where

$$\varkappa_0 = kb \sin\theta \left[\cos^2\varphi + \xi^2(\vartheta)\sin^2\varphi\right]^{1/2},$$

φ is the azimuthal angle of the axis of symmetry, measured from the plane determined by the vectors k, k′. It is easy to see that Eq. (4) transforms into Eq. (7) for small angles of scattering θ, with accuracy to $(n+1)/2 \approx 1$. Evidently the optical approach of Eq. (6) is valid only for very small scattering angles.

In the Born approximation, which is valid for $kR|U|/E \ll 1$ (R is the radius of the nucleus), we can set $n - 1 = 0$ everywhere in Eq. (4) except in the multiplicative factor $n^2 - 1$. We then obtain

$$f(\Omega, \omega) = \sqrt{2\pi}\hbar^{-2}\mu a b^2 U y^{-3/2} J_{3/2}(y), \tag{8}$$

where $y = 2kb \sin(\theta/2)\sqrt{1 - \varepsilon\cos^2\gamma}$. Here the dependence of the amplitude on the angles ω is determined only by the angle γ. Therefore, if we choose as the angle of quantization the direction of the vector k′ − k only such states will be excited for which the projection of the momentum is equal to zero. The cross section of excitation of the lth level can be calculated, not by Eq. (3), but by the simpler formula

$$\sigma_l(\Omega) = (2l+1)\left|\int_0^\pi P_l(\cos\gamma) f(\gamma, \Omega)\, d\cos\gamma\right|^2; \tag{9}$$

the cross sections of excitation of a level with momentum l and projection m in the direction of the vector k, are calculated with the help of the addition theorem for spherical functions. There also follow from Eq. (8) certain conclusions on the angular correlation between the scattered neutron and the photon produced in the transition $l \to 0$. Thus, for example, for $l = 2$, the photon has an angular momentum equal to two and a projection of this momentum in the direction of the vector k′ − k equal to zero; therefore, we get for the angular distribution of photons[7]:

$$I(\alpha) = (15/8\pi)(\cos^2\alpha - \cos^4\alpha), \tag{10}$$

where α is the angle between the direction of the photon and the vector k′ − k.

Starting out with Eq. (8) and the expression for the total cross section of elastic scattering and of the excitation of all rotational levels $\sigma_s(\Omega)$, $= (4\pi)^{-1}\int d\omega\, |f(\omega, \Omega)|^2$, one can show that the angular distributions just obtained are the mean of the angular distributions, taken with a weighting factor for elastic scattering on spherical nuclei having radii from $R_1 = a$ to $R_2 = b$.

For small eccentricity ($|\epsilon| \ll 1$) we get from Eqs. (8) and (9):

$$\sigma_l(\Omega) = \varepsilon^l A_l \left|\left(2kb\sin\frac{\theta}{2}\right)^{(l-3)/2} J_{(l+3)/2}\left(2kb\sin\frac{\theta}{2}\right)\right|^2, \tag{11}$$

$$A_l = 2^{l+1}(2l+1)\pi\left|\frac{\mu a b^2 U (l!)^2}{\hbar^2 (l/2)!\,(2l+1)!}\right|^2.$$

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Possibility of Using Artificial Earth Satellites for the Experimental Verification of the Theory of General Relativity

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AT the present time it is possible to assert that experimental tests have brought about a convincing verification of the general theory of relativity.[1] However, an even further verification of the theory does not appear superfluous. Therefore it is appropriate at this time to point out the possibility of an experimental verification of the general theory of relativity by utilizing artificial earth satellites.

The reception of the radio signals on earth from the satellite can be used to determine the gravitational shift of frequency in the earth's field[2]. The relativistic gravitational change of frequency ν is

$$\frac{\Delta\nu}{\nu}=\frac{\varkappa M_{\eth}}{c^2}\left(\frac{1}{r_{\eth}}-\frac{1}{r_{\eth}+h}\right)\simeq\frac{gh}{c^2}\left(1-\frac{h}{r_{\eth}}\right) \quad (1)$$

