betatron in question, coincides with the expression for the mean square amplitude of small radial phase oscillations in a synchrotron, caused by radiation fluctuations,[4] while the condition $g >> 1$ is equivalent to the condition $E \gg E_c \simeq 10^8$ ev assumed in reference 4. The slow dependence of (9) on time (or energy) is explained by the strong influence of the damping due to radiation.

Remark received during proof: Further investigations by the authors show, that if Eq.(2) is complemented by a term accounting for the law of conservation of momentum in radiation processes, one obtains, instead of Eqs. (4) and (5), the following formula:

$$\bar{\bar{\rho}}^2_{\text{bet}} \leqslant \frac{55\sqrt{3}}{96}\,\frac{\hbar R}{mc}\,\frac{1}{(1-n)^2}\left(\frac{E}{mc^2}\right)^2,$$

which materially differs from the formula of Sokolov and Ternov[1].

This problem is studied in detail in another article by the authors[5].

[1] A. A. Sokolov and I. M. Ternov, J. Exper. Theoret. Phys. **24**, 249 (1953); **28**, 431 (1955); Soviet Phys. JETP **1**, 227 (1955);Doklady Akad. Nauk **97**, 823 (1954).

[2] V. V. Vladimirskii, **18**, 392 (1948).

[3] D. D. Ivanenko and A. A. Solokov, *The classical Theory of Fields*, GITTL, 1949, p. 275.

[4] M. Sands, Phys. Rev. **97**, 470 (1955).

[5] A. A Kolomonskii and A. N. Lebedev, Dokl. Akad. Nauk SSSR **106**, 5 (1956).

Translated by H. Kasha
34

Excitation of Synchrotron due to Electron Radiation Fluctuation in Strong Focusing Accelerators

A. A. KOLOMENSKII
P. N. Lebedev Physical Institute, Academy of Sciences, USSR

(Submitted to JETP editor September 10, 1955)
J. Exper. Theoret. Phys. USSR **30**, 207-209 (January, 1956)

It is well known that the quantum fluctuations of the radiation of a relativistic electron moving in a magnetic field leads to the particular effect of "broadening" of the trajectory. This was noticed first by Sokolov and Ternov[1]. The effect of radiation fluctuation on the excitations of synchrotron oscillations in accelerators with weak focussing has been studied by Sands[2] (see also a paper by Lebedev and the author[3]). The aim of this note is the investigation of this effect in strong focussing accelerators[4] whose designing is now under consideration.

Consider a system composed of sectors with magnetic field (with an angular spread ν each) and sectors free of field (of length L each), forming identical elements of periodicity. The magnetic field gradients (large in absolute magnitude) have opposite signs in neighboring sectors. If the electron energy deviates by ΔE from its equilibrium value, the instantaneous orbit near which the betatron oscillations take place are deformed and deviate from the main equilibrium orbit by a length $\rho_M(\theta)$, which can be written in the form:

$$\rho_M(\theta) = (R\,\psi(\theta)/|n|)\,\Delta E/E, \qquad (1)$$

where R is the radius of the main orbit, n =the index of the magnetic field $(|n| \gg 1)$, θ = the generalized azimuth, $\psi(\theta)$ = a function determined by the parameters of the system and having a period equal to the period of the system $\theta_0 = 2\pi/N$. For the behavior of the function $\psi(\theta)$ in the simplest case, see reference 4. In the case of weak focussing, the expression $\psi(\theta)/|n|$ in (1) is replaced by $1/(1-n)$ where $0 < n < 1$. The mode of the synchrotron is characterized by ΔE, $\rho_M(\theta)$, and also by $\eta = \varphi - \varphi_0$, i e., the difference between the phase φ of the particle (with respect to the accelerating field) and the equilibrium phase l_0. Using (1) we get the relationship:

$$\frac{\bar{\psi}_H}{\sigma|n|}\,\frac{\Delta E}{E} = -\frac{\Delta\omega}{\omega} = \frac{1}{q\omega}\,\frac{d\eta}{dt}, \quad \sigma = 1 + \frac{L}{R\nu}, \qquad (2)$$

where ω is the rotation frequency, q = the acceleration, and $\bar{\psi}_H$=the average of $\psi(\theta)$ over the regions with magnetic field.

Using the well-known expression for the instantaneous power of radiation loss[5]

$$dW/dt = (2e^2c/3R^2_{\text{кр}})(E/mc^2)^4 \qquad (3)$$

One has to take in consideration that, in the given case, the radius of curvature R_c of the trajectory at the point of the instantaneous orbit does not coincide with the radius-vector of the point:

$$R_{\text{кр}} = R\left[1 + \frac{|n| + n(\theta)\,\psi(\theta)}{\psi(\theta)}\,\frac{\rho_M(\theta)}{R}\right]. \qquad (4)$$

