$$\Phi(\mathbf{r}) = \frac{1}{2\sqrt{2\omega}}[i(\mathbf{p}-\mathbf{k})\vec{\gamma} - E'\gamma_4 - m]$$

$$\times [ie\hat{e} + 2\mu'\hat{k}\hat{e}]\, u \frac{e^{i(\mathbf{p}-\mathbf{k})\mathbf{r}}}{p'(p' - |\mathbf{p}-\mathbf{k}|)}$$

and that the current is equal to

$$j = \frac{i}{32\omega pEp'^2(p'-|\mathbf{p}-\mathbf{k}|)^2}\mathrm{Sp}\,\{[-2\mu'\hat{e}\hat{k} - ie\hat{e}] \quad (6)$$

$$\times [i(\mathbf{p}-\mathbf{k})\vec{\gamma} - E'\gamma_4 - m]\vec{\gamma}\mathbf{p}$$

$$\times [i(\mathbf{p}-\mathbf{k})\vec{\gamma} - E'\gamma_4 - m]\,[ie\hat{e} + 2\mu'\hat{k}\hat{e}]$$

$$\times [i\mathbf{p}\vec{\gamma} - E\gamma_4 - m]\}.$$

Carrying out the summation over photon polarizations with the aid of the relation

$$\sum_e \mathbf{ea}\cdot\mathbf{eb} = \mathbf{ab} - k^{-2}(\mathbf{ak})(\mathbf{bk}),$$

where a and b are matrix vectors, we represent j in the form

$$j = \frac{1}{8\omega pEp'^2(p'-|\mathbf{p}-\mathbf{k}|)^2}\mathrm{Sp} \quad (6')$$

$$\times \{[i(\mathbf{pp}_1)\mathbf{p}_1 + \delta\mathbf{p})\vec{\gamma} - \mathbf{pp}_1(E'\gamma_4 + m)]$$

$$\times [2\mu'\hat{k} + ie][-i(\mathbf{pn})(\mathbf{n}\vec{\gamma}) + E\gamma_4 - m]\,[2\mu'\hat{k} + ie]\},$$

where

$$\mathbf{n} = \mathbf{k}/k,\ 2\delta = (E-k)^2 - m^2 - (\mathbf{p}-\mathbf{k})^2,$$

$$E' = E - k,\ \mathbf{p}_1 = \mathbf{p} - \mathbf{k}.$$

The differential cross-section for bremsstrahlung is equal to

$$d\sigma = \frac{R^2}{(2\pi)^2}\frac{d\omega}{\omega}\frac{p^2}{(m^2+p^2\theta^2)^2} \quad (7)$$

$$\times\Big\{e^2\Big[p(p-\omega)\theta^2 + \frac{\omega^2}{2p^2}(m^2+p^2\theta^2)\Big]$$

$$+2\mu'^2\frac{\omega^2}{p^2}(m^2+p^2\theta^2)^2 - me\mu'\frac{\omega^2}{p^2}(m^2+p^2\theta^2)\Big\}\,d^2\theta.$$

The first term in square brackets in Eq. (7) defines the bremsstrahlung for a particle without spin[3], and the second term comes from the spin of the proton. The last two terms define the radiation caused by the presence of the anomalous magnetic moment of the proton.

We see that the influence of spin and of the anomalous magnetic moment of the proton on bremsstrahlung are essential only in the region of high frequencies. One should bear in mind, however, that in the region of high frequencies, it is, strictly speaking, impossible to view the proton as a point charge, because here, due to the interaction of the proton with the meson vacuum, one should describe the 'smearing-out' of the proton, account of which can be taken in some circumstances by a form-factor F, depending on the invariant photon frequency*

$$F = F\left(\frac{E\omega - \mathbf{pk}}{mm_0}\right) = F\left[\frac{\omega}{2E}\left(1 + \frac{E^2}{m^2}\theta^2\right)\frac{m}{m_0}\right].$$

The cross section for radiation, taking into account the proton form-factor, is obtained by multiplying Eq. (7) by $|F|^2$.

We express our sincere gratitude to V. Bar'iakhtar and S. Peletminskii for help in carrying out a series of calculations.

* We consider the dimensions of the proton to be of the order $1/m_0$ (where m_0 is the meson mass).

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Multiple Meson Production in Particle Collisions

L. G. Iakovlev
Moscow State University

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THE results of experiments[1] on multiple meson production by nucleon collisions at 10^9-10^{12} ev are only unsatisfactorily explained by thermodynamical and statistical theories. Following reference 2 we shall consider the multiple production of mesons in light of the field theory of interaction of mesons with nucleons.

