netic meridian (Fig. 1). Let us examine the field at point $z = 0$. The incident wave can be represented in the form:

$$e_x = C \sin\beta \cos(\omega t - \gamma), \quad e_y = C \cos\beta \cos(\omega t - \gamma).$$

For the ordinary wave we can write:

$$e_{x_2} = A_2 \sin(\omega t - \delta), \quad e_{y_2} = B_2 \cos(\omega t - \delta).$$

For the extraordinary wave:

$$e_{x_1} = -A_1 \sin(\omega t - \varphi), \quad e_{y_1} = B_1 \cos(\omega t - \varphi).$$

It is necessary to find the relation between the major and minor semiaxes of the ellipses of polarization of the magnetoionic components ($|B_2/A_1|$ or $|A_2/B_1|$). Solving the equation given above (using the condition of continuity of the field vectors, we find:

$$\left|\frac{B_2}{A_1}\right| = \left(\frac{|K_2|^2 + \mathrm{tg}^2\beta}{1 + |K_2|^2 \mathrm{tg}^2\beta}\right)^{1/2}. \quad (3)$$

Now, using Eq. (3), we can compute how the energy is distributed between the magnetoionic components in the splitting of the plane-polarized wave. For this we must compare the average time values of the streams of the energies $S_{1,2}$ of the two waves:

$$S_2 = (c/4\pi)(A_2^2 + B_2^2), \quad S_1 = (c/4\pi)(A_1^2 + B_1^2).$$

In Fig . 2 we have plotted the graphs of $S_2/S_1 = f(\beta)$ for several values of $|K_2|$. From the plot it is seen that with circular polarization, $|B_2/A_1| = 1$ regardless of the angle β. The maximum dependence of the relative intensity of the components on the angle takes place with plane polarization. With $|K_2| = 1.05$, S_2/S_1 does not exceed 1.1. In other words, with $I \gtrsim 70°$, at the frequencies mentioned above, the average time values of the streams of the energies of the magnetoionic components at the lower boundary of the ionosphere do not differ by more than 10%. With $\beta = 45°$, which corresponds to antenna location in a NE-SW or SE-NW (magnetic direction, the ellipses of polarization and the average time values of the streams of the energies of the magnetoionic components are equal to each other.

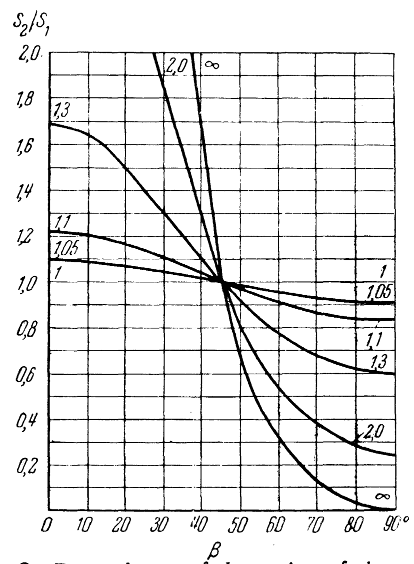

FIG. 2. Dependence of the ratios of time average values of energy flow of the magnetoionic components on the orientation of the transmitting antenna for various $|K_2|$.

The data obtained shows that if, for example, it is desired to have the magnetoionic components excited to the same degree in the ionosphere, which is sometimes important in the measurement of attenuation, the antennas should be disposed in a NW-SE or NE-SW direction. In case it is desired to "invest" the greatest possible energy in the ordinary component, the antenna should be located in a N-S direction.

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Comparison of Stability of Nuclear Shells and Subshells

V. A. KRAVTSOV
M. I. Kalinin Leningrad Polytechnical Institute
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An attempt is made in the present work to compare the stability of various nuclear shells and subshells on the basis of experimental data. To this end, the values of jumps in the curves of the separation energy e_α of an α-particle in light and medium nuclei are studied.

