SOVIET PHYSICS

JETP

SOVIET PHYSICS JETP VOLUME 2, NUMBER 4 JULY, 1956

On the Macroscopic Theory of Superconductivity

V. L. GINZBURG

P. N. Lebedev Physical Institute, Academy of Sciences, USSR

(Submitted to JETP editor August 12, 1954)

J. Exper. Theoret. Phys. USSR **29**, 748-761 (December, 1955)

The question of the basic equations of the macroscopic theory of superconductivity in steady and alternating fields is discussed in connection with some recent researches. Particular attention is paid to clarification of the character of the conclusions which can be made on the basis of measurements of the surface impedance of the metal.

HE macroscopic theory of superconductivity, the basic problem of which consists in establishing the connection between current density j in a superconductor and the electric and magnetic field strengths E and H plays an exceptionally important role in the study of superconductivity. This relation between j and **E**, **H** in the superconducting state has a non-trivial character (in contrast, for instance, to Ohm's law $\mathbf{j} = \sigma \mathbf{E}$, which in most cases determines the relation in the non-superconducting state); moreover, the establishment of the relation is absolutely necessary for quantitative interpretation of experiments in superconductivity. Again, the microtheory of superconductivity can hardly be constructed and developed if we do not know those macroscopic relations which must be obtained as a result of a detailed consideration of the motion of the electrons in the metal. In view of what has been said, it is easy to understand the considerable attention which has been paid to the macrotheory of superconductivity, to which several special monographs and reviews have been devoted (see, for instance, references 1, 2).

An important step in the construction of a macrotheory of superconductivity was the theory of F. and H. London (1935) in which the basic equation[1,2] is

$$\operatorname{curl} \Lambda \mathbf{j}_s = -(1/c)\mathbf{H}. \tag{1}$$

If we restrict ourselves to the case of a steady field, then Eq. (1), together with Maxwell' equations, is sufficient for determining the density $\mathbf{j}_s$ of the superconducting current and the field **H** in the superconductor. Thus, for a massive superconductor $\mathbf{j}_s = (c/4\pi\delta_0)\mathbf{H}$, $\mathbf{H} = \mathbf{H}_0 e^{-z/\delta_0}$, where $\mathbf{H}_0$ is the field at the boundary of the metal, the z axis is directed along the normal of the boundary surface into the metal and $\delta_0 = (\Lambda c^2/4\pi)^{1/2}$. The parameter Λ may be conveniently expressed in terms of the concentration n_s of the "superconducting electrons";

$$\lambda = m/e^2 n_s,$$
$$\delta_0 = \sqrt{\Lambda c^2/4\pi} = \sqrt{mc^2/4\pi e^2 n_s}, \tag{2}$$

where e and m are the charge and the mass of a free electron. The macrotheory based on Eq. (1) turns out to be in qualitative agreement with ex-

---

[1] F. London, *Superfluids, I, Macroscopic Theory of Superconductivity*, New York, 1950.

[2] V. L. Ginzburg, Usp. Fiz. Nauk **42**, 169, 333 (1950).

periments made in weak fields, such that $H \ll H_k$ (H_k is the critical field). In fields $H \sim H_k$, however, such as cocur in the destruction of superconductivity by a field, in the intermediate state, and so on, Eq. (1) and the theory connected with it are completely inapplicable (see reference 2) and require generalization. A macrotheory of superconductivity which can be used in fields of any magnitude was developed only fairly recently[2,3]. Here Eq. (1) is replaced by the relations

$$\frac{1}{2m}\left(-i\hbar\nabla - \frac{e}{c}\mathbf{A}\right)^2\Psi + \alpha\Psi + \beta|\Psi|^2\Psi = 0, \tag{3}$$

$$\mathbf{j}_s = -\frac{ie\hbar}{mc}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) - \frac{e^2}{mc}|\Psi|^2\mathbf{A},$$

where $\mathbf{A}$ is the vector potential, which, if we assume $\operatorname{div}\mathbf{A} = 0$, satisfies the equation $\Delta\mathbf{A} = -(4\pi/c)\mathbf{j}_s$. In a weak field we may consider that $|\Psi|^2 = -\alpha/\beta = |\Psi_\infty|^2 = \text{const}$, and consequently $\mathbf{j}_s = -(e^2/mc)|\Psi|^2\mathbf{A}$, from which, after applying the operation curl, we obtain Eq. (1) with $\Lambda = m/e^2|\Psi|^2$, i.e., $|\Psi|^2 = n_s$.

The further development of the theory[3] and comparison of it with experimental data and, in particular, with the specially designed experiments of Zavaritskii has turned out to be very sucessful[4-10]. However, in the papers of Pippard[11,12] is is asserted that even in weak fields (1) is not applicable, and consequently, that it is essential to make some modification of Eq. (3), which leads to (1) in weak fields. The considerations of Pippard, which are based on experimental material, deserve careful attention. In part 1 of the present article we discuss this question and come to the conclusion that Pippard's replacement of the differential equations (1) or (3) by certain integral equations is not justified. In parts 2 and 3 we discuss the question of the behavior of superconductors in fields which are not steady (as was assumed above in part 1), but are time variable at high frequencies, up to the optical part of the spectrum. Particular attention is devoted to clarifying the character of the conclusions which can be made from the results of measurements of the surface impedance of superconductors.

1. CHARACTER OF THE BASIC EQUATIONS OF THE MACROTHEORY OF SUPERCONDUCTIVITY IN A STEADY FIELD

Pippard's remarks are to a large extent based on interesting experimental data and confirm the conclusion that (1) is not valid in strong fields. At the same time it is easy to see that all the new facts (with one possible exception which will be pointed out below) agree at least qualitatively with the theory[3], in which there appear parameters α and β or the directly measurable quantities:

$$\delta_0 = \sqrt{mc^2\beta/4\pi e^2|\alpha|} = \sqrt{mc^2/4\pi e^2 n_s},\quad H_{\kappa M} = \sqrt{4\pi\alpha^2/\beta}, \tag{4}$$

$$\varkappa = (mc/|e|\hbar)\sqrt{\beta/2\pi} = (\sqrt{2}|e|/\hbar c)H_{\kappa M}\delta_0^2$$

(δ_0 is the penetration depth of a weak field, H_{kM} is the critical field for the bulk metal). The quantity $|\Psi_\infty|^2 \equiv n_s = -\alpha/\beta$ is not directly measurable, so in the expression for δ_0 we may take e^2/m to be the same as for free electrons without making any assumption of a physical nature (for further details, see references 2, 3 and 18). Thus, the "concentration of superconducting electrons" is by definition given by

$$n_s = mc^2/4\pi e^2\delta_0^2 = 2.83\cdot 10^{11}\,\delta_0^{-2}, \tag{5}$$

where δ_0, as in all that follows, is measured in centimeters. The same of course applies also to

---

[3] V. L. Ginzburg and L. D. Landau, J. Exper. Theoret. Phys. USSR **20**, 1064 (1950).

[4] V. P. Silin, J. Exper. Theoret. Phys. USSR **21**, 1330 (1951).

