where $\psi_{1m}^{(0)}$, $\psi_{2m}^{(0)}$ are wave functions of the free molecule $\Phi_{N_0}(N)$, $\Phi_{N_{0+1}}(N)$ are wave functions of the free field, and N is the number of photons. Substituting (14) in (12), multiplying first by $\psi_1^{(0)*}$, then by $\psi_2^{(0)*}$, and integrating, we get two equations. The condition of their consistency gives:

$$\mathcal{E}_{1,2} = \tfrac{1}{2}(\mathcal{E}_1^{(0)} + \mathcal{E}_1^{(0)}) \pm \tfrac{1}{2}\sqrt{(\mathcal{E}_1^{(0)} - \mathcal{E}_2^{(0)})^2 + |V_{12}|^2}$$

Transitions of the system molecule + field with radiation of quanta of energy $\mathcal{E}_1 - \mathcal{E}_2 = h\Omega_0$ are thus possible. It is also not difficult to demonstrate that the matrix element of the dipole moment corresponding to the transition at the frequency Ω_0 is proportional to $\vec{\mu}_{22} - \vec{\mu}_{11}$.

Let us note that the measurements of the frequency Ω_0 offer the possibility of the experimental determination of the matrix element $|V_{12}|$, which is proportional to the product of the dipole moment and the magnitude of the field intensity F. Such measurements offer a method for the precise determination of the field intensity of frequency ω, if the matrix element $|\vec{\mu}_{12}|$ is known. If conversly, the field intensity of frequency ω is known with sufficient precision, it is possible to determine $\vec{\mu}_{12}$ precisely.

Let us evaluate the order of magnitude of possible frequencies $\Omega_0 \sim |\vec{\mu}_{12}| F/h$. Let $|\vec{\mu}_{12}| \sim 10^{10}\,\text{sec}^{-1}$ and $F = 1$ cgs unit = 300 v/cm; then $\Omega_0 \sim 10^9\,\text{sec}^{-1}$. If, however, $F = 10$ cgs units, then $\Omega_0 = 10^{10}\,\text{sec}^{-1}$. By changing the field intensity at the frequency ω, we can change the frequency Ω, which presents some convenience in the experimental handling of the problem.

In conclusion, let us note that radiation at the frequency Ω_0 will be observed only for molecules whose dipole moments $\vec{\mu}_{11}$ and $\vec{\mu}_{22}$ differ from zero.

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Anomalous Skin Effect with Arbitrary Collision Integral

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In previous publications[1,2] an expression was obtained for the surface impedance of metal in the case of anomalous skin effect*. It was then assumed that the integral of collisions can be written with the help of the relaxation time τ in the form:

$$(\partial f/\partial t)_{st} = (f - f_0)/\tau, \tag{1}$$

where f is the electron distribution function, $f_0(\epsilon)$ is the Fermi equilibrium distribution function.

Introduction of the relaxation time can be rigorously established only at high temperatures ($T \gg \Theta$, Θ the Debye temperature). At lower temperatures, the collision integral, in general, cannot be written in form (1), and one must consider an arbitrary collision integral. In the present contribution it is proved that the formula for impedance, obtained in reference 1, is valid for an arbitrary collision integral. Let us note that the left-hand member of (1) is not assumed to be small in comparison with ωf (ω – frequency of external field).

The complete system of equations has the form:

$$\frac{d^2E_\alpha(z)}{dz^2} = \frac{4\pi i\omega}{c^2}\frac{2e^2}{(2\pi\hbar)^3}\int n_\alpha[\psi(z;\mathbf{n}) - \psi(z;-\mathbf{n})]\,dS_p\,^{**}; \tag{2}$$

$$n_z\frac{\partial\psi(z;\mathbf{n})}{\partial z} + \check{L}^{-1}\psi(z;\mathbf{n}) = E_\beta(z)\,n_\beta \equiv E_x n_x + E_y n_y; \tag{3}$$

$$\psi(z;\mathbf{n})|_{z=+\infty} = 0,$$

$$\psi(0;n_x,n_y,n_z) = q\psi(0;n_x,n_y,-n_z);\quad n_z > 0$$

$$\mathbf{n} = \mathbf{v}/v;\quad \mathbf{v} = \nabla_{\mathbf{p}}\varepsilon(\mathbf{p}). \tag{4}$$

Here $-e(\delta f_0/\delta\epsilon)\psi\exp(i\omega t)$ is an addition to the equilibrium distribution function of distribution $f_0(\epsilon)$; $E_\alpha(z)\exp(i\omega t)$ – are the components of the electric field inside the metal; $\check{L}^{-1}=v^{-1}[(\delta/\delta t)_{st}+i\omega\check{I}]$ is operator of the free path length ($\check{I}$ – unit operator)+; ϵ, p, v are the energy, quasi-momentum and velocity of electrons. The parameter q in (4) characterizes reflection of electrons from the surface of metal ($q=0$ corresponds to diffuse and $q=1$ to specular reflection). Integration with respect to dS_p in (2) and below is carried out for all momenta on the Fermi surface $\epsilon(\mathbf{p})=\epsilon_0$, $n_z\geq 0$.

