The most probable number of mesons is proportional to $\Delta E^{2/3}$ in the center of mass system; that is, $n_p \sim (\Delta E_{lab})^{1/3}$ in the lab system. The experiments show that $\Delta E = \eta E$; η changes little between 0.2 - 0.4. One can put therefore*** $np \sim E^{1/3}_{lab}$. In recent experiments[1] the production of mesons by bombardment of protons with neutrons at 2×10^9 ev was investigated. The following reactions were studied: $n + p \to p + p + \pi^-$ and $n + p \to n + p + \pi^+ + \pi^-$. It was found that $W(\pi + \pi^-)/W(\pi^-) \sim 4$. No cases of three meson production were discovered. From (9) one can find:

$$\frac{W(\pi^+, \pi^-)}{W(\pi^-)} = \frac{64}{3\pi} f^2 \left(\frac{g + mf}{g + 2mf}\right)^2 \frac{\Delta E_2^3}{\Delta E_1} \qquad (12)$$

Assuming that in the mean 30% of the total energy in center of mass system was transferred to the mesons[4] (which does not contradict data of reference 1), we find that $\Delta E \doteq 9(1.27 \times 10^9$ ev). The values of g and f we take as usual, namely $g \sim 8$ - 10 and $f \sim 0.2 - 0.6$. All these values for f and g (g does not possess a great influence) give values of W_2/W_1 in good agreement in order of magnitude. Taking into account that here $W_3/W_2 \sim 0$, one should put $0.2 \leq f < 0.3$. If we had only PV coupling ($g = 0$), it would be $f \sim 0.5$.

In an analogous way the probability of meson production in meson-nucleon collisions is calculated:

$$W_n = \frac{2^{n+2} \, | N_n |^2}{\pi^{n-2} (2n)! \, (n-1)!} \left(g + \frac{1}{n+1} 2mf\right)^2 f^{2n} \varepsilon_0^{2n-2} \qquad (13)$$

(ϵ_0 – energy of the incident meson in center of mass system).

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* The use of perturbation theory would be more acceptable if we had only PV coupling, as the constant of this type of coupling is $f < 1$. However, even in the presence of the two types of coupling, the results are reasonable, as obviously the PV coupling has a greater influence on the variation in probability for different numbers of mesons [see (11) and (12)]. A fast nucleon in a collision with another one at rest loses in the mean only 30% of its energy in the center of mass system, which makes the use of perturbation theory somewhat more plausible.

** We make use of dimensionless units: $c = \hbar = \mu_\pi = 1$. The mass of the nucleon $m = 6.7$; the energy unit, 1.4×10^8 ev;

$$x_\mu = x, y, z, it; \quad \varphi_\mu \equiv \partial\varphi/\partial x_\mu, \quad \dot{\varphi} = \partial\varphi/\partial t.$$

*** The used method is approximate only and does not pretend to be rigorous. The part of the matrix element containing the nucleonic functions is not given explicitly. Equation (11) gives only the energy dependence. One can assume, however, that the relative probabilities depend only in a small degree on $|N_n|$ and a more exact calculation has little influence on n_p^n.

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Influence of Quantum Radiation Fluctuations on the Trajectory of an Electron in a Magnetic Field

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THE influence of radiation should be, in general, taken into account in investigating the motion of a relativistic electron in a magnetic field. The quantum mechanical treatment of this problem, based on the Dirac equation, has been given by Sokolov and Ternov.[1] The motion of an electron is described by two quantum numbers. The interaction with the field of virtual photons gives the probability of the change of these numbers, and consequently the change of trajectory in time. In reference 1 a formula was found giving the increase of the mean-square fluctuation of the radius with time, an effect not possessing a classical analogue. At the same time simple physical considerations[2] show, that the quantum (wave) character of the electronic motion should reveal itself only at energies $E \gtrsim E_{1/2} = mc^2(Rmc/\hbar)^{1/2}$ (R = radius of the orbit.), which, for usual values of magnetic field intensity, amounts to the extremely great value of $E_{1/2} \sim 10^{15}$ ev. It is therefore not necessary, in studying electron orbits at $E \ll E_{1/2}$, to use the solution of the

Dirac equation, a procedure connected with tedious approximation by Laguerre polynomials.

We would like here to draw attention to the fact, that the above-mentioned effect of "widening" of the trajectory can be very easily obtained and explained on the basis of the usual classical theory of betratron oscillations, in which we need only to take into account, additionally, the statistical character of radiation. This approach permits us not only to obtain all the results of reference 1, but also to correct certain omissions there from.