[5] V. L. Ginzburg, Dokl. Akad. Nauk SSSR **83**, 385 (1952).

[6] V. N. Zavaritskii, Dokl. Akad. Nauk SSSR **78**, 665 (1951); **85**, 749 (1952).

[7] V. L. Ginzburg, J. Exper. Theoret. Phys. USSR **23**, 236 (1952).

[8] V. L. Ginzburg, Usp. Fiz. Nauk **48**, 25 (1952).

[9] A. A. Abrikosov, Dokl. Akad. Nauk SSSR **86**, 489 (1952).

[10] A. V. Gurevich, J. Exper. Theoret. Phys. USSR **27**, 195 (1954).

[11] A. B. Pippard, Proc. Roy. Soc. (London) **216**, 547 (1953).

[12] A. B. Pippard, Physica **19**, 765 (1953).

(1) snd (2); the suggestion sometimes made that the mass $m = 9.1 \times 10^{-28}$ should be replaced by some effective mass m_{eff} is based to a large extent on a misunderstanding.

In references 2 and 3 it is assumed that the charge e appearing in (3) and in the expression for κ in (4) is equal to the charge of the free electron $|e| = 4.8 \times 10^{-10}$. In this case

$$\varkappa = 2 \cdot 16 \times 10^{7} H_{KM} \delta_0^2 \qquad (6)$$

and only two parameters, δ_0 and H_{kM} enter into the theory. The assumption seems to be a fairly natural one, but may not necessarily be true (this was already noted in reference 8, p. 107). If the charge e in (3) is not equal to that of a free electron* then three parameters (δ_0, H_{kM} and κ) enter into the theory[3]. Verification of the theory is possible in this case also, since the same quantity κ enters, for instance, both into the expression for the surface energy and the expression determining the dependence of the penetration depth on the applied magnetic field. Thus perhaps we should not take it for granted, as is done by Faber[13] and Bardeen[14], that the theory[3] is uniquely linked with (6), although this does seem the most probable hypothesis.

The experimental data given in references 13 and 14 are not very precise, but nevertheless suggest the possibility that the value of κ is two or three times larger than that given by (6); in other words,

$$\varkappa = (\sqrt{2}\,|e_{\mathrm{eff}}|/\hbar c) H_{\kappa M} \delta_0^2 \qquad (7)$$

$$= 2.16 \cdot 10^7 (e_{\mathrm{eff}}/e) H_{\kappa M} \delta_0^2,$$

where $(e_{\mathrm{eff}}/e) \sim 2$ or 3. Since the ratio e_{eff}/e if $e_{\mathrm{eff}} \neq e$ may change from metal to metal there is no reason for supposing[13] that even if $\sqrt{\kappa} \ll 1$ the quantity $\Delta H_{kM} \delta_0$ should have the same value for all metals close to T_k according to the theory[3] (here $\Delta = \sigma_{ns}/(H_{kM}^2/8\pi)$, where σ_{ns} is the surface energy at the boundary between normal and superconducting phases). In support of the hypothesis that $e_{\mathrm{eff}}/e > 1$ there are some data on the dependence of the penetration depth of magnetic field. However, the whole question of the value of the ratio e_{eff}/e remains completely open from an experimental point of view, and it is quite possible that $e_{\mathrm{eff}}/e = 1$, as is more probable, if not indeed necessary, on theoretical grounds. Verification of the theory by measurement of Δ and the field dependence of δ for one and the same specimen is therefore particularly needed, (if the theory is true then the values of κ obtained from both experiments should coincide)*.

---

* It must be pointed out that the introduction of a nonuniversal charge $e_{\mathrm{eff}} \neq e = 4.8 \times 10^{-10}$ is not free from serious objection (the author owes this remark to L. D. Landau). Thus, if the superconductor is nonuniform, e_{eff} will depend on the coordinates and this will upset the gauge invariance of the theory. However, to take account of nonuniformity of the metal might require just such a generalization of the theory[3], which was developed only for uniform media where this difficulty does not arise.

13 T. E. Faber, Proc. Roy. Soc. (London) **223**, 174 (1954).

14 J. Bardeen, Phys. Rev. **94**, 554 (1954).

* It should be noted that the theory[3] has been worked out only for temperatures close to T_k, since it is only in this region that the dependence of δ_0 and H_{kM} on $(T_k - T)$, and the expression for the free energy in terms of $|\Psi|^2$, are reliably known. For lower temperatures, down to $T = 0$, the generalization of the theory[3] is not uniquely indicated, even if we assume on empirical grounds particular forms of the dependence of δ_0 and H_{kM} on T. One variant of such a possible generalization of the theory[3] has been considered recently by Bardeen[14] who assumed that $H_{kM} = H_0(1 - (T/T_k)^2)$, $F_{n0} = -(H_0^2/4\pi)(T/T_k)^2$ and $\delta_0^2 = \delta_{00}^2/(1 - (T/T_k)^4)$. The same forms of temperature dependence of H_{kM} and δ_0, which are in agreement with experiment to a first approximation, can be obtained by assuming (as in reference 3) that

$$F_{s0} = F_{n0} + \alpha|\Psi|^2 + \tfrac{1}{2}\beta|\Psi|^4,$$

but supposing, as in reference 14, that the dependence of α and β on T has the form:

$$\alpha = -A\,\frac{1 - (T/T_k)^2}{1 + (T/T_k)^2}, \qquad A = \frac{e^2 H_0^2 \delta_{00}^2}{mc^2},$$

In this case all the formulas of the theory[3] remain in force, if we understand by α and β, the above functions of T. However, such a generalization of the theory, as also that given by Bardeen[14], has only a limited value, since the exact form of the dependence of F_{s0} on $|\Psi|^2$ is unknown, and the expressions given for

Within the framework of the phenomenological theory the parameters δ_0 and H_{kM} are completely independent. From the microscopic point of view, as far as can be said in the present state of the theory[8], only two parameters should enter into the theory: the breadth of the gap in the energy spectrum determining in the first place the field H_{kM} $(T=0)$,and the concentration $n_s(T=0) \equiv n_{s0}$, determining the penetration depth at $T=0$. The quantity n_{s0}, according to the available data (see reference 2 and below) is several times smaller than the concentration of the conduction electrons in the normal state and may depend on the treatment of the metal, presence of impurities, etc. Moreover, if we examine the path by which the quantity n_{s0} enters the microtheory (see reference 8, pp. 40-42), we have some reason for supposing that the value of n_{s0} is the higher the nearer the metal is to an ideal one. Pippard's experiments[11] support this supposition since in the case he investigated, the addition of impurities caused an increase of δ_0 (i.e., a diminution of n_{s0}). The fact that H_{kM} and T_k are only slightly changed cannot be regarded as surprising in view of what has been said, and in any case does not contradict existing theoretical ideas. Thus, the contrary assertion by Pippard does not appear to us to be well founded. The only known experimental result which evidently contradicts the theory[3] (also the London theory, which is a special case of it) is the non-monotonic dependence of penetration depth on the angle between the current and the tetragonal axis for mono-crystalline tin (see references 11 and 15 and the literature quoted there)*. However, the experiments in question[11] were carried out at high frequencies and not for the static conditions which have been implied in the preceding discussion. Moreover, as pointed out by Shoenberg (reference 15, p. 161) and Khaikin[16], there are various reasons for doubting the validity of the experimental data and thus the question of the anisotropy of the penetration depth (in particular, in a static or low frequency field) remains open.