Axis OZ coincides with the direction of internal normal to the surface of metal.

Writing equation (3) separately for $\psi(z;\mathbf{n})$ and $\psi(z;-\mathbf{n})$ and eliminating $\psi(z;\mathbf{n})+(z;-\mathbf{n})$, we will obtain an equation for the function $\psi_-(z;\mathbf{n})=\psi(z;\mathbf{n})-\psi(z;-\mathbf{n})$++, which determines this current density

$$\frac{\partial^2\psi_-(z;\mathbf{n})}{\partial z^2}-\frac{1}{n_z^2}\check{L}^{-2}\psi_-(z;\mathbf{n})=\frac{2}{n_z^2}E_\beta(z)\check{L}^{-1}n_\beta. \quad (5)$$

In this equation the fact that the main contribution to the current density in the anomalous skin effect comes from electrons with small n_z has been taken into account (see references 1,2). Therefore we may consider that the operator $\check{L}$ acts on functions whose values are taken on the curve $n_z=0$ on the the Fermi surface.

Continuing functions $E_\alpha(z)$ and $\psi_-(z;\mathbf{n})$, as even functions, into the region of negative z, and going over to Fourier transforms, we find:

$$\mathcal{E}_\beta(t)\left\{t^2\delta_{\alpha\beta}+\frac{4\pi i\omega}{c^2}\frac{2e^2}{(2\pi\hbar)^3}\times\int 2n_\alpha[\check{I}+t^2n_z^2\check{L}^2]^{-1}\check{L}n_\beta\,dS_{\mathbf{p}}\right\}=-2E'_\alpha(0)+\frac{4\pi i\omega}{c^2}\frac{2e^2}{(2\pi\hbar)^3}\times\int 2n_z^2n_\alpha[\check{I}+t^2n_z^2\check{L}^2]^{-1}\check{L}^2\psi'(0;\mathbf{n})\,dS_{\mathbf{p}}, \quad (6)$$

where $\mathcal{E}_\alpha(t)=\int_{\infty}^{\infty}E_\alpha(z)\exp(-itz)\,dz$ is the Fourier transform of the component of the electric field ($\alpha=x,y$).

Analogously to what was done in references 1,2 it can be shown that the asymptotic expression for the surface impedance, in the limiting case of the anomalous skin effect, is obtained by replacing the integrals in (6) by their asymptotic expressions for large t. In the process the operator $\check{L}$ drops out. In fact, for large t, since operators $\check{L}$ and $[\check{I}+t^2n_z^2\check{L}^2]^{-1}$ commute,

$$\int n_\alpha[\check{I}+t^2n_z^2\check{L}^2]^{-1}\check{L}n_\beta\,dS_{\mathbf{p}}\approx\frac{1}{t}\int_0^{2\pi}\frac{n_\alpha(\varphi)\,d\varphi}{K(\varphi,\pi/2)}\left\{\int_0^\infty dx[\check{I}+x^2\check{L}^2]^{-1}\check{L}\right\}n_\beta(\varphi);$$

$$\left\{\int_0^\infty dx[\check{I}+x^2\check{L}^2]^{-1}\check{L}\right\}n_\beta(\varphi)=\{\operatorname{arc\,tg}x\check{L}\}_{x\to\infty}n_\beta(\varphi)=\frac{\pi}{2}n_\beta(\varphi).$$

Here $K(\varphi,\vartheta)$ – Gauss curvature of the Fermi surface; α,ϑ – angles of a spherical coordinate system in the velocity space: $\mathbf{n}=(\sin\vartheta\cos\varphi;\ \sin\vartheta\sin\varphi;\ \cos\vartheta)$;

$$n_x(\varphi)=\cos\varphi;\quad n_y(\varphi)=\sin\varphi;\quad dS_{\mathbf{p}}=\sin\vartheta\,d\vartheta\,d\varphi/K(\varphi,\vartheta).$$

In a similar way one can calculate the asymptotic value of the second integral in Eq. (6).