$$=1{,}09\cdot 10^{-18}h\left(1-\frac{h}{r_{\eth}}\right),$$

where $\varkappa = 6.670 \times 10^{-8}$ is the gravitational constant, $M = 5.98 \times 10^{27}$ is the mass of the earth, $r = 6.38 \times 10^8$ is the earth's radius, $g = 981$ the gravitational acceleration, h is the height of the satellite above the earth's surface. For $h \approx 800$ km, $\Delta\gamma/\gamma = 7.6 \times 10^{-11}$; if $h >> r$, then $\Delta\nu/\nu = 7 \times 10^{-10}$. (the observed shift in the earth's field is of course, violet and not red). Furthermore, in the radio band it is in principle possible to achieve an accuracy of one part in 10^{12} using atomic clocks[3], and an accuracy of greater than a part in 10^{10} has already been achieved[4]. Equation (1) does not give the total change in frequency of the radiation emitted from the satellite since it is necessary to include the Doppler shift as well. It may be easily shown from the general expression for the Doppler effect in the gravitational field (cf. reference 5, sec. 116) that to the pertinent accuracy of the order of $\sim \nu^2/c^2$ the gravitational and the Doppler shifts of frequency simply add. It is then possible to use for the Doppler shift the usual formula

$$\left(\frac{\Delta\nu}{\nu}\right)_\partial=\frac{\sqrt{1-(v^2/c^2)}}{1-(v/c)\cos\theta}$$

$$\approx 1+\frac{v}{c}\cos\theta-\frac{v^2}{2c^2}(1-2\cos^2\theta),$$

where v is the velocity of the satellite relative to the earth's surface and θ is the angle between the velocity and the line of sight. For v and θ it is possible to use their instantaneous values and the acceleration of the source does not enter anywhere into the result. For satellites close to the earth, the corrections connected with the rotation of the earth (one is considering the system in which measurements are performed as fixed on the earth) are small and may be neglected (these corrections are $\lesssim \nu_0{}^2/c^2 \sim 2 \times 10^{-12}$ where $v_0 = 4.6 \times 10^4$ is the velocity of the earth's surface at the equator). In observations of the satellite at the angle $\theta = \pi/2$, the linear Doppler effect disappears, but the quadratic effect remains, and is equal to $(\Delta\nu/\nu)_\partial = -v^2/2c^2$

$$=-\varkappa M_{\eth}/2c^2(r_{\eth}+h)\cong -3.5\cdot 10^{-10}\ [1-(h/r_{\eth})],$$

where the orbit is considered circular for the sake of simplicity. For $h = 800$ km $(\Delta\nu/\nu)\partial = -3 \times 10^{-10}$ i.e., the quadratic Doppler effect is already 4 times greater than the gravitational frequency shift. For that and other reasons the difficulties standing in the way of the discussed experiment should certainly not be underestimated. More distant satellites are more favorable for such an experiment.

The second effect of the general theory of relativity that may be verified with the help of a satellite is the precession of the perihelion of the orbit. For the earth's satellites this precession (in angular seconds per century) is, according to the formula of Einstein, equal to

$$\Psi=\frac{5\pi^2a^2Y}{24c^2T^2(1-e^2)}=8.35\cdot 10^{-19}\frac{a^2}{T^3(1-e^2)} \quad (2)$$

$$=\frac{1.74\cdot 10^{25}}{a^{5/2}(1-e^2)},$$

where $Y = 365.25$ is the number of days in a year, a is the major semi-axis of the orbit in centimeters, e is the eccentricity of the orbit and T is the period of rotation of the satellite in days (for the earth in particular only the last expression applies, of course).

Equation (2) does not include the effect of the sun on the motion of the perihelion of the satellite which is 7.6″ per century. The effect of Eq. (2) may be very large for the earth's satellites, as was already emphasized[6]. For satellites close to earth it reaches approximately 1500″ per century, while for Mercury $\Psi = 43''$ per century. More detailed considerations show that, from the point of view of feasibility of measuring the effect, the observations of the satellite over one year may be more favorable than observations of Mercury over one hundred years. It is not impossible that employment of the radio method for the measurement of the perihelion precession will result in even more favorable results. Still, as is shown below, there is a tempting possibility of discovering the influence of the earth's rotation on the precession of the satellite's perihelion and also on the rotation of the modes of its orbit.