In view of all that had been said, we do not see any reason, for the time being, to believe the conclusion that (1) is already invalid at low fields. Moreover, the equation proposed by Pippard to replace it,

$$\Lambda \mathbf{j}_s = -\frac{3}{4\pi\xi_0}\int \frac{\mathbf{r}\,(\mathbf{r}\mathbf{A})\, e^{-r/\xi} dV}{r^4} \tag{8}$$

seems to be unsuitable for a variety of reasons. Indeed, as is clear from Eq. (8) or similar ones*, the density of superconducting current $\mathbf{j}_s(\mathbf{r})$ is determined by the field in a region of dimensions of order $\sim \xi$ around the point $\mathbf{r}$ and the properties of specimens of dimensions $d \ll \xi$ must differ radically from the properties of the bulk metal. Further, the parameters ξ_0 and ξ in (8) are such that for an ideal metal $\xi_0 = \xi \sim 10^{-4}$, and for addition of impurities the parameter ξ diminishes so that $0 \leq \xi \leq \xi_0$. From this it follows that films of pure metal with the same value of T_k as bulk specimens should behave anomalously if their thickness d is such that $d \lesssim \xi_0 \sim 10^{-4}$. However, it is known from experiments[6,15] that such films behave in agreement with the theory[3] and the penetration depth δ_0 is about the same for them as for the bulk metal. The situation is similar also for superconducting colloids, as Pippard himself points out[11]. The properties of specimens of small dimensions indicate that in weak fields the current density $\mathbf{j}_s(\mathbf{r})$ is determined by a region around the point $\mathbf{r}$ of dimensions $\xi \ll d_{min} \sim 10^{-6}$ (d_{min} is the minimum thickness of a film with values of T_k and δ_0 the same as for bulk metal). In the framework of the macrotheory this means that the connection between

the functions $H_{kM}(T)$ and $\delta_0(T)$ are also only approximate. Nevertheless, this generalization may be useful, since the exact form of the function $F_{s0}(|\Psi|^2)$, as is clear from Bardeen, is of little importance within certain limits, for instance, from the point of view of calculating the quantity σ_{ns} (though of course in calculating other quantities the form of the function $F_{s0}(|\Psi|^2)$ may be more important). A more detailed discussion of the application of the theory[3] at all temperatures will be given in a forthcoming paper by the author.

* Generalization of Eqs. (1) and (3) to the anisotropic case leads[7] to a monotonic dependence of δ_0 for change of θ from 0 to $\pi/2$ (θ is the angle between the current and the symmetry axis).

[15] D. Shoenberg, *Superconductivity*, Cambridge, 1952.

* Equation (8) is not gauge invariant and consequently must be replaced, for instance, by

$$\operatorname{curl} \Lambda \mathbf{j}_s = -(3/4\pi\xi_0)\int r^{-4}\,\mathbf{r}(\mathbf{r}\mathbf{H})\,e^{-r/\xi}dV$$

(reference 12, p. 772). We shall not go into this question in detail, since it is not particularly relevant to the discussion in the text.

[16] M. S. Khaikin, J. Exper. Theoret. Phys. USSR **28**, 115 (1955); Soviet Physics JETP **1**, 164 (1955)

$\mathbf{j}_s$ and the field is not of an integral character, and for a weak field, where the theory must be linear, we are led in a natural manner to Eq. (1)[1,2]. Equation (1) has, moreover, a clear meaning from the point of view of quantum theory*. Equations of the type (8) are on the contrary obtained by Pippard only by an analogy with the theory of the anomalous skin effect, although there do not appear to be any grounds for developing an analogy between the superconducting current and the normal current in the conditions of either the normal or the anomalous skin effect.

Thus, there are many convincing considerations in favor of the validity of Eq. (1), and the question of the nonvalidity of (1) in weak fields need, in our opinion, only be seriously discussed when this is demanded by new experiments (which should in the first place clear up the question of the character of the anisotropy of penetration depth of static, or sufficiently low frequency fields).

2. BEHAVIOR OF SUPERCONDUCTORS IN A HIGH FREQUENCY FIELD (GENERAL CONSIDERATIONS)

In part 1 we discussed the behavior of superconductors in a constant field. In an alternating field, and in particular, in a high-frequency field, the situation is in general a good deal more complicated, in the first place because of losses and the anomalous character of the skin-effect in metals at low temperatutes. Bearing in mind this complication and the detailed arrangements of the relevant experiments, we shall limit ourselves, in the case of the high frequency fields, to the situation of weak fields ($H \ll H_k$), where the problem can be treated as linear, and moreover, we shall be concerned only with the infleunce of the metal on the field outside it (for instance, in a resonator). In these conditions, and allowing for the smallness of the penetration depth of the field into the metal, the effect of the metal can be described by a single complex quantity which may depend on the frequency ω. Such a quantity is the surface impedance Z (or a function of it):

$$Z(\omega) = R(\omega) + iX(\omega) = \frac{4\pi}{c}\left[\frac{E_x}{H_y}\right]_0, \qquad (9)$$

where the subscript 0 indicates that the components E_x and H_y are to be taken at the surface of the metal (the z-axis is directed into the metal).

If the relation between the current and the field in the metal is such that we can introduce a complex dielectric constant $\epsilon'(\omega)$, then

$$Z(\omega) = \frac{4\pi}{c\sqrt{\varepsilon'(\omega)}}, \qquad (10)$$

$$\varepsilon' = \varepsilon - i\frac{4\pi\sigma}{\omega} = \frac{16\pi^2}{c^2 Z^2}$$

$$= \frac{16\pi^2}{c^2}\frac{(R^2 - X^2) - 2iXR}{(X^2 + R^2)^2}.$$

For normal incidence of a plane wave on a plane boundary between a vacuum and the medium, Eq. (10) is exact, but for oblique incidence of the wave on the boundary, and generally for an arbitrary field in the vacuum at the boundary, the impedance will in general depend on the character of this field, i.e., it is not a characteristic of the medium alone. However, for metals, where the field in the metal diminishes very rapidly, the quantity Z, for a plane boundary, does not depend on the character of the field, if only terms up to the order of $1/\epsilon'$ are considered (see reference 17, Sec. 19). Thus, in practice, if the condition