Writing the balance between the energy received by the particle from the accelerating field and the energy used up by radiation in one period (taking into account fluctuations), and using equations (2)-(4), we obtain the linearized phase relation in the form

$$\frac{d^2\eta}{dt^2} + \frac{1}{E}\left(\frac{Q}{\sigma}\,\frac{dW}{dt} + \dot{E}\right)\frac{d\eta}{dt} + \frac{q\omega^2\bar{\psi}_H eV_0 \sin\varphi_0}{\sigma|n|E}\,\eta = \frac{q\omega\bar{\psi}_H}{\sigma^2|n|E}(a_1 - \bar{a}_1)\bar{\varepsilon}, \qquad (5)$$

where $V_0 \cos\varphi$ is the accelerating potential; $\bar{\epsilon}$ – the mean value of the quanta emitted by the electron at the given energy E; $a_1 - \bar{a}_1$ – the deviation (fluctuation) of the number of quanta emitted per unit time, from its mean value; the coefficient Q is determined by the accommodation parameters

$$Q = 2 + \bar{\psi}_\Phi - \bar{\psi}_Д + \bar{\psi}_H / |n|, \qquad (6)$$

where $\bar{\psi}_F$ and ψ_D are the averages of $\psi(\theta)$ over focussing ($n < 0$) and defocussing ($n > 0$) sectors. The calculations show that $Q \simeq 4$. Note that the coefficient E of $d\eta/dt$ in (5) can be neglected in that range of energy where radiation (and its fluctuation) is important. Assuming a linear time dependence, we can solve Eq. (5). For the mean square of η, we find:

$$\bar{\eta^2} = \frac{55\sqrt{3}}{64} \frac{\hbar c}{e^2} \frac{q}{\sigma} \frac{\bar{\psi}_H}{Q|n|} \operatorname{ctg} \varphi_0 \frac{mc^2}{E}, \qquad (7)$$

where the factor $\bar{\psi}_H / Q|n|$ is approximately equal to $1/|n|$. Equation (7) can be used to find the azimuthal dimensions of the electron concentration, which are of some interest for evaluation of loss by coherent radiation.

The largest radial deviation ρ_{max} of the instantaneous orbit is determined by (1), when $\psi(\theta)$ is given its maximum value $\psi_{max}(\theta)$. Using (1), (2), (5), and (7) we obtain the mean square value:

$$\bar{\rho^2_{max}} = (55\sqrt{3}/96)(\hbar R/mc)(\psi^2_{max}/Q|n|^2)(E/mc^2)^2. \qquad (8)$$

Note that exactly the same correction characterizes the instantaneous orbit in a strong focussing betatron, in which the radiation losses compensate on the average. The evaluations show that, near the center of steadiness ($\sqrt{|n|}\,\nu \simeq \pi/2$) the factor $\psi^2_{max}/Q|n|^2 \simeq 10/|n|^2$, while in the case of weak focussing[2,3] it is replaced by the expression $1/(1-n)(3-4n)$, which, for $n \sim 0.6$-0.7 is $\simeq 10$. The small dependence of θ^2_{max} on E or t is explained by the influence of powerful extinction linked to the large magnitude of the mean radiation losses. This extinction has a simple physical meaning. It can be shown that it corresponds to the fact that when the orbit is displaced along the radius, the particle radiates in such a way that the change of its energy tends to restore the instantaneous orbit in its equilibrium position.

If, for instance, we let $H_{max} \approx 10^4$ oersteds, then, according to Eq. (8), we get the evaluation $\left(\bar{\rho^2_{max}}\right)^{1/2}_{\text{cm}} \lesssim E^{3/2}_{\text{BeV}}/|n|$, which shows that even for $E \simeq 5 \div 10$ BeV is only of the order of a centimeter. The considered effect has thus, by itself, no appreciable effect on acceleration.

---

[1] A. A. Sokolov and I. M. Ternov, J. Exper. Theoret. Phys. USSR 24, 249 (1953); 28, 431 (1955); Dokl. Akad. Nauk SSSR 97, 823 (1954). Soviet Phys. JETP 1, 227 (1955)

[2] M. Sands, Phys. Rev. 97, 470 (1955).

[3] A. A. Kolomenskii and A. N. Lebedev, J. Exper. Theoret. Phys. USSR 30, 205 (1956). (English Translation: Soviet Phys. JETP 2, 130 (1956).

[4] E. Courant, M. Livingston and H. Snyder, Phys. Rev. 88, 1190 (1952).

[5] D. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, 1953.

Translated by E. S. Troubetzkoy
35.

Auger Effect in Heavy Atoms

V. N. MOKHOV AND M. G. URIN
Moscow Physico-engineering Institute

(Submitted to JETP editor September 15, 1955)
J. Exper. Theoret. Phys. USSR 30, 209-210
(January, 1956)

EXCITED atoms, in which one of the interior electrons (say a K-electron) is missing, undergo transitions to a lower energy state by means of radiation of a quantum or by a nonradiative transition with the emission of an electron (Auger Effect). The total number of transitions per unit time $(1/c)_\epsilon$ has been obtained for the nonrelativistic case for arbitrary Z with the aid of the Coulomb function.[1] Only in the case of the interaction of L-electrons (for $Z = 47$) has the screening of the atomic nucleus been taken into account.[2]

In first order perturbation theory,

$$\frac{1}{\tau} = \frac{2\pi}{k} | V(n_1 l_1 m_1,\ n_2 l_2 m_2 \,|\, 100,\ kl'm') - V(n_2 l_2 m_2,\ n_1 l_1 m_1 \,|\, 100,\ kl'm')|^2$$

and

$$\frac{1}{\tau} = \frac{2\pi}{k} | V(n_1 l_1 m_1,\ n_2 l_2 m_2 \,|\, 100,\ kl'm')|^2$$

for the interaction of electrons with parallel and antiparallel spins, respectively. Here we have the matrix elements of the operator $V = 1/|\mathbf{r}_1 - \mathbf{r}_2|$, which corresponds to a transition from a state with quantum numbers n_1, l_1, M_1; m_2, l_2, m_2 to a state