where

$$N_c(x) = \frac{-1}{(8\pi^4)^2} \int e^{ipx} I_{1,2c}(p)\, dp. \quad (8)$$

In the momentum representation, the equation for a two nucleon system in the center of mass takes the following form

$$(2E - W) A^{rs}(k, j) = [\cdots] - g^2 F(E) A^{rs}(k, J), \quad (9)$$

where [. . .] denotes the terms corresponding to the noncovariant Tamm theory[2].

The last terms of Eq. (9) arise by separating finite parts from the divergent terms of the equation;

$$F(E) \sim \int \frac{(E - m) I_c(p_4\gamma_4 - \vec{k\gamma})}{(3E - W - p_4)(E - W + p_4)} dp_4. \quad (10)$$

The denominator in Eq. (10) is obtained from the finite limits of integration with respect to time in the divergent terms of Eq. (3). Asymptotically, for large values of p_4, $I_{1c}(p) \sim p_4$(pseudoscalar coupling), $I_{2c} \sim p_4^3$ (pseudoscalar coupling). Therefore, for pseudovector coupling, the integral (10) diverges, and accordingly, the integral equation (9) is not free of divergences. Thus the proposed renormalization program does not lead to a definite result for pseudovector coupling. We note that in perturbation theory, divergences of this type do not arise, thanks to the infinite limits of integration with respect to time.

$F(E)$ is finite for pseudoscalar coupling.

We thus find for pseudoscalar coupling a) for large k, i.e., for small distances between particles, $F(E) \to 0$, and accordingly the form of the interaction does not differ from the results of reference 2; b) for small k , i.e., for large distances between particles,

$$F(E) = \frac{k^2}{m} \varphi(\gamma),$$

where

$$\varphi(\gamma) = \frac{3f^2}{4\pi(2\pi)^3} \quad (11)$$

$$\times \int_0^1 dx \frac{[\gamma x^2 - (x-1)^2][2\gamma x - (\gamma+1)x^2 - x + 2]\, x}{\sqrt{(1-\gamma)x^3 + (\gamma-2)x^2 + x}\,[\gamma x + (x-1)^2]};$$

$\gamma \equiv \mu^2 / m^2$. For small values of γ, $\varphi(\gamma) > 0$, in particular for $\gamma = 1/36$, $\varphi(\gamma) > 0$.

It therefore follows that the computation of renormalized terms in the equations for a two nucleon system with pseudoscalar coupling is equivalent to a decrease in the nucleon mass

$$m^* = m / 1 + \varphi(\gamma). \quad (12)$$

After this analysis was completed, there appeared an article[4] in which it was shown that higher Tamm approximations in the equations of a two nucleon system with pseudoscalar coupling lead to divergences similar to those described above for the pseudovector coupling. This shows that the proposed renormalization program in principle does not remove divergence difficulties which arise for the pseudoscalar as well as for the pseudovector coupling.

In conclusion, I wish to take this occasion to express my gratitude to V. B. Silin for suggesting this problem and for his constant help during the analysis.

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The Connection between the Distribution of a Quasi-monochromatic Stationary Process and the Distribution of Its Envelope

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(Submitted to JETP editor July 23, 1955)
J. Exper. Theoret. Phys. USSR 29, 702-703
(November, 1955)

Blanc-Lapierre and coworkers[1] have recently shown that if one knows the characteristic function $f_\xi(u)$ of a quasi-monochromatic stationary random process $\xi(t) = A\cos(\omega_0 t - \theta)$ (A and θ are functions of t, slowly varying compared to $\cos\omega_0 t$) then the probability distribution $W_A(A)$ of the envelope $A(t)$ can be obtained using the Hankel transform:

$$\frac{w_A(A)}{A} = \int_0^\infty f_\xi(u) J_0(Au)\, u\, du, \quad (1)$$

where J_0 is the zero order Bessel function. From this it follows that if ξ is distributed normally, then A follows the Rayleigh distribution.