$$16\pi^2 / c^2 |Z|^2 = |\varepsilon'| \gg 1, \qquad (11)$$

---

* In reference 3 the current density $\mathbf{j}_s(\mathbf{r})$ depends on field over the whole region, since the function $\Psi(\mathbf{r})$ is not constant [as it is in the limiting form (1)]. However, the connection between remote regions established in this way is realized in a way which is characteristic of quantum mechanics, and not by the introduction of integral operators. It should be noted in this connection that in (3) the term $(\hbar^2/2m)\nabla^2\Psi$, if Ψ changes appreciably over a distance δ_0/κ, is of order $\alpha\Psi$, which is large. Thus, this term is also large and cannot be considered simply as the first term in an expansion of the type $a\nabla^2\Psi + b\nabla^2(\nabla^2\Psi)+\ldots$ In our opinion the prominence of the expression $(\hbar^2/2m)\nabla^2\Psi$ is explained by the fact that the Ψ-function introduced in reference 3 is closely connected with the true Ψ-function of the electrons in the metal; in the equation for the latter, however, there enters only the differential operator of the kinetic energy-- $(\hbar^2/2m)\nabla^2$. It should be noted in this connection, that the expression obtained in reference 3 for the surface energy, $\sigma_{ns} = H_{kM}^2\delta_0/\sqrt{23}\pi\kappa$ is obtained also, as far as order of magnitude is concerned, from the uncertainty principle if we take account of the fact that the thickness of the transition layer between phases is $\sim \delta_0/\kappa$ (from the uncertainty principle it is evident that $\sigma_{ns} \sim \frac{\hbar_{ns}^2}{2m(\delta_0/\kappa)^2}\,\frac{\delta_0}{\kappa} \sim \frac{H_{kM}^2\delta_0}{2\pi\kappa}$).

[17] Ia. L. Alpert, V. L. Ginzburg and E. L. Feinberg, *Propagation of Radio Waves*, Gostekhizdat, 1953.

is fulfilled, as it is with a large margin for metals, the impedance may be considered as characteristic of the metal alone, as was mentioned above*.

This conclusion is valid even if it is not possible to introduce a quantity ϵ' characterizing the metal, but the metal may still be considered as a "good conductor". This is always the case, at least up to frequencies higher than those of visible light (see reference 18, Sec. 2).

It should be noted too, that where the impedance is not universal, it can still be used if the character of the external field is determined. The question of the existence of a universal impedance independent of the character of the external field can be decided experimentally, for instance, by a study of the reflection of plane waves of incidence for various angles.

The impedance can always be written in the form

$$Z(\omega) = \frac{4\pi}{c\sqrt{\varepsilon'_{\text{eff}}(\omega)}}, \tag{12}$$

$$\varepsilon'_{\text{eff}} \equiv \varepsilon_{\text{eff}} - i\frac{4\pi\sigma_{\text{eff}}}{\omega}$$

$$\equiv \varepsilon_{1\,\text{eff}} - i\varepsilon_{2\,\text{eff}} = \frac{16\pi^2}{c^2 Z^2},$$

where $\epsilon'_{\text{eff}}(\omega)$ is a new complex quantity playing the role of an effective dielectric constant of the metal. In the special case of normal skin-effect, of course, $\epsilon'_{\text{eff}} = \epsilon'$. Generally, however, ϵ'_{eff} coincides with the complex dielectric constant of a medium that would have exactly the same effect on the external field as the metal if it were put in its place. From this it is clear that from measurement of the impedance alone, bringing in no additional considerations, it is impossible to conclude that the metal cannot be characterized by a parameter ϵ'. Such a conclusion indeed cannot be made without going outside the framework of phenomenological concepts, but only from a determination of the field in the metal itself (the possibility of such measurements is not excluded in principle, but there is no need to consider them as yet). These remarks make evident the unsoundness of attempts[19] to establish, on the basis of impedance measurements, that the relation between field and current in a superconductor is not a differential one. We should, incidentally, qualify what has just been said by pointing out that we have been talking up to now of an isotropic body; if there is anisotropy, i.e., no equivalence between the x and y axes, the question of the possibility of characterizing the field in the metal by some tensor ϵ'_{ik} can be decided by investigating the dependence of $Z(\omega)$ on the angle between the crystal axis and the field (see Pippard[11]). Because of this, experiments on anisotropic specimens at high frequencies (e.g., tin), are particularly important, as has already been mentioned in Sec. 1.

We shall now discuss some general properties of the functions $Z(\omega)$ and $\epsilon'_{\text{eff}}(\omega)$. It can be shown that the quantities $Z(\omega)$ and $\epsilon'_{\text{eff}}(\omega)$, considered as functions of a complex variable ω, have neither poles nor zeros in the lower half-plane (the time dependence is determined by the factor $e^{i\omega t}$) or on the real axis, with the exception, perhaps, only of the point $\omega = 0$. We shall not here discuss the proof of this statement, which follows essentially from the requirement of satisfying the principle of causality, particularly as it has recently been discussed by the author elsewhere[20] (see also reference 17, Sec. 83).

In the static case, for superconductors $[E_{x,y}]_0 = 0$ and consequently $Z(0) = 0$; for nonsuperconductors the quantity $Z(0)$ is likewise finite. Thus the function $Z(\omega)$ has no poles along the whole real axis and below it; hence,

$$\oint_L \frac{Z(\omega') - Z(\infty)}{\omega' - \omega}\, d\omega' = 0, \tag{13}$$

in particular, if the closed contour L is made up of the real axis and the path along a semicircle below the point $\omega' = \omega$ (where ω is some real frequency) and an infinite lower semicircle. The integral around the infinite semicircle vanishes, and consequently (13) can be written in the form

$$i\pi\{Z(\omega) - Z(\infty)\} + \int_{-\infty}^{+\infty} \frac{Z(\omega') - Z(\infty)}{\omega' - \omega}\, d\omega' = 0, \tag{14}$$

* Under condition (11) we have, for any angle of incidence of the wave on the surface

$$Z(\omega) = (4\pi/c)[E_x/H_y]_0 = -(4\pi/c)[E_y/H_x]_0.$$

[18] V. L. Ginzburg and G. P. Motulevich, Usp. Fiz. Nauk 55, 469 (1955).

[19] A. A. Galkin and M. I. Kaganov, J. Exper. Theoret. Phys. USSR 25, 761 (1953).

[20] V. L. Ginzburg, Akust. Zh. 1, 31 (1955).

where, as in all that follows, the principal value of the integral should be understood. Further, from the requirements that $[H_{x,y}]_0$ should be real for $[E_{x,y}]_0$ real, it follows that on the real axis $Z(-\omega) = Z^*(\omega)$, i.e., $R(-\omega) = R(\omega)$ and $X(-\omega) = -X(\omega)$. Thus, if we separate (14) into its real and imaginary parts we obtain without difficulty

$$R(\omega) - R(\infty) = -\frac{2}{\pi}\int_0^\infty \frac{\omega'[X(\omega') - X(\infty)]}{\omega'^2 - \omega^2}d\omega', \tag{15}$$

$$X(\omega) - X(\infty) = \frac{2\omega}{\pi}\int_0^\infty \frac{[R(\omega') - R(\infty)]}{\omega'^2 - \omega^2}d\omega'.$$