We shall use the pseudoscalar charge-symmetric theory with pseudoscalar (PS) and pseudovector (PV) coupling. Only the meson field is doublequantized; the virtual creation and annihilation of pairs is not taken into account. Canonical transformation of the total Hamiltonian creates nonlinear (in φ) terms. The perturbation theory is used in the first approximation; we assume that the term in the interaction Hamiltonian containing φ^n is the one mainly responsible for the production of n mesons.* From Lagrange Function**

$$L = \chi'^{+}\left[i\alpha_\mu \frac{\partial}{\partial x_\mu} - \rho_3 m - g\rho_2(\vec{\tau}, \vec{\varphi}) - f\sigma_\mu(\vec{\tau}, \vec{\varphi}_\mu)\right]\chi' + \nabla\varphi^+\nabla\varphi + \dot{\varphi}^+\dot{\varphi} - \varphi^+\varphi + {}^1/_2(\nabla\varphi_0)^2 + {}^1/_2\dot{\varphi}_0^2 + {}^1/_2\varphi_0^2 \quad (1)$$

With help of the variation principle we obtain the equation

$$[D_0 - U']\chi' = 0, \quad (2)$$

where

$$D_0 \equiv i\alpha_\mu\partial/\partial x_\mu - \rho_3 m, \quad U' \equiv g\rho_2(\vec{\tau}, \vec{\varphi}) + f\sigma_\mu(\vec{\tau}, \vec{\varphi}_\mu), \quad (3)$$

$$\varphi = \sum_{k=1}^{\infty}\left(\frac{2\pi}{\varepsilon_k L^3}\right)^{1/2}[a_{k_+}\exp\{i(p_k x)\} + a_{k_-}^{+}\exp\{-i(p_k x)\}], \quad \text{etc}$$

Performing the canonical transformation[3]

$$\chi = e^s\chi', e^{-s}[D_0 - U']e^s, \quad s = -i\rho_1 f(\vec{\tau}, \vec{\varphi}) \quad (4)$$

and dropping contact terms, we have, instead of (2)

$$[D_0 - U]\chi = 0; \quad (5)$$

D_0 has the former meaning, and the interaction Hamiltonian density has the following form:

$$U = g\rho_2(\vec{\tau}, \vec{\varphi}) + (e^{-s} - 1)(m\rho_3 + g\rho_2(\vec{\tau}, \vec{\varphi})) \quad (6)$$

Expanding e^{-s} in series, we get U in the following form:

$$U = \sum_{n=1}^{\infty}\rho_i g_n(\vec{\tau}, \vec{\varphi})^n, \quad (7)$$

$$i = 2 \text{ for } n = 2k+1, \quad i = 3 \text{ for } n = 2k.$$

$$g_n = (-1)^k\frac{(2f)^{n-1}}{(n-1)!}\left(g + \frac{2}{n}mf\right).$$

According to the formula

$$W_n d\varepsilon_1 d\varepsilon_2 \ldots d\varepsilon_n = 2^{n-1}\pi^{1-2n}|H_n'|^2\prod_{i=1}^{n}\varepsilon_i(\varepsilon_i - 1)^{1/2}d\varepsilon_i, \quad (8)$$

where $H_n' = \int\chi_m^+ U_n\chi_l\,(dr)$ $[U_n = \rho_i g_n(\vec{\tau}, \vec{\varphi})^n]$ is the matrix element of the transition of the system from two nucleons to a system of two nucleons and n mesons, we find the probability of production, in unit time, n π-mesons in the energy interval

$$W_n d\varepsilon_1 \ldots d\varepsilon_n = 2^{1-n}\pi^{1-2n}g_n^2\left|\int\chi_m^+(\vec{\tau}, \vec{\varphi})^n\rho_i\chi_l\,(dr)\right|^2 \times\prod_{i=1}^{n}\varepsilon_i(\varepsilon_i - 1)^{1/2}d\varepsilon_i, \quad (9)$$

The integral in (9) depends on the nature of the colliding and produced particles. We shall assume that all the mesons and also all the nucleons are of the same type. Then the total probability of production of n mesons ~~is~~:

$$W_n = \frac{(2f)^{2n-2}n!}{\pi^{n-1}(n-1)!(n-1)!}\left(g + \frac{1}{n}2mf\right)^2|N_n|^2 \times\int_1^{\Delta E}(\varepsilon_1^2 - 1)^{1/2}d\varepsilon_1 \ldots \int_1^{\Delta E - \sum_1^{n-1}\varepsilon_k}(\varepsilon_n^2 - 1)^{1/2}\delta\left(\Delta E - \sum_{k=1}^{n}\varepsilon_k\right)d\varepsilon_n. \quad (10)$$

$|N_n|$ is the part of the matrix element for the present not given explicitly. We assume only that it is of the same order for various transitions. ΔE = total energy (in the center of mass system) transferred from the nucleons to mesons.

