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Electron Plasma Oscillations in an External Electric Field

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THIS letter is concerned with the determination of the frequency of oscillations of electron plasma placed in a constant and uniform electric field E_0*.

We shall denote by $F(\mathbf{r}, \mathbf{v}, t)$ a distribution function of the plasma electrons. This function satisfies the kinetic equation

$$\frac{\partial F}{\partial t} + \mathbf{v}\,\mathrm{grad}\,F + \frac{e}{m}(\mathbf{E}_0 + \mathbf{E})\frac{\partial F}{\partial \mathbf{v}} + J[F] = 0,$$

where $J(F)$ is the collision integral and $\mathbf{E}$ is the electric field specified by the plasma oscillations and satisfying the relation

$$\mathrm{div}\,\mathbf{E} = 4\pi e\int F d\mathbf{v} - 4\pi e n_0$$

where n_0 is the equilibrium density of the ions. The equilibrium distribution function F_0 of the electrons in the absence of oscillations has the form[2]

$$F_0 = f_0(v^2) + \mathbf{E}_0\mathbf{v} f_1(v^2),$$

$$f_0(v^2) = C\exp\left\{-w + \int_0^w \left[1 + \frac{w}{\xi}\right]^{-1} dw\right\},$$

$$f_1(v^2) = -\frac{el}{m}\frac{\partial f_0}{\partial v},$$

where $w = mv^2/2T$, $\xi = (M/6m)(eE_0 l/T)^2$, m and M are the masses of electrons and ions respectively, l is the mean free path of electrons and T the temperature. If we assume that the distribution function F differs only slightly from F_0, we obtain the following equations for $f = F - F_0$ and the field $\mathbf{E}$:

$$\frac{\partial f}{\partial t} + \mathbf{v}\,\mathrm{grad}\,f + \frac{e\mathbf{E}_0}{m}\frac{\partial f}{\partial \mathbf{v}} + \frac{e\mathbf{E}}{m}\frac{\partial F_0}{\partial \mathbf{v}} + \frac{1}{\tau}f = 0, \tag{1}$$

$$\mathrm{div}\,\mathbf{E} = 4\pi e\int f d\mathbf{v},$$

where the term f/τ phenomenologically takes into account the presence of collisions (τ is the average time between collisions).

We seek a solution for the set of equations (1) according to Landau[3] in the form

$$f(\mathbf{r}, \mathbf{v}, t) = \int f_{\mathbf{k}}(\mathbf{v}, t)\, e^{i\mathbf{k}\mathbf{r}} d\mathbf{k},\quad f_{\mathbf{k}}(\mathbf{v}, t) \tag{2}$$

$$= \frac{1}{2\pi i}\int_{-i\infty+\sigma}^{i\infty+\sigma} f_p(\mathbf{v})\, e^{pt} dp,$$

$$\mathbf{E}(\mathbf{r}, t) = -\,\mathrm{grad}\,\varphi = -i\int \mathbf{k}\varphi_{\mathbf{k}}(t)\, e^{i\mathbf{k}\mathbf{r}} d\mathbf{k},$$

$$\varphi_{\mathbf{k}}(t) = \frac{1}{2\pi i}\int_{-i\infty+\sigma}^{i\infty+\sigma} \varphi_p e^{pt} dp,\quad \sigma > 0,$$

and we obtain

$$\varphi_p = \frac{\frac{4\pi m}{E_0}\int \exp\left\{-\frac{m}{eE_0}\int_0^{v_x}\left(p + i\mathbf{k}\mathbf{v} + \frac{1}{\tau}\right)dv_x\right\}\left\{\int_{-\infty}^{v_x} g(\mathbf{v})\exp\left\{\frac{m}{eE_0}\int_0^{v_x}\left(p + i\mathbf{k}\mathbf{v} + \frac{1}{\tau}\right)dv_x\right\}dv_x\right\}d\mathbf{v}}{k^2 - \frac{4\pi e}{E_0}\int \exp\left\{\frac{-m}{eE_0}\int_0^{v_x}\left(p + i\mathbf{k}\mathbf{v} + \frac{1}{\tau}\right)dv_x\right\}\left\{\int_{-\infty}^{v_x} i\mathbf{k}\frac{\partial F_0}{\partial \mathbf{v}}\exp\left[\frac{m}{eE_0}\int_0^{v_x}\left(p + i\mathbf{k}\mathbf{v} + \frac{1}{\tau}\right)dv_x\right]dv_x\right\}d\mathbf{v}}, \tag{3}$$

where $g(\mathbf{v}) = f_k(\mathbf{v}, 0)$ and the field E_0 is assumed to be along the x axis.