For $\omega \to \infty$, the metal has no effect on the propagation of the wave, and consequently $X(\infty) = 0$ and $R(\infty) = 4\pi/c$. However, the transition to the limit of very high frequencies must be considered with some care in view of the limited applicability of the formulas for surface impedance[20]. In this connection we note that the equation $X(\infty) = 0$ follows directly from the second of the equations (15) and the fact that $X(0) = 0$. Moreover, the value of $R(\infty)$ is of little importance, since in the second of the equations (15), $\int_0^\infty \frac{R(\infty)}{\omega'^2 - \omega^2}d\omega' = 0$ (the integral to be taken as the principal value), so that $R(\infty)$ does not really enter, while in the first of the equations (15), $R(\infty)$ can be elimiated by taking the difference of the expression for two frequencies. Thus,

$$R(\omega_2) - R(\omega_1) = -\frac{2}{\pi}\int_0^\infty \left\{\frac{1}{\omega'^2 - \omega_2^2} - \frac{1}{\omega'^2 - \omega_1^2}\right\} X(\omega')\,\omega' d\omega', \tag{16}$$

$$X(\omega) = \frac{2\omega}{\pi}\int_0^\infty \frac{R(\omega')\,d\omega'}{\omega'^2 - \omega^2}.$$

The second of these relations is particularly useful for quantitative estimates, since the quantity $R(\omega)$ is easier to measure, and moreover is always positive. This is evident from the fact that R is directly connected with the heat developed in the metal

$$Q = (c/4\pi)^2 |H_0|^2 R/2, \tag{17}$$

where Q is the mean quantity of heat developed per unit time per unit area of the metal surface (the metal specimen is considered as sufficiently thick to absorb the wave completely) and H_0 is the amplitude of the high-frequency magnetic field at the surface ($[H]_0 = H_0 e^{i\omega t}$).

If we introduce resistive and reactive skin-depths $\delta_r(\omega)$ and $\delta_0(\omega)$ in the usual way, defined by

$$R(\omega) = (2\pi\omega/c^2)\,\delta_r(\omega), \qquad X(\omega) = (4\pi\omega/c^2)\,\delta_0(\omega), \tag{18}$$

then, for instance, the second of the equations (18) may be rewritten in the form

$$\delta_0(\omega) = \frac{1}{\pi}\int_0^\infty \frac{\omega'\delta_r(\omega')\,d\omega'}{\omega'^2 - \omega^2}. \tag{19}$$

To improve the convergence of the integral in the region of high frequency (in this connection see reference 20) it is appropriate to determine not the quantity $\delta_0(\omega)$ itself, but the difference $\delta_0(\omega_2) - \delta_0(\omega_1)$, in terms of $\delta_r(\omega')$, or, in particular, the difference:

$$\delta_0(\omega) - \delta_0(0) = \frac{1}{\pi}\int_0^\infty \left\{\frac{1}{\omega'^2 - \omega^2} - \frac{1}{\omega'^2}\right\}\delta_r(\omega')\,\omega' d\omega', \tag{20}$$

where $\delta_0(0) \equiv \delta_0$ is the penetration depth of a weak static magnetic field as used in Sec. 1. Equation (20) was given earlier by Pippard[11], and used successfully by him for estimating the dependence of δ_0 on ω.

It is also useful to obtain formulas analogous to those given for the quantity $\epsilon'_{eff}(\omega)$. According to (12)

$$R^2 = \frac{8\pi^2}{c^2}\,\frac{\varepsilon_{eff} + \sqrt{\varepsilon_{eff}^2 + (4\pi\sigma_{eff}/\omega)^2}}{[\varepsilon_{eff}^2 + (4\pi\sigma_{eff}/\omega)^2]},$$

$$X^2 = \frac{8\pi^2}{c^2}\,\frac{-\varepsilon_{eff} + \sqrt{\varepsilon_{eff}^2 + (4\pi\sigma_{eff}/\omega)^2}}{[\varepsilon_{eff}^2 + (4\pi\sigma_{eff}/\omega)^2]} \tag{21}$$

$$\varepsilon_{eff} = -\frac{16\pi^2}{c^2}\frac{X^2 - R^2}{(X^2 + R^2)^2}, \quad \sigma_{eff} = \frac{8\pi\omega}{c^2}\frac{XR}{(R^2 + X^2)^2},$$

where the root must always be taken as positive. For $\omega \to 0$, the penetration depth of the field into the superconductor is given by $\delta_0(\omega) \to \delta_0(0) \equiv \delta_0$ and is finite; consequently, $X(\omega) = \text{const}\ \omega$. Further, the quantity σ_{eff} for $\omega \to 0$ is certainly finite; (the static conductivity $\sigma(0)$ is finite; σ_{eff} in the theory of the anomalous skin effect is also finite, and finally because the function $\sigma_{eff}(\omega)$ is even, σ_{eff} would vary as ω^{-2} if it had a pole, and thus would lead to $X \sim \omega^{3/2}$ instead of $X \sim \omega$). From this and from (21) it is clear that $\epsilon_{eff} = -\omega_s^2/\omega^2$, $R = \text{const} \cdot \omega^2$ and

$$\varepsilon'_{eff} = -\omega_s^2/\omega^2 \equiv -c^2/\delta_0^2\omega^2 \quad (\text{for} \quad \omega \to 0), \tag{22}$$

if we consider only the leading term.

It should be noted that if the differential relation (1) is valid for an alternating field (for $T \to 0$, and before the onset of quantum absorption) we have

$$\varepsilon'_{eff} = \varepsilon' = \varepsilon = \varepsilon_0(\omega) \tag{23}$$

$$-\frac{4\pi e^2 n_s}{m\omega^2} \equiv \varepsilon_0(\omega) - \frac{c^2}{\delta_0^2\omega^2},$$

where $\epsilon_0(\omega)$ is the part of ϵ not connected with the superconducting electrons (for further details, see references 2 and 21). Equation (23) is of course consistent with (22).

The presence of a pole proportional to ω^{-2} distinguishes a superconductor from normal conductors for which $\epsilon' = \epsilon - i4\pi\sigma/\omega$, where $\epsilon(0)$ and $\sigma(0)$ are finite. It is just because of this that the proof given below and the corresponding equations are somewhat different from those ordinarily used for nonsuperconductors (see, for instance, reference 17, Sec. 83, and reference 20).