Assuming $\epsilon_k > 1$, we have from (10):

$$W_n = \frac{n^3|N_n|^2 2^{2n-1}}{\pi^{n-1}n!(2n)!}f^{2n+2}\left(g + \frac{2mf}{n}\right)^2(\Delta E)^{2n-1}. \quad (11)$$

The relative probability, for large n, is

$$W_n/W_{n-1} \doteq \pi^{-1}f^2 n^{-3}\Delta E^2.$$

The most probable number of mesons is proportional to $\Delta E^{2/3}$ in the center of mass system; that is, $n_p \sim (\Delta E_{lab})^{1/3}$ in the lab system. The experiments show that $\Delta E = \eta E$; η changes little between 0.2 - 0.4. One can put therefore*** $np \sim E^{1/3}_{lab}$. In recent experiments[1] the production of mesons by bombardment of protons with neutrons at 2×10^9 ev was investigated. The following reactions were studied: $n + p \to p + p + \pi^-$ and $n + p \to n + p + \pi^+ + \pi^-$. It was found that $W(\pi + \pi^-)/W(\pi^-) \sim 4$. No cases of three meson production were discovered. From (9) one can find:

$$\frac{W(\pi^+, \pi^-)}{W(\pi^-)} = \frac{64}{3\pi} f^2 \left(\frac{g + mf}{g + 2mf}\right)^2 \frac{\Delta E_2^3}{\Delta E_1} \qquad (12)$$

Assuming that in the mean 30% of the total energy in center of mass system was transferred to the mesons[4] (which does not contradict data of reference 1), we find that $\Delta E \neq 9(1.27 \times 10^9$ ev). The values of g and f we take as usual, namely $g \sim 8$ - 10 and $f \sim 0.2 - 0.6$. All these values for f and g (g does not possess a great influence) give values of W_2/W_1 in good agreement in order of magnitude. Taking into account that here $W_3/W_2 \sim 0$, one should put $0.2 \leq f < 0.3$. If we had only PV coupling ($g = 0$), it would be $f \sim 0.5$.

In an analogous way the probability of meson production in meson-nucleon collisions is calculated:

$$W_n = \frac{2^{n+2} |N_n|^2}{\pi^{n-2} (2n)!\,(n-1)!} \left(g + \frac{1}{n+1} 2mf\right)^2 f^{2n} \varepsilon_0^{2n-2} \qquad (13)$$

(ϵ_0 – energy of the incident meson in center of mass system).

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* The use of perturbation theory would be more acceptable if we had only PV coupling, as the constant of this type of coupling is $f < 1$. However, even in the presence of the two types of coupling, the results are reasonable, as obviously the PV coupling has a greater influence on the variation in probability for different numbers of mesons [see (11) and (12)]. A fast nucleon in a collision with another one at rest loses in the mean only 30% of its energy in the center of mass system, which makes the use of perturbation theory somewhat more plausible.

** We make use of dimensionless units: $c = \hbar = \mu_\pi = 1$. The mass of the nucleon $m = 6.7$; the energy unit, 1.4×10^8 ev;

$$x_\mu = x, y, z, it; \quad \varphi_\mu \equiv \partial\varphi/\partial x_\mu, \quad \dot{\varphi} = \partial\varphi/\partial t.$$

*** The used method is approximate only and does not pretend to be rigorous. The part of the matrix element containing the nucleonic functions is not given explicitly. Equation (11) gives only the energy dependence. One can assume, however, that the relative probabilities depend only in a small degree on $|N_n|$ and a more exact calculation has little influence on n_p^n.

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Influence of Quantum Radiation Fluctuations on the Trajectory of an Electron in a Magnetic Field

A. A. KOLOMENSKII AND A. N. LEBEDEV

P. N. Lebedev Physical Institute, Academy of Sciences, USSR

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THE influence of radiation should be, in general, taken into account in investigating the motion of a relativistic electron in a magnetic field. The quantum mechanical treatment of this problem, based on the Dirac equation, has been given by Sokolov and Ternov.[1] The motion of an electron is described by two quantum numbers. The interaction with the field of virtual photons gives the probability of the change of these numbers, and consequently the change of trajectory in time. In reference 1 a formula was found giving the increase of the mean-square fluctuation of the radius with time, an effect not possessing a classical analogue. At the same time simple physical considerations[2] show, that the quantum (wave) character of the electronic motion should reveal itself only at energies $E \gtrsim E_{1/2} = mc^2(Rmc/\hbar)^{1/2}$ (R = radius of the orbit.), which, for usual values of magnetic field intensity, amounts to the extremely great value of $E_{1/2} \sim 10^{15}$ ev. It is therefore not necessary, in studying electron orbits at $E \ll E_{1/2}$, to use the solution of the