Letters to the Editor

Solution of the Equations of Pseudo-scalar Meson Theory with Pseudo-scalar Coupling

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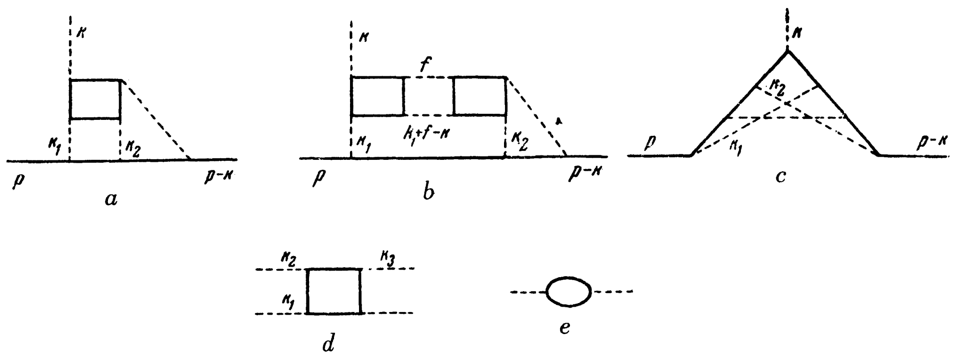
THE problem of renormalization of the meson charge g was analyzed in references 1 and 2. It was observed thereby that the expressions determined by Abrikosov and Khalatnikov[3] for the vertex portion Γ, for the nucleon Green's function G, and for the meson Green's function D, are correct only for $g_1^2 \ll 1$ (g_1 is the meson charge prior to re-normalization), as suggested in reference 3, but also for all values of g_1^2.

The proof of this statement contains one point which was not fully analyzed. This concerns the effect of meson-meson scattering at the vertex portion. Even though the simplest diagram a, which includes the effect of interest to us (proportional to $[\ln \lambda_p^2 \lambda_k^{-2}]^{-1})^*$, is small and cannot play an effective role, nevertheless diagram b, which contains one more meson scattering act, turns out to be of the same order as a. This raises the problem of summing a large number of various diagrams produced by meson-meson scattering. Although the aggregate of the diagrams can hardly change the conclusion reached in references 1 and 2 that the renormalized meson charge g vanishes , nevertheless reference 2 calls attention to the importance of examining the meson-meson scattering.



References 1 and 2 employed the double-limit technique developed by Abrikosov and Khalatnikov[3] of cutting-off the divergent terms of the equations that determine G, Γ, and D. The relationship used there for the limiting cut-off momenta for the integration with respect to the nucleon momenta λ_p and the meson momenta λ_k was:

$$\lambda_k: \ln \lambda_p^2 \lambda_k^{-2} \gg 1.$$

If this equality is strengthened substantially:

$$\ln \lambda_p^2 m^{-2} \gg \ln \lambda_k^2 m^{-2},\ \lambda_p \to \infty,\ \lambda_k \to \infty, \tag{1}$$

it becomes possible to obtain exact solutions for the equations for G, D and Γ, fully accounting for the meson-meson scattering in such an approximation of a point interaction, considered as a limit of the "spread" interaction[4] with finite λ_p and λ_k. We shall now prove that if equation (1) is used in the pseudo-scalar meson theory with pseudo-scalar coupling, G, D and Γ assume the following values:

$$G = \hat{p}^{-1},\ p^2 \gg m^2. \tag{2}$$

$$D = k^{-2}[1 + \pi^{-1} g_1^2 \ln \lambda_p^2 k^{-2}]^{-1},\ \lambda_k^2 \gg k^2 \gg m^2, \qquad D = k^{-2},\ k^2 \gg \lambda_k^2; \tag{3}$$

$$\Gamma(p, p-k; k) = \Gamma_5,\ \lambda_p^2 \gg p^2 \gg m^2, \tag{4}$$

$$\lambda_p^2 \gg (p-k)^2 \gg m^2,\ \lambda_k^2 \gg k^2 \gg m^2,$$

$$\Gamma(p, p-k; k) = 0,\ p^2 > \lambda_p^2 \quad \text{or} \quad k^2 > k_k^2.$$

The only diagram used in equations (2)- (4) is diagram e, contained in the expression for D. We shall now verify that an arbitrary unaccounted diagram is of no significance. Let us examine first an arbitrary n-fold overlapping diagram, contained in $\Gamma(p, p-k; k)$. Let $p^2 \sim (p-k)^2 \sim k^2$. Taking into account that to such a diagram c there corresponds only one logarithmic integration $[p^2 \ll k_1^2, k_2^2 \ldots k_n^2 \ll \lambda_k^2]$, we find the value of the operator b, which contains $(n+1)$ D-functions, $(2n+3)$ Γ-functions, and $2(n+1)$ G-functions:

$$b \sim g_1^{2(n+1)} \int_{p^2}^{\lambda_k^2} \frac{dk_1^2}{k_1^2} \left[1 + \frac{1}{\pi} g_1^2 \ln \lambda_p^2 k^{-2}\right]^{-n-1} \quad (5)$$

$$\leqslant \frac{\ln \lambda_k^2 p^{-2}}{[\ln \lambda_p^2 \lambda_k^{-2}]^{n+1}} \ll 1$$

$$[\ln \lambda_p^2 \lambda_k^{-2} = \ln \lambda_p^2 m^{-2} - \ln \lambda_k^2 m_2^{-2} \approx \ln \lambda_p^2 m^{-2}].$$

Here we did not assume g_1^2 to be small compared with unity. If $n = 0$, we have an aggregate of nonoverlapping diagrams equal in the order of magnitude to $\ln (\lambda_k^2 p^{-2}) / \ln (\lambda_p^2 \lambda_k^{-2}) \ll 1$. We see that all the overlapping and nonoverlapping diagrams make a negligible contribution to Γ if (1) is used.

Let us now turn to the meson-meson scattering. The elementary small square (see diagram d) equals $g_1^4 \ln \lambda_p^2 \lambda_k^{-2}$, where k^2 is the largest from among k_1^2, k_2^2, k_3^2, $(k_1 + k_2 - k_3)^2$. Accordingly, and also taking into account that only one logarithmic integration corresponds to diagram a [assuming that $p^2 \sim (p-k)^2 \sim k^2$] we obtain in the region $p^2 \ll k_1^2 \approx k_2^2 \ll \lambda_k^2$ the following estimate of the operator a:

$$a \sim g_1^6 \int_{p^2}^{\lambda_k^2} \left[1 + \frac{1}{\pi} g_1^2 \ln \lambda_p^2 k_1^{-2}\right]^{-3} [\ln \lambda_p^2 k_1^{-2}] \frac{dk_1^2}{k_1^2}.$$

$$\leqslant \frac{\ln \lambda_k^2 m^{-2}}{[\ln (\lambda_p^2 m^{-2})]^{-2}} \ll 1.$$

The transition from a to b, involves one more logarithmic integration with respect to f^3, resulting in the factor:

$$g_1^4 \int_{q^2}^{\lambda_k^2} \frac{df^2}{f^2} \left[1 + \frac{1}{\pi} g_1^2 \lambda_p^2 f^{-2}\right]^{-2} \ln \lambda_p^2 f^{-2}$$

$$\approx \frac{\ln \lambda_k^2 m^{-2}}{\ln \lambda_p^2 m^{-2}} \ll 1.$$

We see that condition (1) insures a rapid convergence of the series of consecutive meson-meson scattering acts, making it unnecessary to examine in detail all the meson-meson scattering diagrams.

The above estimates prove the correctness of equations (1)-(3) for any value of g_1^2. Unlike the case in references 1 and 2, meson-meson scattering is known here to play a negligible role. A consequence of (1)-(2) is the vanishing of the renormalized meson charge.

$$g^2 = g_1^2 [1 + \pi^{-1} g_1^2 \ln \lambda_p^2 m^{-2}]^{-1}$$

$$\rightarrow \pi [\ln \lambda_p^2 m^{-2}]^{-2} \rightarrow 0.$$

Let us remark that the use of condition (1) simplifies most radically the problem of finding G, D and Γ. The examination given here shows that meson-meson son scattering (without involving the term $\lambda\phi^4$) does not change the conclusion reached in references 1 and 2, that the renormalized meson charge g vanishes.

Recently Ter-Martirosian, Diatlov, and Sudakov, using a very clever technique, determined the value of aggregate of substantial meson-meson scattering diagrams, and established that in the case when the meson momenta are small compared with certain values of λ, the entire aggregate of the diagrams differs from the simplest diagram d by only a numerical factor. No conditions whatever were imposed here on λ_p and λ_k. The result obtained by Ter-Martirosian, Diatlov, and Sudakov show that the proof that $g = 0$ is independent of the relationship between λ_p and λ_k.