Bearing in mind what has been said, we see that the integral

$$\oint \frac{\omega'\{\varepsilon'_{eff}(\omega') - \varepsilon'_{eff}(\infty)\}\,d\omega'}{\omega' - \omega} = 0$$

taken along the same contour as in (13), but with an additional detour around the semicircle below the point $\omega' = 0$, vanishes. Proceeding just as we did in going from (13) to (15), we obtain

$$\varepsilon_{eff}(\omega) = 1 - \frac{\omega_s^2}{\omega^2} + \frac{2}{\pi}\int_0^\infty \frac{\omega'\varepsilon'_{2eff}(\omega')\,d\omega'}{\omega'^2 - \omega^2}$$

$$\equiv 1 - \frac{\omega_s^2}{\omega^2} + 8\int_0^\infty \frac{\sigma_{eff}(\omega')\,d\omega'}{\omega'^2 - \omega^2}, \tag{24}$$

$$\sigma_{eff}(\omega) = -\frac{1}{2\pi^2}\int_0^\infty \frac{\omega'^2\{\varepsilon_{eff}(\omega') - 1\}\,d\omega'}{\omega'^2 - \omega^2} \tag{25}$$

$$= -\frac{\omega^2}{2\pi^2}\int_0^\infty \frac{\varepsilon_{eff}(\omega') - 1}{\omega'^2 - \omega^2}\,d\omega'$$

$$-\frac{1}{2\pi^2}\int_0^\infty \{\varepsilon_{eff}(\omega') - 1\}\,d\omega',$$

where we have also taken into account that along the real axis $\epsilon'_{eff}(-\omega) = \epsilon^*_{eff}(\omega)$ and that $\epsilon'_{eff}(\infty) = 1$ (vacuum). Equations (24) and (25) are valid of course for any medium. In the non-superconducting state, where $\epsilon_{eff}(\omega)$ has no singularity at $\omega = 0$, we have from (25)

$$\sigma_{eff}(0) = -\frac{1}{2\pi^2}\int_0^\infty [\varepsilon_{eff}(\omega') - 1]\,d\omega' \tag{26}$$

Where it is possible to introduce the quantity $\epsilon'(\omega)$ for all frequencies, (24) and (25) give the relation between $\epsilon(\omega)$ and $\sigma(\omega)$. It has already been pointed out that $R(\omega)$ is always positive; for the same reason $\sigma(\omega)$ is always positive and therefore, as is clear from (10), in the region where we can make use of the concept ϵ', $X(\omega) > 0$. From the meaning of the quantities $X(\omega)$ and $\delta_0 = c^2X/4\pi\omega$ [22], we must suppose that for a plane metallic surface $X(\omega) > 0$ and consequently, $\sigma_{eff}(\omega) > 0$ [see Eq. (21)].

In the infra-red part of the spectrum, at least for metals such as Ag, Au and Cu, there is a region in which

$$\varepsilon'_{eff} \approx \varepsilon \approx -4\pi e^2 n_0/m\omega^2 = -\omega_0^2/\omega^2, \tag{27}$$

where n_0 is by definition the concentration of free electrons in the metal (see references 2, 18, where

[21] V. L. Ginzburg, J. Exper. Theoret. Phys. USSR **21**, 979 (1951).

[22] E. Maxwell, P. W. Marcus and J. C. Slater, Phys. Rev. **76**, 1332 (1949).

n_0 is denoted by N). Unfortunately, it has not been strictly established for any superconductor whether a region exists in which (27) holds sufficiently accurately, but this is very probable and will be assumed in what follows.

According to the available data (which need improvement), we have, from optical measurements, $n_0 = 6 \times 10^{22}$ for tin[2,18], while the estimate from (17) (based on reference 21) gives $n_0 = 2.7 \times 10^{22}$ for tin, and this estimate is if anything, too low.

Comparing (27) and (24), where it is sufficiently accurate to neglect unity, we obtain the summation rule*,

$$n_0 = n_s + \int_0^\infty n_c(\omega')\, d\omega'; \tag{28}$$

$$n_c(\omega) = 2m\sigma_{\text{eff}}(\omega)/\pi e^2,$$

since $\omega_0^2 = 4\pi e^2 n_0/m$ and $\omega_s^2 = 4\pi e^2 n_s/m$ (to avoid a frequently occurring misconception, we emphasize again that everywhere above e and m are by definition just the charge and mass of the free electron). Equation (28), like all the other equations obtained, is valid for any temperature T; n_0 is practically independent of temperature, while $n_s = 2.83 \times 10^{11}\delta_0^{-2}$ [see Eq. (5)] and varies in such a way that for $T = T_k$, $n_s = 0$, while for $T = 0$, n_s has its maximum value n_{s0}. For tin, for $T \to 0, \delta_0 \sim 5 \times 10^{-6}$ and $n_{s0} \sim 1.1 \times 10^{22}$ (the use of alternative data leads to an even smaller value of n_{s0}). Thus, according to all the available data, $n_{s0} < n_0$, from which it follows that for $T \to 0$ and for sufficiently low frequencies, where absorption is absent, part of the free electrons becomes superconducting and part goes into some kind of bound state. From this evident limitation and lack of foundation of the so-called "two-fluid model" of superconductors become clear (see, for instance, reference 15, p. 194).

3. BEHAVIOR OF SUPERCONDUCTORS IN HIGH-FREQUENCY FIELDS (CALCULATION OF SURFACE IMPEDANCE)

In the general case, taking account of (24), we may write

$$\varepsilon'_{\text{eff}}(\omega) = \varepsilon_c(\omega) - (\omega_s^2/\omega^2) - 4\pi i \sigma_{\text{eff}}(\omega)/\omega \tag{29}$$

$$\varepsilon_c(\omega) = 1 + 8\int_0^\infty \frac{\sigma_{\text{eff}}(\omega')}{\omega'^2 - \omega^2}\, d\omega' .$$

For $T \to 0$, and not too high frequencies, $R \to 0$ and consequently, $\sigma_{\text{eff}} \to 0$. In this case (29) coincides with (23), obtained from (1), but with the difference that according to (23), the field in the metal decays exponentially, (since $\epsilon'_{\text{eff}} = \epsilon'$), while nothing can be said about the law of decay from (29). If the considerations pointing to the validity of (1) in a static field are indeed correct (see Sec. 1), then for frequencies such that $\sigma_{\text{eff}} \to 0$ for $T \to 0$, it is justified to apply (1) and (23) (this follows in particular from the fact that $\delta_0(\omega)$ changes very little in comparison with δ_0 right up to frequencies of order 10^{10} (see reference 11). With increasing frequency, absorption must set in, analogous to the inner photo-effect (we shall call this "quantum absorption") and we should expect that the threshold frequency, ω_k, for quantum absorption should be given by

$$\omega_k \sim kT_k/\hbar \sim 10^{11} \text{ to } 10^{12}; \tag{30}$$

$$\lambda_k = 2\pi c/\omega_k \sim 0.1 \text{ to } 1 \text{ cm}$$

For tin, according to Fawcett[23], $\lambda_k < 0.83$ cm. There are unfortunately no measurements in the region of shorter waves or for other metals in spite of the fact that such experiments are possible, and their importance was emphasized more

* Actually in (28) the upper limit should not be infinity but of the order of the maximum frequency for which (27) is still valid. It is supposed, however, as is usually valid, that to the accuracy with which we are working, it is permissable to replace the finite limit by infinity.