In the following we shall be interested only in the plasma oscillatory properties which are determined by the so-called dispersion relation. The latter is obtained by setting the denominator of Eq. (3) equal to zero.

It is easy to establish that even in the case of strong fields when $\xi \gg 1$, the inequality $m\Omega\bar{v}/eE_0 \gg 1$ holds, where $\Omega = \sqrt{4\pi n_0 e^2/m}$ is the Langmuir frequency and $\bar{v}$ the average velocity of the electrons. Therefore we shall assume that $mp\bar{v}/eE_0 \gg 1$. Neglecting orders of E_0 higher than the first, and carrying out the integration in the dispersion relation over the velocity components perpendicular to the direction of the wave vector k we have

$$k^2 - \frac{8\pi^2 e^2}{m}\left\{\int_{-\infty}^{\infty} \frac{u f_0(u^2)}{\omega'/k - u} du \right. \tag{4}$$

$$- \frac{1}{2} \mathbf{E}_0\mathbf{n} \int_{-\infty}^{\infty} \frac{\Phi_1(u^2)}{\omega'/k - u} du$$

$$+ \mathbf{E}_0\mathbf{n} \int_{-\infty}^{\infty} \frac{u^2 f_1(u^2)}{\omega'/k - u} du$$

$$\left. + i\frac{e\mathbf{E}_0\mathbf{n}}{mk} \int_{-\infty}^{\infty} \frac{u f_0(u^2)}{(\omega'/k - u)^3} du \right\} = 0,$$

where $\omega' = i(p + \tau^{-1})$, $\mathbf{n} = \mathbf{k}/k$; u is the component of velocity along the wave vector k and

$$\Phi_1(u^2) = \int_{|u|}^{\infty} f_1(u^2)\, 2u\,du.$$

Expanding Eq. (4) into a series of powers of ku/ω', and retaining only the terms up to the fourth order of ku/ω' we finally obtain the dispersion relation in the form

$$k^2 - k^2\frac{\Omega^2}{\omega'^2} - k^4\overline{v^2}\,\frac{\Omega^2}{\omega'^4} \tag{5}$$

$$- 2\Omega^2\frac{k^3}{\omega'^3}(\mathbf{n}\mathbf{v}) - 3i\Omega^2\frac{e\mathbf{E}_0\mathbf{n}}{mk}\frac{k^4}{\omega'^4} = 0,$$

where $\overline{v^2}$ and $\mathbf{v}$ are the average values of the square of the velocity and of the velocity vector of the electron.

$$\overline{v^2} = \frac{1}{n_0}\int v^2 f_0(v^2)\,dv, \quad \mathbf{v} = \frac{\mathbf{E}_0}{3n_0}\int v^2 f_1(v^2)\,dv. \tag{6}$$

Solving Eq. (5) for ω' we obtain

$$\omega' = \Omega + k^2\overline{v^2}/2\Omega + (\mathbf{k}\mathbf{v}) + i3e\mathbf{E}_0\mathbf{k}/2m\Omega. \tag{7}$$

From Eq. (2) the Fourier components of the electric field vary according to the relation $E = \text{const.} \exp(i\mathbf{k}\mathbf{r} + pt)$. Since $\omega' = i(p + \tau^{-1})$, then $E_k = \text{const} \exp\{-\gamma t + i\mathbf{k}\mathbf{r} - i\omega t\}$, where

$$\omega = \Omega + \frac{1}{2}\frac{k^2\overline{v^2}}{\Omega} + (\mathbf{k}\mathbf{v}), \quad \gamma = \frac{1}{\tau} - \frac{3e\mathbf{E}_0\mathbf{k}}{2m\Omega}. \tag{8}$$