From the point of view of the theory of betatron oscillations, the emission of a quantum of the energy $\epsilon = \hbar\omega$ is accompanied by a discrete shortening of the radius of the so-called instantaneous orbit, about which the oscillations occur, by the amount

$$\Delta\rho_{M} = (\varepsilon / E) R / (1 - n), \tag{1}$$

where $n = -(R/H)\, \partial H/\partial R$ is the magnetic field factor. The amplitude of the oscillations changes as well by a given value ΔA. The random changes $\Delta\rho_M$ and ΔA, added statistically, bring about the "widening" of the trajectory. The situation here is analogous to pendulum oscillations (betatron oscillations), the point of suspension of which (instantaneous orbit) is subjected to random impulses, governed by a given distribution law.

We shall consider an electron, having a constant energy (in the mean), moving in an axially-symmetric magnetic field. The equation of the radial motion of such an electron can be written;

$$\rho'' + \rho = \frac{1}{eH(1-n)} \left[\int_{\tau_0}^{\tau} \left(\sum_{i=1}^{k} \varepsilon_i \,\delta(\tau - \tau_i) \right) d\tau - D\tau \right], \tag{2}$$

where $\tau = (c/R)\sqrt{1-n}\, t$, $\varepsilon_i = \hbar\omega_i$ is the energy of the ith emitted photon, δ = delta function, ρ = deviation of the particle from the basic orbit of radius R, which is determined by the condition, that the mean (in time) energy $\bar{E} = eH(R)R$. The ϵi and τi in Eq. (2) should be regarded as independent random variables. The right side of Eq. (2) describes the influence of radiation fluctuations, the constant D, describing the compensation of losses, being selected so that $\bar{\rho} = 0$. Finding the oscillatory part of the solution of Eq. (2), ρ^2_{bet}, and taking its mean value in time, we obtain

$$\tilde{\rho}^2_{бет} = \frac{1}{2e^2 H^2 (1-n)^2} \sum_{j=1}^{\infty} \varepsilon_j^2 \tilde{n}_j, \tag{3}$$

where $\tilde{n}_j$ is the mean number of photons of jth frequency emitted during the phase τ. Passing, with the help of well-known formulas (see e.g. reference 3) to the continuous spectrum, and taking the mean we obtain

$$\tilde{\rho}^2_{бет} = \frac{55}{48\sqrt{3}} \frac{1}{(1-n)^2} \frac{e^2}{mc} \frac{\hbar}{Rmc} \left(\frac{E}{mc^2}\right)^5 t. \tag{4}$$

This result is easily generalized for the case of a slowly (adiabatically) varying field. Since the betatron oscillations are damped as $H^{-1/2} \sim E^{-1/2}$, it follows immediately from Eq. (4)

$$\tilde{\rho}^2_{бет} = \frac{55}{48\sqrt{3}} \frac{1}{(1-n)^2} \frac{e^2}{mc} \frac{\hbar}{Rmc} \frac{mc^2}{E} \int_{t_0}^{t} \left(\frac{E}{mc^2}\right)^6 dt, \tag{5}$$

which, as well as (4), exactly coincides with the results of reference 1.

Apart from the excitation of the betatron orbits, still another effect, not mentioned in reference 1 and consisting in statistical "widening" of the instantaneous orbit r_M itself should be taken into account. We have then only $\tilde{r}_M = R$, and not $r_M = R$, as it has been assumed in reference 1. The expression for $\tilde{\rho}^2$ can be written therefore

$$\tilde{\rho}^2 = \tilde{\rho}^2_{M} + \tilde{\rho}^2_{бет}. \tag{6}$$

We shall note, that one can, of course, deduce the expression for $\tilde{\rho}^2_M$ directly from (1).

For the explanation of the behavior of fluctuations of the instantaneous orbit we shall consider the acceleration of an electron in a hypothetical betatron, in which radiative losses are compensated in the mean by the vortex field. Writing down the energy change of particle on an instantaneous orbit, ρ_M, we obtain the equation

$$\frac{d\rho_M}{dt} = \frac{\bar{\varepsilon}\,[\tilde{a}_1 - \sum_i \delta(t - t_i)]}{eH(1-n)} - \frac{\dot{H}}{H}(1+g)\,\rho_M, \tag{7}$$

where $\bar{\epsilon}$ — is the mean value of the energy of quanta emitted at a given energy E, $\tilde{a}_1$ is the mean number of such quanta, emitted in unit time, and

$$g = \frac{2}{3} \frac{3-4n}{1-n} \frac{e^2 c}{R^2} \left(\frac{E}{mc^2}\right)^4 \frac{1}{\dot{E}}. \tag{8}$$

Solving (7), we obtain, for $g \gg 1$ and linear increase of energy with time,

$$\tilde{\rho}^2_{M} = \frac{55\sqrt{3}}{96} \frac{\hbar R}{mc} \frac{1}{(1-n)(3-4n)} \left(\frac{E}{mc^2}\right)^2. \tag{9}$$

It is interesting to note, that (9), obtained for the betatron in question, coincides with the expression for the mean square amplitude of small radial phase oscillations in a synchrotron, caused by radiation fluctuations,[4] while the condition $g \gg 1$ is equivalent to the condition $E \gg E_c \simeq 10^8$ ev assumed in reference 4. The slow dependence of (9) on time (or energy) is explained by the strong influence of the damping due to radiation.