In the Galilean system of coordinates the field of a rotating body has non-zero components of the type $g_{0\alpha}$ $(\alpha = 1, 2, 3)$. Then we have for the case of a weak field at a large distance from the body (cf. reference 7, Sec. 100):

$$\mathbf{g} \simeq -\frac{2\varkappa}{c^3r^3}[\mathbf{Ir}],\quad \mathbf{I}=\int[\mathbf{r}',\ \mu\mathbf{v}]\,dV, \quad (3)$$

Where $g_\alpha = -g_{0a}/g_{00} \cong g_{0\alpha}$, μ is the mass density at the point $\mathbf{r}'$ moving with velocity V and $\mathbf{r}$ is the distance from the center of the body to the point of observation. For a sphere the Eq. (3) is applicable everywhere outside the sphere and for μ const., $I = 2/5Mr_0^2\,\omega$, where M is the mass of the sphere, r_0 its radius and ω the angular velocity of rotation. Lense and Thirring[8] have shown that the field of Eq. (3) results in an additional precession of the perihelion of the satellite(planet) by the angle (in angular seconds per century)

$$\Psi_R = -\frac{\pi^2 r_0^2 Y}{9c^2 \tau T^2 (1-e^2)^{3/2}}, \qquad (4)$$

$$\Delta = \frac{|\Psi_R|}{\Psi} = \frac{8}{15}\left(\frac{r_0}{a}\right)^2 \frac{T}{\tau(1-e^2)^{1/2}},$$

where τ (in days) is the period of rotation of the central sphere producing the field. In Eq. (4) it is assumed, for simplicity, that the plane of the orbit coincides with the equator of the rotating sphere and that the rotation of both the satellite and the sphere take place in the same direction. In a general case[8] a multiplicative factor $(1-3\sin^2(i/2))$, appears in Eq. (4), where i is the angle between the equatorial plane and the plane of the orbit. The angle of rotation of the nodes is smaller by a factor of 2 than the angle of Eq. (4) and has an opposite sign. To obtain the total effect it is only necessary to add algebraically the precession of the perihelion of Eq. (4) with the precession of Eq. (2).

In the case of Mercury ($a = 5.8 \times 10^{12}$, $T = 88$ days, $r_0 = r_\odot = 6{,}96 \cdot 10^{10}$ and $\tau = \tau_\odot \simeq$ 28 days) $\Delta \approx 2.5 \times 10^{-4}$ and $\Psi_r = 0.01''$. At the present time the accuracy of measurement of the precession of the perihelion has reached the order of $1''$. For a nearby satellite of earth the picture is quite different. For h=400 km, $T \approx 1.54$ hours: $\Delta \approx 3 \times 10^{-2}$ and $\Psi_r \approx -43''$ per century. Thus the relativistic effect connected with the rotation of the earth is of the same magnitude as the total relativistic effect for Mercury. Thus it appears desirable to give attention to the possibility of a measurement of this relativistic "rotation effect".

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Regularized Theory of Field System

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IN the papers[1,2], it is shown that the meaning of the relativistically invariant regularization removing the divergences in the current field theory consists in replacing the usual field equations by equations with higher order derivatives. However, because of the known difficulties related to negative energies[3], the problem of interpretation of the field theory with higher order derivatives is not solved.

For sake of simplicity let us consider a neutral scalar field, subject to the equation:

$$\prod_{i=0}^{n} (\Box - \varkappa_i^2)\,\Phi(x) = -e'\rho(x), \qquad (1)$$

where $\varkappa_0 < \varkappa_1 < \ldots \varkappa_n$, and $\rho(x)$ is the density of the field sources. Equation (1) is equivalent to the system of equations

$$(\Box - \varkappa_i^2)\,\Phi_i(x) = -e'C_i\rho(x), \quad i = 0, 1, 2, \ldots, n, \qquad (2)$$

where the constants C_i are

$$C_i = \left[\prod_{i \neq j=0}^{n} (\varkappa_j^2 - \varkappa_i^2)\right]^{-1} \qquad (3)$$

and satisfy the Pauli-Villars regularization conditions. In this case the solution of Eq. (1) has the form

$$\Phi(x) = \sum_{i=0}^{n} \Phi_i(x). \qquad (4)$$

In the absence of sources, the solution of (2) can be written in the form of a Fourier expansion:

$$\Phi_i(x) = \left(\frac{\hbar c^2 |C_i|}{2L^3}\right)^{1/2} \sum_k \omega_i^{-1/2} \{a_i(k) \exp(-i\omega_i t + i\mathbf{kr}) + a_i^+(k) \exp(i\omega_i t - i\mathbf{kr}), \qquad (5)$$

where $\omega_i = (k^2 + \varkappa_i^2)^{1/2}$. The expression for the