Equation (8) determines the frequency and the damping of the plasma oscillations in the presence of the electric field. These are correct if $k^2\overline{v^2} \ll \Omega^2$, $\mathbf{k}\mathbf{v} \ll \Omega$. It is evident that the electric field decreases damping if the angle between the wave vector k and the electric field E_0 is less than $\pi/2$. However, even in the case of strong fields, when $\xi \gg 1$ the inequality $3e\mathbf{E}_0\mathbf{k}/2m\Omega < 1/\tau$ is true. Indeed $\tau \simeq l/\bar{v}$, where, in the case of strong fields, $\bar{v} \simeq (M/m)^{1/4}(eE_0 l/m)^{1/2}$, and since $k \simeq \Omega/\bar{v}$, then $(3e\mathbf{E}_0\mathbf{k}/2m\Omega)\tau \simeq \sqrt{m/M} < 1$.

We shall give the expressions for the frequency of the plasma oscillations in the limiting cases of weak and strong fields, assuming that the mean free path does not depend on the velocity of the electrons. In the case of weak fields ($\xi \ll 1$):

$$\omega = \Omega + \frac{k^2 s^2}{2\Omega} + \sqrt{\frac{8}{3\pi}}\frac{e\mathbf{E}_0\mathbf{k}l}{ms}, \quad s = \sqrt{\frac{3T}{m}}.$$

For the case of strong fields ($\xi \gg 1$):

$$\omega = \Omega + \frac{1}{2}\frac{\Gamma(5/4)}{\Gamma(3/4)}\frac{k^2\sigma^2}{\Omega} + \frac{4}{3}\frac{\Gamma(3/2)}{\Gamma(3/4)}\frac{e\mathbf{E}_0\mathbf{k}l}{m\sigma},$$

$$\sigma = \left(\frac{4M}{3m}\right)^{1/4}\left(\frac{eE_0 l}{m}\right)^{1/2}.$$

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* Gordeev[1] has devoted an article to this problem. However, he uses an equilibrium function which does not satisfy the kinetic equation.

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K-Meson Charge Exchange in Hydrogen and Deuterium

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COMPARISON of cross sections for charge exchange of *K*-mesons in hydrogen and deuterium can give valuable information about their spins and parities. Theoretical investigation can be carried out similarly to the charge exchange of π-mesons in hydrogen and deuterium[1]. At the present time, apparently, the existence of at most two different *K*-mesons has been established: θ and τ. θ-meson decays into two π-mesons and consequently should have even parity with even spin and odd parity with odd spin. Analysis carried out by Dali[2] indicates that τ-meson apparently has odd parity and even spin. We shall consider the charge exchange of mesons with spins 0 and 1.

In a general case the amplitude of charge exchange of a meson with a proton is equal to $u_p = a + \mathbf{b}\vec{\sigma}$. Here a and $\mathbf{b}$ are functions of momenta and spins of meson before and after the charge exchange; $\vec{\sigma}$ is the spin operator. The amplitude of charge exchange with deuteron is expressed by u_p:

$$u_d = \sqrt{2}\int \psi^*(\mathbf{r})\, e^{i\vec{\varkappa}\mathbf{r}}\, u_p \psi_d(\mathbf{r})\, d\mathbf{r}.$$

Here ψ_d is the wave function of the deuteron, $\psi(\mathbf{r})$ is the wave function of two neutrons appearing as a result of the collision, $\vec{\varkappa} = (\mathbf{k} - \mathbf{k}')/2$ and $\mathbf{k}$ and $\mathbf{k}'$ are the momenta of the meson before and after the collision ($\hbar = c = 1$).

