Letters to the Editor

Interaction of Nucleons through a Pseudoscalar Meson Field

A. A. RUKHADZE
P. N. Lebedev Physical Institute, Academy of Sciences, USSR
(Submitted to JETP editor February 9, 1955)
J. Exper. Theoret. Phys. USSR **29**, 709-711 (November, 1955)

IN 1945 Tamm[1] (see also reference 2) suggested a quantum mechanical method for investigating relativistic problems in particle interactions. A covariant formulation of Tamm's method was later given by Cini[3].

We have applied Tamm's method to the problem of the interaction of nucleons through a pseudoscalar meson field. In the first Tamm approximation for pseudoscalar coupling, the integral equation for a two nucleon system is obtained from the Hamiltonian:

$$H(x) = -\frac{if}{2}[\overline{\psi}_\alpha(x), \psi_\beta(x)](\gamma_5\tau_i)^i_{\beta\alpha}\varphi_i(x), \quad (1)$$

and for pseudovector coupling, from the Hamiltonian:

$$H(x) = \overset{i}{\underset{\mu}{j}}(x)\frac{\partial\varphi_i(x)}{\partial x_\mu} + (j_\mu n_\mu)^2, \quad (2)$$

where

$$\overset{i}{\underset{\mu}{j}}(x) = -\frac{ig}{2}[\overline{\psi}_\alpha(x), \psi_\beta(x)](\gamma_5\gamma_\mu\tau i)_{\beta\alpha}. \quad (3)$$

In the first Tamm approximation the following probability amplitudes are taken different from zero:

$$\langle\Psi_0^*|\psi_\alpha^+\psi_\beta^+|\Psi(\sigma)\rangle = A^\sigma_{\alpha\beta}(x_1, x_2) \text{ (amplitude sought)}$$

$${}_0\langle\psi_\alpha^+\psi_\beta^+\varphi_i^+|\Psi(\sigma)\rangle \quad \text{and} \quad {}_0\langle\overline{\psi}_\alpha^+\psi_\beta^+\psi_\gamma^+\psi_\delta^+\varphi_i^+|\Psi(\sigma)\rangle$$

The equations are investigated by the following renormalization program: All divergent terms in the equation due to unobservable vacuum fluctuations ("vacuum diagrams") are removed. The divergent terms remaining in the equations correspond to self-energies of the nucleons, and are written as

$$-\frac{3ig^2}{4}\int_{-\infty}^{\sigma}dx_1\int_{-\infty}^{\sigma_1}dx_2\{S^+(x''-x_1) \times N(x_2-x_1)A^{\sigma_2}(x_2, x') - S^+(x'-x_1)N(x_2-x_1)A^{\sigma_2}(x_2, x'') + S^+(x''-x_2)N(x_1-x_2)A^{\sigma_2}(x_1, x') - S^+(x'-x_2)N(x_1-x_2)A^{\sigma_2}(x_1, x'')\}. \quad (3')$$

For pseudoscalar coupling,

$$N(x_2-x_1) \equiv \gamma_5 S_F(x_2-x_1)\gamma_5\Delta_F(x_2-x_1) \equiv -\frac{4}{(2\pi)^8}\int I_1(p)e^{ip(x_2-x_1)}dp = -\frac{4}{(2\pi)^8}\int dp\, e^{ip(x_2-x_1)}\int\frac{\gamma_5(\hat{p}-\hat{k}+m)\gamma_5 dk}{[(p-k)^2-m^2](k^2-\mu^2)} \quad (4)$$

For pseudovector coupling,

$$N(x_2-x_1) \equiv \gamma_5\gamma_\mu S_F(x_2-x_1)\gamma_5\gamma_\nu\frac{\partial^2\Delta_F(x_2-x_1)}{\partial x_{1\mu}\partial x_{2\nu}} + m\Delta^{(1)}(0) = -\frac{4}{(2\pi)^8}\int e^{ip(x_2-x_1)}I_2(p)\,dp + m\Delta^{(1)}(0). \quad (5)$$

The quantity $I(p)$ can be written in the form

$$I(p) = I(m) + \sum_{n=1}^{N}(p-m)^n A_n - I_c(p) \quad (6)$$

and therefore

$$N(x_2-x_1) = B_0\delta(x_2-x_1) + \sum_{n=1}^{N}B_n\int e^{ip(x_2-x_1)}(p-m)^n dp + N_c(x_2-x_1) \quad (7)$$

(for pseudoscalar coupling $N = 1$, for pseudovector coupling $N = 3$).