Remark received during proof: Further investigations by the authors show, that if Eq.(2) is complemented by a term accounting for the law of conservation of momentum in radiation processes, one obtains, instead of Eqs. (4) and (5), the following formula:

$$\bar{\bar{\rho}}^2_{\text{bet}} \lesssim \frac{55\sqrt{3}}{96} \frac{\hbar R}{mc} \frac{1}{(1-n)^2} \left(\frac{E}{mc^2}\right)^2,$$

which materially differs from the formula of Sokolov and Ternov[1].

This problem is studied in detail in another article by the authors[5].

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Excitation of Synchrotron due to Electron Radiation Fluctuation in Strong Focusing Accelerators

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IT is well known that the quantum fluctuations of the radiation of a relativistic electron moving in a magnetic field leads to the particular effect of "broadening" of the trajectory. This was noticed first by Sokolov and Ternov[1]. The effect of radiation fluctuation on the excitations of synchrotron oscillations in accelerators with weak focussing has been studied by Sands[2] (see also a paper by Lebedev and the author[3]). The aim of this note is the investigation of this effect in strong focussing accelerators[4] whose designing is now under consideration.

Consider a system composed of sectors with magnetic field (with an angular spread ν each) and sectors free of field (of length L each), forming identical elements of periodicity. The magnetic field gradients (large in absolute magnitude) have opposite signs in neighboring sectors. If the electron energy deviates by ΔE from its equilibrium value, the instantaneous orbit near which the betatron oscillations take place are deformed and deviate from the main equilibrium orbit by a length $\rho_M(\theta)$, which can be written in the form:

$$\rho_M(\theta) = (R\,\psi(\theta) / |n|)\,\Delta E / E, \qquad (1)$$

where R is the radius of the main orbit, n = the index of the magnetic field $(|n| \gg 1)$, θ = the generalized azimuth, $\psi(\theta)$ = a function determined by the parameters of the system and having a period equal to the period of the system $\theta_0 = 2\pi / N$. For the behavior of the function $\psi(\theta)$ in the simplest case, see reference 4. In the case of weak focussing, the expression $\psi(\theta)/|n|$ in (1) is replaced by $1/(1-n)$ where $0 < n < 1$. The mode of the synchrotron is characterized by ΔE, $\rho_M(\theta)$, and also by $\eta = \varphi - \varphi_0$, i e., the difference between the phase φ of the particle (with respect to the accelerating field) and the equilibrium phase l_0. Using (1) we get the relationship:

$$\frac{\bar{\psi}_H}{\sigma|n|}\frac{\Delta E}{E} = -\frac{\Delta\omega}{\omega} = \frac{1}{q\omega}\frac{d\eta}{dt}, \quad \sigma = 1 + \frac{L}{R\nu}, \qquad (2)$$

where ω is the rotation frequency, q = the acceleration, and $\bar{\psi}_H$=the average of $\psi(\theta)$ over the regions with magnetic field.

Using the well-known expression for the instantaneous power of radiation loss[5]

$$dW / dt = (2e^2c / 3R^2_{\text{кр}})(E / mc^2)^4 \qquad (3)$$

One has to take in consideration that, in the given case, the radius of curvature R_c of the trajectory at the point of the instantaneous orbit does not coincide with the radius-vector of the point:

$$R_{\text{кр}} = R\left[1 + \frac{|n| + n(\theta)\,\psi(\theta)}{\psi(\theta)}\frac{\rho_M(\theta)}{R}\right]. \qquad (4)$$

Writing the balance between the energy received by the particle from the accelerating field and the energy used up by radiation in one period (taking into account fluctuations), and using equations (2)-(4), we obtain the linearized phase relation in the form

$$\frac{d^2\eta}{dt^2} + \frac{1}{E}\left(\frac{Q}{\sigma}\frac{dW}{dt} + \dot{E}\right)\frac{d\eta}{dt} + \frac{q\omega^2\bar{\psi}_H eV_0 \sin\varphi_0}{\sigma|n|E}\eta = \frac{q\omega\bar{\psi}_H}{\sigma^2|n|E}(a_1 - \bar{a}_1)\bar{\varepsilon}, \qquad (5)$$