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* We shall use here the symbols of references 1 and 2, except that λ will be used for Λ'

[1] I. Ia. Pomeranchuk, Dokl. Akad. Nauk SSSR 104, 51 (1955)

[2] I. Ia. Pomeranchuk, Dokl. Akad. Nauk. SSSR 105, 3 (1955)

[3] A. A. Abrikosov and I. M. Khalatnikov, Dokl. Akad. Nauk. SSSR 103, 993 (1955)

[4] L. D. Landau, A. A. Abrikosov and I. M. Khalatnikov Dokl. Akad. Nauk SSSR, 95, 497 (1954)

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Orientation of Planes in Double V° Decay Events

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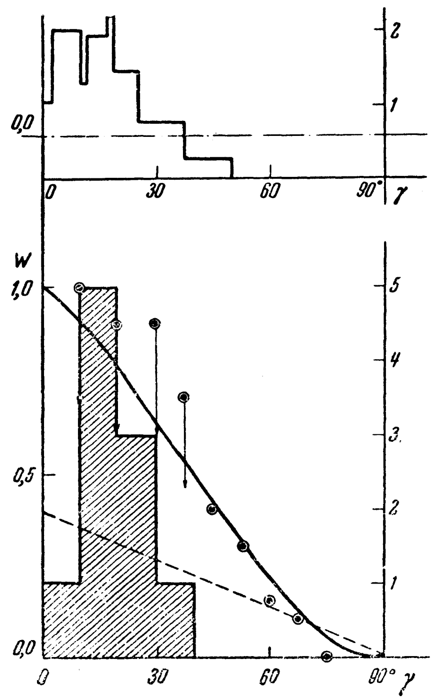
IN a letter under the same title, Bellam[1] et al considered ten so-called double V° events obtained with a Wilson cloud chamber, two V° decays appearing on the same photograph. The authors assume that both V° particles originated from the same disintegration, and they consider the angles between the plane of their lines of flight and the planes of their decay products. Denoting η_a as the lesser and η_b as the greater of the two angles for a double V° event, the authors plot all ten events as points on an $\eta_b - \eta_a$ plane. Under these circumstances, it turns out that the majority of the points lie in the lower right corner of the graph, i.e., η_a is comparatively small ($< 40°$) as a rule, and η_b is large ($> 40°$ in most cases). On the basis, despite the poor statistics, the authors assume that there is a possibility of some correlation between the directions of the decay planes of the secondary particles and the plane of the V° particles. From this the authors conclude that at least one V° particle has a spin greater than $1/2$[2].

Actually, on the basis of the available statistics and the positions of the points on the graph of reference 1, no conclusions can be made concerning a correlation. From elementary considerations it is apparent that if one does not distinguish the angles for some physical reason, the lesser angles will always be concentrated about a small value, and the larger angles about a large value. For η_a in the interval η_a to $\eta_a + d\eta_a$, and η_b in the interval $\eta_b + d\eta_b$, and for the conditions $\eta_a \leq \eta_b$ and isotropic decay, the probability is

$$\frac{6}{5} \cdot \frac{64}{\pi^4}\left(\frac{\pi}{2} - \eta_a\right)\eta_b \, d\eta_a \, d\eta_b.$$

The figure shows the integral curve of the probabilities W for a number of events lying in the lower right corner of the $\eta_b - \eta_a$ plane, to the right of a line cutting off the axes η_b and η_a with segments γ and $\pi/2 - \gamma$, depending on the magnitude of γ, and plotted with experimental points from reference 1. The arrows show the errors as defined by $\sqrt{N}$. The majority of the experimental points lie satisfactorily close to the calculated curve.



Nevertheless, one can draw certain conclusions concerning the orientations of decay planes if one considers the distribution for angle η_a only. The differential curve of the distribution will appear as a straight line (dotted line in the figure.) Experimental data from reference 1 are shown in the figure in the form of a crosshatched histogram. One may note a grouping of experimental points in the neighborhood of the angle $\sim 20°$. These values are analogous to those obtained earlier in reference 3 where particles of a definite type $\Lambda_0 \Lambda^-$