levels with the filling of which (group) the period begins, is equal to the number of the period, one can consider that Eq. (10) as coming from the rule of systematic filling (Aufbau principle) of the $(n + l)$ groups, an expression of the relation between the number of the period in the system of Mendeleev and the charge of the nucleus of the atoms of the element beginning the period.

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Gamma Radiation Accompanying the Absorption of Fast Protons by Nuclei

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THE absorption of fast protons by nuclei can be accompanied by the radiation of photons through two mechanisms. On the one hand, the absorption of a proton by the nucleus causes a diffraction perturbation of the proton wave, enabling it to radiate (diffraction radiation)[1]; on the other hand, radiation can be caused by the direct absorption of the proton (bremsstrahlung radiation)[2]. The second mechanism is more essential. In the present note we wish to estimate the role of the anomalous magnetic moment of the proton in bremsstrahlung radiation, our earlier calculation of which[2] was not exact.

We will assume that for an estimate of the bremsstrahlung the proton can be described by the Dirac equation with an anomalous magnetic moment μ':

$$(\gamma_\nu \partial/\partial x_\nu - ie\gamma_\nu A_\nu - i\mu'\gamma_\nu\gamma_\rho F_{\nu\rho} + m)\,\psi = 0, \qquad (1)$$

here $F_{\nu\rho} = \partial A_\rho/\partial x_\nu - \partial A_\nu/\partial x_\rho$ is the field tensor and A_ν is the vector potential, equal to $A_\nu = (2\omega)^{-1/2} e_\nu \exp\{-i(\mathbf{kr} - \omega t)\}$ (e_ν is the unit polarization vector, ω the frequency, $\mathbf{k}$ the wave vector of the photon; we employ the system of units in which $c = \hbar = 1$). Since $e \ll 1$, in terms containing the electromagnetic field, it is possible to replace ψ by $\psi_0 = ue^{i(\mathbf{pr} - Et)}$, where u is the spinor amplitude of the incident plane wave of the proton with momentum $\mathbf{p}$ and energy E. In this way we obtain an inhomogeneous equation for

$$(\vec{\gamma}\partial/\partial\mathbf{r} - \gamma_4 E' + m)\,\Phi = (2\omega)^{-1/2}(ie\hat{e} + 2\mu'\hat{k}\hat{e})\,\Phi_0(\mathbf{r})\,e^{-i\mathbf{kr}}, \qquad (2)$$

where

$$E' = E - \omega, \quad \hat{a} = \gamma_\nu a_\nu \quad (\nu = 1, 2, 3, 4), \quad \Phi_0(\mathbf{r}) = ue^{i\mathbf{pr}}.$$

Obtaining $\Phi(\mathbf{r})$ from this equation on the surface of the nucleus, which is assumed completely black with respect to the incident proton, it is possible to define the current of protons, absorbed by the nucleus, which has one photon at infinity. This current density is defined by the formula

$$j = \left(\overline{\Phi}\frac{\vec{\gamma}\mathbf{p}}{p}\Phi\right)_{r=R}, \quad \overline{\Phi} = \Phi^*\gamma_4,$$

and the bremsstrahlung cross-section is equal to

$$d\sigma = (j\pi R^2/v)\,(2\pi)^{-3}\omega^2 d\omega d^2\theta \qquad (3)$$

where v is the proton velocity, $d^2\theta$ the solid angle in which the photon is emitted.

The solution of Eq. (2) has the form

$$\Phi(\mathbf{r}) = -(2\omega)^{-1/2}\int G_0(\mathbf{r}, \mathbf{r}') \times (ie\hat{e} + 2\mu'\hat{k}\hat{e})\,e^{-i\mathbf{kr}'}\Phi_0(\mathbf{r}')\,d\mathbf{r}', \qquad (4)$$

where G_0 is the Green's function for the Dirac equation[1]:

$$G_0(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi}\left(\vec{\gamma}\frac{\partial}{\partial\mathbf{r}} - \gamma_4 E' - m\right)\frac{e^{ip'|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}, \qquad (5)$$

$$p' = \sqrt{E'^2 - m^2}.$$

Employing Eq. (5) it is possible to show that $\Phi(\mathbf{r})$ is given by the formula

$$\Phi(\mathbf{r}) = \frac{1}{2\sqrt{2\omega}} [i(\mathbf{p}-\mathbf{k})\vec{\gamma} - E'\gamma_4 - m]$$