The cross section for the charge exchange with a proton in terms of the amplitude is $\sigma_p = \sigma_a + \sigma_b$; $\sigma_a = \overline{a^2}$, $\sigma_b = \overline{b^2}$. The cross section of charge exchange with deuteron, summed over states of the two neutrons after collisions*

$$\sigma_d = (\sigma_a + {}^2/_3\, \sigma_b)\, F_- + {}^1/_3\, \sigma_b F_+,$$
$$F_\pm = 1 \pm (\alpha / \varkappa)\, \mathrm{arc\,tg}\, (\varkappa / \alpha).$$

For $\varkappa = 0$: $F_+ = 2$, $F_- = 0$. Here $\epsilon = \alpha^2/M$ is the binding energy of the deuteron and the bars denote averaging and summations over the spin states of the meson (if it has spin).

We shall consider charge exchange with scattering at small angles. In this case $\varkappa \ll k$ and it may be assumed that $\mathbf{k}$ and $\mathbf{k}'$ have the same direction. We shall now consider several cases in more detail:

1. Charge exchange of *K*-meson with spin 0 without a change in parity. In this case a is a scalar, $\mathbf{b}$ is a pseudovector. However, since there is only one vector in the problem, $\mathbf{k}$, and for the construction of a pseudovector at least two vectors are necessary, then $\mathbf{b} = 0$. We obtain $\sigma_p = \sigma_a$, $\sigma_d = \sigma_a F_-$.

For $\varkappa = 0$: $\sigma_d / \sigma_p = 0$.

2. Charge exchange of *K*-meson with spin 1 without a change in parity. Beside the vector $\mathbf{k}$ there are two other vectors (pseudovectors): $\mathbf{j}$ and $\mathbf{j}'$, determining the direction of the meson spin before and after the charge exchange. In this case $\mathbf{b} \sim [\mathbf{j}\mathbf{j}']$

$$\sigma_p = \sigma_a + \sigma_b, \qquad \sigma_d = (\sigma_a + {}^2/_3\, \sigma_b)\, F_- + {}^1/_3\, \sigma_o F_+.$$

For $\varkappa = 0$: $\sigma_d / \sigma_p = 2\sigma_b / 3\,(\sigma_a + \sigma_b)$.

3. Charge exchange of *K*-meson with spin 1 into *K*-meson with spin 0 without a change in parity. In this case $a = 0$, $\mathbf{b} \sim \mathbf{j}$,

$$\sigma_p = \sigma_b, \qquad \sigma_d = {}^2/_3\, \sigma_b F_- + {}^1/_3\, \sigma_b F_+.$$

For $\varkappa = 0$: $\sigma_d/\sigma_p = {}^2/_3$.

4. Charge exchange of *K*-meson with spin 0 with a change in parity. In this case $a = 0$, $\mathbf{b}$ is a vector ($\mathbf{b} \sim \mathbf{k}$) and

$$\sigma_p = \sigma_b,\ \sigma_d = {}^2/_3\, \sigma_b F_- + {}^1/_3\, \sigma_b F_+.$$

For $\varkappa = 0$: $\sigma_d / \sigma_p = {}^2/_3$.

5. Charge exchange of *K*-mesons with spin 1 with a change in parity. In this case $\mathbf{b} \sim \mathbf{k} \times [\mathbf{j}\mathbf{j}']$,

$$\sigma_p = \sigma_a + \sigma_b,\ \sigma_d = (\sigma_a + {}^2/_3\, \sigma_b)\, F_- + {}^1/_3\, \sigma_b F_+.$$

For $\varkappa = 0$: $\sigma_d/\sigma_p = 2\sigma_b/3\,(\sigma_a + \sigma_b)$.

6. Charge exchange of *K*-meson with spin 1 into *K*-meson with spin 0 with a change in parity. In this case $\mathbf{b} \sim [\mathbf{j}\mathbf{k}]$,

$$\sigma_p = \sigma_a + \sigma_b,\ \sigma_d = (\sigma_a + {}^2/_3\, \sigma_b)\, F_- + {}^1/_3\, \sigma_b F_+.$$

For $\varkappa = 0$: $\sigma_d / \sigma_p = 2\sigma_b / 3(\sigma_a + \sigma_b)$.