As a result we arrive at the following integral equation for $\mathcal{E}_\alpha(t)$:

$$\mathcal{E}_\alpha(t)=\frac{-2E'_\alpha(0)+(1-q)\,i\dfrac{6\omega}{c^2}B_\alpha\displaystyle\int_0^\infty\frac{\ln t-\ln\tau}{t^2-\tau^2}\mathcal{E}_\alpha(\tau)\,d\tau}{t^2+i(3\pi^2\omega/c^2)(B_\alpha/t)}, \quad (7)$$

where B_α is the principal value of the tensor

$$B_{\alpha\beta} = \frac{8e^2}{3(2\pi\hbar)^3} \int_0^{2\pi} \frac{n_\alpha(\varphi)\, n_\beta(\varphi)\, d\varphi}{K(\varphi, \pi/2)}, \qquad (8)$$

and the axes X and Y are chosen along its principal axes.

The integral equation (7) is independent of the form of the collision integral and, in particular, coincides with the equation that was obtained when we introduced the relaxation time τ. Therefore we may use immediately the results of the previous work[1] and write down the surface impedance Z_α in the limiting cases of diffuse ($q = 0$) and specular ($q = 1$) reflections of electrons from the surface of the metal:

$$Z_\alpha = R_\alpha + iX_\alpha = -\frac{4\pi i\omega}{c^2}\frac{E_\alpha(0)}{E'_\alpha(0)} \qquad (9)$$

$$= \begin{cases} \left(\sqrt{3}\,\dfrac{\pi\omega^2}{c^4 B_\alpha}\right)^{1/3}(1 + i\sqrt{3}); & q = 0. \\ \dfrac{8}{9}\left(\sqrt{3}\,\dfrac{\pi\omega^2}{c^4 B_\alpha}\right)^{1/3}(1 + i\sqrt{3}); & q = 1. \end{cases}$$

$$X_\alpha / R_\alpha = \sqrt{3}. \qquad (10)$$

Examination of the integral equation (7) shows that formulas (9) and (10) for the surface impedance are valid with accuracy to within a small numerical factor of the order of unity, for arbitrary linear relation between $\psi(0; n_x, n_y, n_z)$ and $\Psi(0; n_x, n_y, -n_z)$ on the boundary metal - vacuum.

Thus, in the region of the anomalous skin-effect dependence of the surface impedance upon frequency ($Z_\alpha \sim \omega^{2/3}$), its independence of temperature, and relation (10) are all valid not only for arbitrary law of dispersion of electrons $\epsilon = \epsilon(p)$, but also for arbitrary collision integral and any relation $\psi(0; n_x, n_y, n_z[= Q\,\psi(0; n_x, n_y, -n_z)$ at the boundary metal-vacuum.

In conclusion the authors wish to avail themselves of this opportunity to thank I. M. Lifshitz for the discussion of their results.

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* Anomalous skin effect occurs at high frequencies and low temperatures, when the mean free path of electrons is large compared with the depth of penetration of the field into metal.

**Here use is made of the spherical symmetry of Fermi surface.

\+ All operators operate on functions in momentum space.

++ In so doing we consider that the collision operator has a center of symmetry.

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The Internal Compton Effect

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THIS letter deals with some conclusions which can be drawn from the theoretical analysis of the internal Compton effect.

In the internal Compton effect, a nucleus jumps from an excited to the ground state, with the result that the atom is ionized and a gamma ray radiated. The emission of an electron and a gamma ray when the nucleus de-excites itself is due to the interaction of the electron with the nucleus and the electromagnetic field: i.e., it is a third order effect. This is presumably why the effect has not been observed till recently. It was in 1953 that Brown and Stump[1] saw a continuous spectrum accompanying internal conversion.

The internal Compton effect was analyzed in reference 1 by use of nonrelativistic perturbation theory, i.e., with the restriction that the energy difference between the excited and ground states was much less than the electron rest energy. In the perturbation theory approach, the internal Compton effect is of third order and considered to take place in the following steps: a virtual gamma ray emitted by the nucleus is scattered from the atomic electrons and as a result a real gamma ray is radiated and an electron ionized.

The initial state (1) of the system consists of an excited nucleus, an electron in its ground state and no quanta. In the final state (2) the nucleus is in its ground state, the electron is in an excited (ionized) state and a gamma ray has been emitted. The perturbation was taken to be the nonrelativistic interaction between a charged