$$\times [ie\hat{e} + 2\mu'\hat{k}\hat{e}]\, u \frac{e^{i(\mathbf{p}-\mathbf{k})\mathbf{r}}}{p'(p' - |\mathbf{p}-\mathbf{k}|)}$$

and that the current is equal to

$$j = \frac{i}{32\omega p E p'^2 (p' - |\mathbf{p}-\mathbf{k}|)^2} \mathrm{Sp}\, \{[-2\mu'\hat{e}\hat{k} - ie\hat{e}] \quad (6)$$

$$\times [i(\mathbf{p}-\mathbf{k})\vec{\gamma} - E'\gamma_4 - m]\vec{\gamma}\mathbf{p}$$

$$\times [i(\mathbf{p}-\mathbf{k})\vec{\gamma} - E'\gamma_4 - m]\,[ie\hat{e} + 2\mu'\hat{k}\hat{e}]$$

$$\times [i\mathbf{p}\vec{\gamma} - E\gamma_4 - m]\}.$$

Carrying out the summation over photon polarizations with the aid of the relation

$$\sum_e \mathbf{ea}\cdot\mathbf{eb} = \mathbf{ab} - k^{-2}(\mathbf{ak})(\mathbf{bk}),$$

where a and b are matrix vectors, we represent j in the form

$$j = \frac{1}{8\omega p E p'^2 (p' - |\mathbf{p}-\mathbf{k}|)^2} \mathrm{Sp} \quad (6')$$

$$\times \{[i(\mathbf{pp}_1)\,\mathbf{p}_1 + \delta\mathbf{p})\vec{\gamma} - \mathbf{pp}_1(E'\gamma_4 + m)]$$

$$\times [2\mu'\hat{k} + ie][-i(\mathbf{pn})(\mathbf{n}\vec{\gamma}) + E\gamma_4 - m]\,[2\mu'\hat{k} + ie]\},$$

where

$$\mathbf{n} = \mathbf{k}/k,\ 2\delta = (E-k)^2 - m^2 - (\mathbf{p}-\mathbf{k})^2,$$

$$E' = E - k,\ \mathbf{p}_1 = \mathbf{p} - \mathbf{k}.$$

The differential cross-section for bremsstrahlung is equal to

$$d\sigma = \frac{R^2}{(2\pi)^2}\frac{d\omega}{\omega}\frac{p^2}{(m^2 + p^2\theta^2)^2} \quad (7)$$

$$\times \Big\{ e^2 \Big[p(p-\omega)\,\theta^2 + \frac{\omega^2}{2p^2}(m^2 + p^2\theta^2)\Big]$$

$$+ 2\mu'^2 \frac{\omega^2}{p^2}(m^2 + p^2\theta^2)^2 - me\mu' \frac{\omega^2}{p^2}(m^2 + p^2\theta^2)\Big\}\, d^2\theta.$$

The first term in square brackets in Eq. (7) defines the bremsstrahlung for a particle without spin[3], and the second term comes from the spin of the proton. The last two terms define the radiation caused by the presence of the anomalous magnetic moment of the proton.

We see that the influence of spin and of the anomalous magnetic moment of the proton on bremsstrahlung are essential only in the region of high frequencies. One should bear in mind, however, that in the region of high frequencies, it is, strictly speaking, impossible to view the proton as a point charge, because here, due to the interaction of the proton with the meson vacuum, one should describe the 'smearing-out' of the proton, account of which can be taken in some circumstances by a form-factor F, depending on the invariant photon frequency*

$$F = F\Big(\frac{E\omega - \mathbf{pk}}{mm_0}\Big) = F\Big[\frac{\omega}{2E}\Big(1 + \frac{E^2}{m^2}\theta^2\Big)\frac{m}{m_0}\Big].$$

The cross section for radiation, taking into account the proton form-factor, is obtained by multiplying Eq. (7) by $|F|^2$.

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* We consider the dimensions of the proton to be of the order $1/m_0$ (where m_0 is the meson mass).

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Multiple Meson Production in Particle Collisions

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THE results of experiments[1] on multiple meson production by nucleon collisions at 10^9-10^{12} ev are only unsatisfactorily explained by thermodynamical and statistical theories. Following reference 2 we shall consider the multiple production of mesons in light of the field theory of interaction of mesons with nucleons.

We shall use the pseudoscalar charge-symmetric theory with pseudoscalar (PS) and pseudovector (PV) coupling. Only the meson field is double-