The introduction of terms B_0 into the equations for a two nucleon system is prevented by mass renormalization, and terms which contain B_n are rejected. As a result of this, we can make the following substitution in Eq. (3)

$$N(x) \rightarrow N_c(x),$$

where

$$N_c(x) = \frac{-1}{(8\pi^4)^2} \int e^{ipx} I_{1,2c}(p)\, dp. \tag{8}$$

In the momentum representation, the equation for a two nucleon system in the center of mass takes the following form

$$(2E - W) A^{rs}(k, j) = [\cdots] - g^2 F(E) A^{rs}(k, J), \tag{9}$$

where [. . .] denotes the terms corresponding to the noncovariant Tamm theory[2].

The last terms of Eq. (9) arise by separating finite parts from the divergent terms of the equation;

$$F(E) \sim \int \frac{(E-m) I_c(p_4\gamma_4 - \vec{k\gamma})}{(3E - W - p_4)(E - W + p_4)} dp_4. \tag{10}$$

The denominator in Eq. (10) is obtained from the finite limits of integration with respect to time in the divergent terms of Eq. (3). Asymptotically, for large values of p_4, $I_{1c}(p) \sim p_4$(pseudoscalar coupling), $I_{2c} \sim p_4^3$ (pseudoscalar coupling). Therefore, for pseudovector coupling, the integral (10) diverges, and accordingly, the integral equation (9) is not free of divergences. Thus the proposed renormalization program does not lead to a definite result for pseudovector coupling. We note that in perturbation theory, divergences of this type do not arise, thanks to the infinite limits of integration with respect to time.

$F(E)$ is finite for pseudoscalar coupling.

We thus find for pseudoscalar coupling a) for large k, i.e., for small distances between particles, $F(E) \to 0$, and accordingly the form of the interaction does not differ from the results of reference 2; b) for small k , i.e., for large distances between particles,

$$F(E) = \frac{k^2}{m} \varphi(\gamma),$$

where

$$\varphi(\gamma) = \frac{3f^2}{4\pi(2\pi)^3} \tag{11}$$
$$\times \int_0^1 dx \frac{[\gamma x^2 - (x-1)^2][2\gamma x - (\gamma+1)x^2 - x + 2]x}{\sqrt{(1-\gamma)x^3 + (\gamma-2)x^2 + x}\,[\gamma x + (x-1)^2]};$$

$\gamma \equiv \mu^2/m^2$. For small values of γ, $\varphi(\gamma) > 0$, in particular for $\gamma = 1/36$, $\varphi(\gamma) > 0$.

It therefore follows that the computation of renormalized terms in the equations for a two nucleon system with pseudoscalar coupling is equivalent to a decrease in the nucleon mass

$$m^* = m / 1 + \varphi(\gamma). \tag{12}$$

After this analysis was completed, there appeared an article[4] in which it was shown that higher Tamm approximations in the equations of a two nucleon system with pseudoscalar coupling lead to divergences similar to those described above for the pseudovector coupling. This shows that the proposed renormalization program in principle does not remove divergence difficulties which arise for the pseudoscalar as well as for the pseudovector coupling.

In conclusion, I wish to take this occasion to express my gratitude to V. B. Silin for suggesting this problem and for his constant help during the analysis.

[1] I. E. Tamm, J. of Phys. USSR **5**, 1 (1945).

[2] I. E. Tamm, V. P. Silin and V. Ia. Feinberg, J. Exper. Theoret. Phys. USSR **24**, 3 (1953).

[3] M. Cini, Nuovo Cimento **10**, 526, 614 (1953).

[4] H. Lehman, Z. Naturforsch **8a**, 579 (1953).

Translated by M. A. Melkanoff
265

The Connection between the Distribution of a Quasi-monochromatic Stationary Process and the Distribution of Its Envelope

S. M. Rytov
P. N. Lebedev Physical Institute
Academy of Sciences, USSR

(Submitted to JETP editor July 23, 1955)
J. Exper. Theoret. Phys. USSR **29**, 702-703
(November, 1955)

Blanc-Lapierre and coworkers[1] have recently shown that if one knows the characteristic function $f_\xi(u)$ of a quasi-monochromatic stationary random process $\xi(t) = A\cos(\omega_0 t - \theta)$ (A and θ are functions of t, slowly varying compared to $\cos\omega_0 t$) then the probability distribution $W_A(A)$ of the envelope $A(t)$ can be obtained using the Hankel transform:

$$\frac{w_A(A)}{A} = \int_0^\infty f_\xi(u) J_0(Au)\, u\, du, \tag{1}$$