Hence, knowing $f_\xi(u)$, one can use the Fourier transform to obtain the distribution of ξ:

$$w_\xi(\xi) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f_\xi(u) e^{-iu\xi} du, \tag{2}$$

while the Hankel transform gives the distribution of the amplitude A. This elegant result is based on the following conditions: 1) A and θ are independent and 2) θ is distributed uniformly in the interval $(0, 2\pi)$. It is not difficult to show that both these are necessary conditions for $\xi(t)$ to be stationary, taking the two-dimensional distribution $w(A, \theta)$ to be stationary. The latter must have the form $w(A, \theta)\, dA d\theta = w_A(A)\, dA d\theta/2\pi$ or else the process $\xi(t)$ will not be stationary.

In the present note I should like to show that (1) and (2) can be used to derive a formula which gives $w_\xi(\xi)$ directly in terms of $w_A(A)$. Indeed, according to (1) we have

$$f_\xi(u) = \int_0^\infty w_A(A) J_0(Au)\, dA.$$

In conjunction with (2) this gives

$$w_\xi(\xi) = \int_0^\infty w_A(A)\, dA \frac{1}{2\pi}\int_{-\infty}^{+\infty} J_0(Au) e^{-i\xi u} du.$$

The inner integral is $1/\pi\sqrt{A^2-\xi^2}$ when $A \geq |\xi|$ and zero for $A < |\xi|^2$, so that

$$w_\xi(\xi) = \frac{1}{\pi}\int_{|\xi|}^\infty \frac{w_A(A)\, dA}{\sqrt{A^2-\xi^2}}, \tag{3}$$

or, introducing the new variable of integration x, $A = |\xi| \operatorname{ch} x$

$$w_\xi(\xi) = \frac{1}{\pi}\int_0^\infty w_A(|\xi| \cosh x) dx \tag{4}$$

It is easy to verify that the Rayleigh distribution for A:

$$w_A(A) = \frac{A}{\sigma^2} e^{-A^2/\sigma^2}$$

implies by formula (4), the normal distribution for ξ with mean square $\overline{\xi^2} = \sigma^2$. If the amplitude is fixed, i.e., $w_A(A) = \delta(A - A_0)$ then from (3) it follows that

$$w_\xi(\xi) = \begin{cases} 1/\pi\sqrt{A_0^2-\xi^2} & (|\xi| \leqslant A_0), \\ 0 & (|\xi| > A_0). \end{cases}$$

The uniform distribution for A, i.e., $w_A(A) = 1/A_0$ for $A \leq A_0$ and $w_A(A) = 0$ for $A > A_0$ gives, according to (4)

$$w_\xi(\xi) = \begin{cases} \dfrac{1}{2\pi A_0} \ln\left(\dfrac{A_0 + \sqrt{A_0^2-\xi^2}}{A_0 - \sqrt{A_0^2-\xi^2}}\right) & (|\xi| \leqslant A_0) \\ 0 & (|\xi| > A_0). \end{cases}$$

The exponential distribution $w_A(A) = \alpha e^{-\alpha A}$ for the amplitude A leads to a Macdonald function of zero order for the distribution of ξ.

$$w_\xi(\xi) = \frac{\alpha}{\pi}\int_0^\infty e^{-\alpha|\xi|\cosh x} dx = \frac{\alpha}{\pi} K_0(\alpha\xi).$$

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Translated by R. Krotkov
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The Fermi-Yang Hypothesis

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(Submitted to JETP editor December 18, 1954)
J. Exper. Theoret. Phys. USSR 29, 707-708
(November, 1955)

According to the Fermi-Yang hypothesis a π-meson is considered as a composite particle consisting of a proton and an anti-neutron in a bound state. In the work of Fermi and Yang[1] the interaction between a nucleon and an anti-nucleon is approximated by a potential well whose width is equal to the nuclear Compton wavelength, $r_0 = \hbar/Mc$, and whose depth is determined by the requirement that the lowest energy eigenvalue of the system equals the meson rest energy $E = \mu c^2$. With this condition they obtain the value $V_0 = 26.5\, Mc^2$ for the depth of the well in the