^2}{3(a+\tau_1)\tau_1^2 v} + \frac{2a\tau_1^2(a-\tau_1)}{3\tau_1^4 v^2}\right] \times L_3\left(\frac{\theta^2}{2(\lambda-\lambda_0)}\right) + \dots \right\}, $$

where

$$ v = \tau_1(\lambda - \lambda_0)\,x. $$

In contrast to the case of constant absorption coefficient, the coefficient ratio of adjacent Chebishev-Laguerre polynomials in Eq. (17) does not tend to zero with increasing x. The reason for this has already been considered: photons scattered through large angles have a larger absorption probability than photons scattered through small angles. The "homogeneity" of absorption is destroyed, and the angular distribution must fall off more steeply than a gaussian. Similarly, if $0 < \tau_1 < a$ expression (17) can be used. Thus if 1.5 mev γ-rays are incident on carbon ($\lambda_0 = 1/3$, $\tau_1 = 0.15\ \text{cm}^{-1}$), then at a depth of $v = 20$, the coefficient of the second polynomial is less than one sixth the coefficient of the zeroth polynomial.

The condition $\tau_1 < a$ is satisfied in a wide energy interval for light elements, in the interval 0.5-3 mev for medium elements, and not at all for the heavy elements. Figure 3 shows a graph of the normalized angular distributions at two depths of penetration. The small rise in the $v = 10$ curve depends on singly scattered radiation.

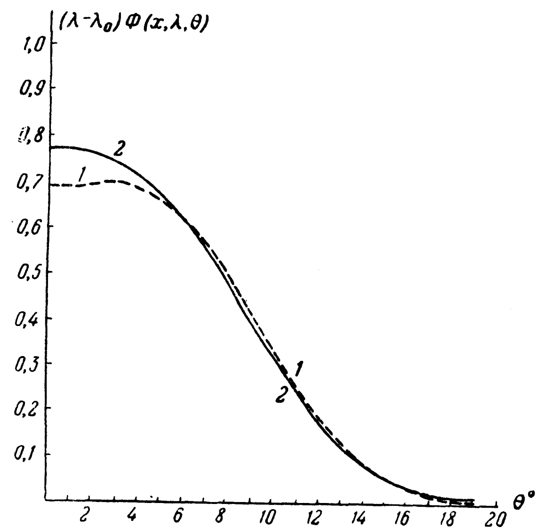

FIG. 3. Normalized angular distributions of scattered γ-radiation of energy 2.8 mev in carbon at depths of $v = 10$ and $v = 40$. Initial energy is 3 mev, $\tau_1 = 0.12\ \text{cm}^{-1}$. *1.* $v = 10$. *2.* $v = 40$.

7. In the above, and in reference 3 we have considered the propagation of γ-rays in a homogeneous medium. The problem of multiple γ-ray scattering in an inhomogeneous medium is of interest (the atmosphere, transition effects at counter walls, etc.).

---

[8] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, Gov. Publisher USSR, 1948, p. 557

Let us examine the passage of a parallel monochromatic bundle of γ-rays through a medium which consists of plane parallel layers of various materials; the absorption coefficient can be considered independent of energy in each material. For simplicity let the medium consist of two layers: the first of material A ($\tau_0 = \tau_A$, $a = a_A$) thickness and the second of material B ($\tau_0 = \tau_B$, $a = a_B$). The absorption coefficient and constant a both depend on the material, and can be conveniently written:

$$a = a_A + (a_B - a_A)\, u\,(x - x_A), \qquad (18)$$

$$\tau_0 = \tau_A + (\tau_B - \tau_A)\, u\,(x - x_A);$$

where $u\,(x)$ is a unit step function.

The kinetic equation in variables l and s [reference 3, Eq. (13)] is written:

$$\frac{\partial F_e(x, s)}{\partial x} + [\tau_A + (\tau_B - \tau_A)\, u\,(x - x_A)]\, F_e(x, s) = \frac{a_A + (a_B - a_A)\, u\,(x - x_A)}{s}\, F_e(x, s) + \delta(x). \qquad (19)$$

The solution of this equation differs from Eq. (8) in that the exponential describing the damping of the unscattered radiation has the argument $\tau_A\, x_A + \tau_B\,(x - x_A)$ instead of $\tau_0\, x$, while the exponential describing the scattered radiation has the argument $a_A\, x_A + a_B\,(x - x_A)$ instead of ax. Analogously, if the medium consists of layers: A thickness x_A, B thickness x_B, ... , M thickness x_M. then substitutions must be made in the homogeneous medium equations:

$$\tau_A\, x_A + \tau_B\, x_B + \ldots + \tau_M\, x_M \text{ for } \tau_0\, x$$

and

$$a_A\, x_A + a_B\, x_B + \ldots + a_M\, x_M \text{ for } ax. \qquad (20)$$

Thus, in the case of constant absorption coefficient, the final result of multiple scattering does not depend on the order of layers, and the layers may be shuffled. This is not true for an arbitrary absorption coefficient.

Let us now examine the passage of γ-rays through a medium whose density is a function of the depth of penetration x: $\rho\,(x) = f\,(x)$. The atmosphere will serve as an example. Since the absorption coefficient and the number of electrons are both proportional to the density of the material in the energy region where Compton scattering predominates over photoeffect and pair production, we can write

$$\tau\,(\lambda, x) = \overline{\tau\,(\lambda)}\, f\,(x)\,;\ a\,(x) = \bar{a} f\,(x). \qquad (21)$$

It is easy to show that in this case we may use the results for a homogeneous medium with the substitutions $\overline{\tau\,(\lambda)}$ and $\bar{a}$ for $\tau\,(\lambda)$ and a, and $z = \int_0^x f\,(x)\,dx$ for the depth of penetration x.

In conclusion, I would like to express gratitude to Prof. C. Z. Belen'kii for valuable comments, and to Academician I. E. Tamm and Prof. E. L. Feinberg for interest in this work.

Translated by G. L. Gerstein
226