[23] E. Fawcett, Proc. Phys. Soc. (London) **66A**, 1071 (1953).

than 10 years ago[24]. For $T > 0$, $\sigma_{\mathrm{eff}} > 0$ even for $\omega < \omega_k$, on account of the appearance in the superconductor of "normal" electrons (excitations), so that for $T \to T_k$, $\sigma_{\mathrm{eff}} \to \sigma_{\mathrm{eff}n}$, where $\sigma_{\mathrm{eff}n}$ is the effective conductivity in the normal state. The "normal" electrons in the superconductor also make their contribution to $\epsilon_c(\omega)$, and it is convenient to write (though this has a rather formal character)

$$\varepsilon_c(\omega) = \varepsilon_0(\omega) + \varepsilon_n(\omega), \tag{31}$$

where ϵ_n is the contribution to ϵ_c from the "normal" electrons*, and ϵ_0 is the contribution of the "bound" electrons. For $T \to 0$ and $\omega < \omega_k$, $\epsilon_n(\omega) \to 0$, while ϵ_0 for $\omega \ll \omega_k$ is constant and may be estimated with the help of (28):

$$\varepsilon_0(\omega) \approx 8 \int_0^\infty \frac{\sigma_{\mathrm{eff}}(\omega')\, d\omega'}{\omega'^2 - \omega^2} \tag{32}$$

$$= \frac{4\pi e^2}{m} \int_0^\infty \frac{n_c(\omega')\, d\omega'}{\omega'^2 - \omega^2} \sim \frac{4\pi e^2 (n_0 - n_{s0})}{m\omega_k^2}$$

$$\sim \frac{4\pi e^2 n_0}{m\omega_k^2} \sim \frac{4\pi e^2 n_0}{m\,(kT_k/\hbar)^2} \sim 10^8 \text{ to } 10^{10}.$$

For $\omega \sim \omega_k$ the dependence of ϵ_0 on ω must already be appreciable. The estimate (32) coincides, as is to be expected, with that given in reference 2; the nature of the estimate is such that even the value $\epsilon_0 \sim 10^7$ would not apparently be contradictory. From the experimental data[19,22] for tin it is probably possible to deduce only that for $\lambda \sim 1$ cm, $\epsilon_0 \ll \omega_s^2/\omega^2 \sim 10^9$; a value $\epsilon_0 \sim 10^8$ is quite compatible with (32) and with this estimate. For $T \to T_k$, ϵ_0 and ϵ_n tend to their respective values in the normal state, where $\epsilon_0 \sim 1$.

It is natural to try to calculate the quantities $\epsilon_n(\omega)$ and $\sigma_{\mathrm{eff}}(\omega)$ connected with the "normal" electrons in the same way as is done in the theory of the anomalous skin-effect in metals in the normal state[25,26]. We need only consider the case of the extreme anomalous skin-effect, where the mean free path is much greater than the skin depth ($l \gg \delta$). Under these conditions the calculation of Z for superconductors has been carried out both on the basis of the kinetic equations for the normal electrons[22,27], and on the basis[2,21] of the simpler "ineffectiveness concept"[26] which is essentially of a dimensional character. Although they are more complicated, the calculations[22,27] are more consistent with the model assumed (free electrons in a medium with $\epsilon = \epsilon_0 - c^2/\delta_0^2\omega^2$). On the other hand this model itself, in its application to the "normal" electrons in a superconductor, is completely without foundation, particularly if we take account of the possibility of quantum absorption, which for $T > 0$ may take place even for $\omega < \omega_k$*. In this connection the calculations in reference 21 are to be preferred, as being less tied to a particular model. In general it can be said that reliable calculations of Z or ϵ_{eff} in superconductors are not yet possible, and it is therefore useful to carry out the calculations by a variety of methods. From comparison of the results it is possible to assess to a certain extent how reliable the calculations are. In the simplest, and therefore most convenient variant of the calculations, using the method of dimensions, it is assumed[21] that

---

* To avoid misunderstanding we must point out that in references 2 and 21 and in a number of other papers, the whole contribution of the "normal" electrons is included in σ_{eff}, which is a complex quantity (in the present paper σ_{eff} is real). It is evident that $-4\pi i\sigma_{\mathrm{eff}}/\omega$ in references 2 and 21 is equal to $\epsilon_n - 4\pi i\sigma_{\mathrm{eff}}/\omega$ in the notation of the present paper [see Eqs. (29) and (31)].

[24] V. L. Ginzburg, J. Exper. Theoret. Phys. USSR **14**, 134 (1944).

* For quantum absorption in a particular range of frequencies the relevant part of σ_{eff} is presumably σ_k, where σ_k is a quantity analogous to the ordinary conductivity. In this case $\sigma_{\mathrm{eff}} = \sigma_k + \sigma'_{\mathrm{eff}}$ where σ'_{eff} is an effective conductivity connected with the normal electrons, which in the region of the anomalous skin-effect is completely different from the "ordinary" (static) conductivity of the "normal" electrons [see Eq. (33)].

[25] G. Reuter and E. H. Sondheimer, Proc. Roy. Soc. (London) **195A**, 336 (1948); E. H. Sondheimer, Proc. Roy. Soc. (London) **224A**, 260 (1954).

[26] A. B. Pippard, Proc. Roy. Soc. (London) **191A**, 385, 399 (1947); **224A**, 273 (1954).

[27] A. A. Abrikosov, Dokl. Akad. Nauk SSSR **86**, 43 (1952).

$$\varepsilon'_{\text{eff}} = \varepsilon_s - i\frac{4\pi\sigma}{\omega}\frac{2\pi}{\sqrt{3}}\frac{\delta'}{l}, \tag{33}$$

$$\delta' = -\frac{ic}{\omega\sqrt{\varepsilon'_{\text{eff}}}},$$

$$\varepsilon_s = \varepsilon_0 - \frac{c^2}{\delta_0^2\omega^2},$$

where l is the mean free path and σ the static conductivity of the "normal" electrons in the superconductor (this means that for normal skin-effect conditions the normal current density in the superconductor would be $j_n = \sigma E$). The conductivity due to quantum absorption is of course not taken into account in (33). From (33) it follows that the quantity $y = \sqrt{\epsilon'_{\text{eff}}} = 4\pi/cZ$ is determined by the equation

$$y^3 - \varepsilon_s y + 8\pi^2 c\sigma/\sqrt{3}\omega^2 l = 0.$$

Hence, as was shown in reference 8, we have

$$\varepsilon_s \equiv \varepsilon_0 - \frac{c^2}{\delta_0^2\omega^2} = -\frac{16\pi^2}{c^2}\frac{(X^2-3R^2)}{(X^2+R^2)^2}, \tag{34}$$

$$\sigma/l = (16\pi\omega^2/c^4)\sqrt{3}R(X^2+R^2)^{-2}.$$

The coefficient in front of δ'/l in (33) is chosen so that in the normal state (for $\epsilon_s \to 0$) we should have

$$Z = Z_n = (\sqrt{3}\pi\omega^2 l/c^4\sigma)^{1/3} \tag{35}$$

$$\times (1+\sqrt{3}i);\ X = \sqrt{3}R,$$

i.e., so that the relation should apply which is obtained in the more rigorous theory (for the normal state) for diffuse reflection of the electrons at the surface.

With the same assumption, the kinetic calculation[27] gives

$$\varepsilon_s = \varepsilon_0 - c^2/\delta_0^2\omega^2 \tag{36}$$

$$= -(4\pi^2/c^2)X^2(X^2+R^2)^{-2}(3-\eta),$$

$$\sigma/l = (16\pi\omega^2/3c^4)\cdot X^3(X^2+R^2)^{-3}(1+\eta),$$

$$R/X = \frac{1}{2\pi}\left[2\sqrt{\eta}\tan^{-1}\sqrt{\eta} - \ln\left(\frac{1+\eta}{4}\right)\right],$$

where for $\eta < 0$ we must replace $\sqrt{\eta}\tan^{-1}\sqrt{\eta}$ by $-\sqrt{|\eta|}\tanh^{-1}\sqrt{|\eta|}$; a graph of the function $\eta(R/X)$ is given by Abrikosov[27]. For $\eta = 3$, it is evident that $\epsilon_s = 0$, and (36) goes over into (35) just as does (34): for $\eta = -1$ and $\epsilon_s < 0$ we have $\sigma = 0$ and $\epsilon_s = -16\pi^2/c^2X^2$; this last result follows directly from (34) for $\sigma = 0$*. To facilitate comparison between (34) and (36) it is convenient to express them in the forms:

$$\varepsilon_s = -16\pi^2 c^{-2}X^{-2}\varphi_1(R/X), \tag{34a}$$

$$\sigma/l = 16\sqrt{3}\pi\omega^2 c^{-4}X^{-3}f_1(R/X),$$

$$\varepsilon_s = -16\pi^2 c^{-2}X^{-2}\varphi_2(R/X), \tag{36a}$$

$$\sigma/l = 16\sqrt{3}\pi\omega^2 c^{-4}X^{-3}f_2(R/X),$$

where the form of the functions $\varphi_{1,2}$ and $f_{1,2}$ is evident from comparison with (34) and (36). These functions are plotted in Figs. 1 and 2 (the points $R/X = 0$ and $R/X = 1/\sqrt{3} = 0.58$ correspond to $T = 0$ and $T = T_k$), and it can be seen that the differences between (34) and (36) are not large and indeed probably within the precision which either formula can claim (except possibly in the regions close to $T = T_k$ and $T = 0$ where φ_1, φ_2 and f_1, f_2, respectively, approach zero). From the point of view of the microtheory of superconductivity, the

---

* Equation (33) is valid only if the wave decays rapidly as one goes into the metal, so that for $\sigma = 0$ it is necessary that $R = 0$ and $\epsilon_s < 0$; in the case of (36), for $\sigma = 0$ and $\epsilon_s > 0$, $\eta \to \infty$, $X = 0$ and $R = 4\pi/c\sqrt{\epsilon_s}$, which corresponds to a transparent body. As is clear from (34), for instance, ϵ_s can in principle change sign. At the point where $\epsilon_s = 0$, $X = \sqrt{3}R$ and if $\omega < \omega_k$ the superconductor becomes transparent. In all probability, the frequency ω_0 for which $\epsilon_s = 0$ is greater than ω_k, so that transparency does not occur (in this connection it is important that in the relevant rrequency range ϵ_0 may depend appreciably on ω). The introduction of a universal impedance is possible only if $|\epsilon'_{\text{eff}}| \gg 1$, and consequently the region close to the point $\epsilon_s = 0$ requires special consideration if σ_{eff} is close to zero. Generally speaking, the formulas given above are not valid in this region,but their use for deciding the question of the existence of this region is quite permissable.

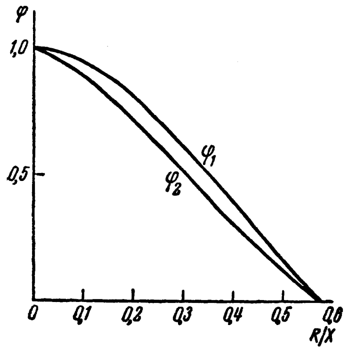

FIG. 1

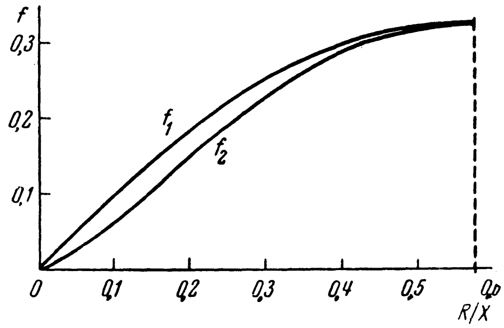

FIG. 2

most interesting feature is the value of $\epsilon_s(\omega)$ at low temperatures; this can be determined from measurements of $X(\omega)$ (R has only secondary importance in the determination of ϵ_s, since in the region concerned $R^2/X^2 \ll 1$ for $\omega < \omega_k$ and, for instance, $\varphi_1 = 1 - 5(R/X)^2$. In this region, we have from (34) $\sigma/l = 16\sqrt{3} \times \pi\omega^2 c^{-4} X^{-4} R \simeq \sqrt{3}\,\omega^2 \epsilon_s^2 R/16\pi^3$, so by measuring R (calorimetrically, for instance), we can find σ/l*, using the static value of δ_0 (which is permissible as long as $|\epsilon_s| \gg \epsilon_0$). The accuracy of such a determination cannot of course exceed that of the formulas (34) and (36) themselves. We shall not discuss here the question of the measurement of Z for superconducting films (see references 8 and 21).

In conclusion, we should like once more to emphasize the importance, from the point of view of studying superconductivity, of optical measurements of n_0 [see (27)] and measurements of $\epsilon'_{\mathrm{eff}}(\omega)$ in the region of centimeter and millimeter waves for various superconductors.

* In the region $\epsilon_s = -c^2/\delta_0\omega^2$, it follows from the relation given for σ/l and (35) that $R/R_n \sim \delta_0^2\omega^{4/3}$. According to Galkin and Bezuglyi[28], just such a dependence is found in tin for $\omega < 3 \times 10^{11}$. A deviation from this dependence, within the framework of the approximation used, would indicate an influence of the term ϵ_0 in the expression $\epsilon_s = \epsilon_0 - (c/\delta_0\omega)^2$.

[28] A. A. Galkin and P. A. Bezuglyi, Dokl. Akad. Nauk SSSR **97**, 217 (1954).

Translated by D. Shoenberg
269

ERRATA
(both of our own and of JETP)

Vol.	Page	Column	Line	Reads	Should read
2	434	2	22	27.3 μ	23.7 μ
2	557	Fig. 10			On the right hand side, abscissa values should read 0, 200 400, 600, 800, 1000.
2	591	2	7	$A = \frac{e^2 H_0^2 \delta_{00}}{mc^2}$	$A = \frac{e^2 H_{00}^2 \delta_{00}^2}{mc^2}$
2	754	1	3 ff.	————	[14] B. B. Kinsey and G. A. Bartholomew, Phys. Rev. **82**, 380 (1951). [15] B. B. Kinsey and G. A. Bartholomew, Phys. Rev. **83**, 234 (1951).
2	771	1	10	Intermediate State	Intermediate State of Tin
	771	1	19	sphere of lead	sphere of tin
3	145	1	1	$R = 10\ ec$	$R = 1/ec$