Figure 1a shows a typical curve, first given by Elsasser[1], of the dependence of the energy of α-decay on the number of neutrons in the nucleus, or the separation energy e_α of α-particles on the number of neutrons in nuclei with equal Z. It should be noted that, if the decay energy E_α is in the positive direction of the Y axis, the value of e_α, having the opposite sign, increases in the negative direction of the Y axis. There is a jump Δe_α of the curve $E = f(N)$, mainly in the interval from N_m to $N_m + 2$, where N_m is the number of neutrons forming a shell. As has been shown[2], the value of the jump Δe_α measured along the energy axis equals the sum of the depressions on the energy surface for the nuclei (Z, N_m) and $(Z-2, N_m)$, caused by their increased stability due to the closing of a shell or a subshell of N_m neutrons. Consequently, Δe_α is proportional to the stability of the neutron shells or subshells.

As has been shown by Sebgupt[3], the existence of a proton shell or subshell consisting of Z_m protons can be found from the curve which shows the dependence of E_α (or e_α) on the number Z of protons in nuclei with N = const, (see Fig. 1b), where the value of the shift of the curve on the Y axis, Δe_α, is also the measure of stability of proton shells and subshells.

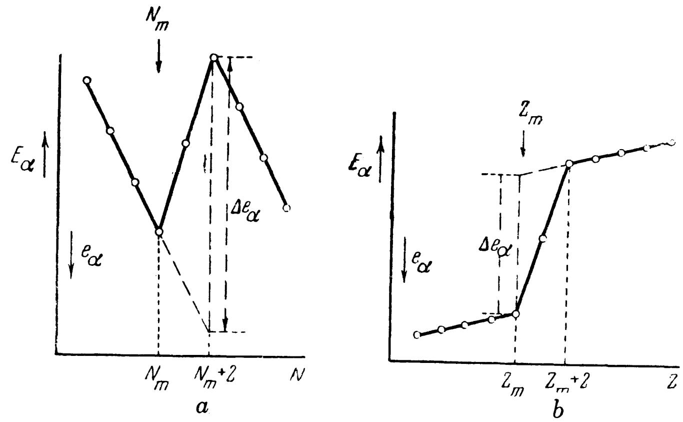

FIG. 1. a — Z=const, b — N=const

In the case of α-stable nuclei, the energy $e(Z, N)$ needed for separation of an α-particle from the nucleus can be calculated from the binding energy of the nucleons. Denoting this value by $E(Z, N)$ we have:

$$e_\alpha(Z, N) = E(Z, N) - E(Z-2, N-2) - E(2,2). \quad (1)$$

Curves showing the dependence of e_α on N (Z = const.) and of e_α on Z (N = const.), where e_α was calculated using Eq. (1), were plotted for nuclei with Z from 6 to 54 with the help of tables[4,5,6], based on experimental data.

In these curves, which are to be published separately, jumps and knees can be observed, caused by shells and subshells consisting of 8, 14, 20, 28, 32, 34, 40, 50, 58, 64 and 70 neutrons and of 8, 14, 20, 24, 28, 32, 38, 46 and 50 protons.

The values of the measure of stability Δe_α were calculated for each curve, according to the scheme of Fig. 1, for all neutron and proton shells and subshells. The curves showing the dependence of Δe_α on Z for neutron shells, and on N for proton shells, show a maximum for a certain value of Z or N (Fig. 2). This once more confirms the statement of the author[7] that the stability of a neutron shell depends on the number of protons, and the stability of a proton shell on the number of the neutrons in the nucleus.

The mean values of shell and subshell stability Δe_α and the even values of Z or N, for which Δe_α are greatest, are given in the Table (see below). Comparison of the $\overline{\Delta e_\alpha}$ in the Table shows that the most stable shells consist of 8, 50, and 82 neutrons and protons and 126 neutrons. The subshells $Z_m = 24$, $Z_m = 46$ and $N_m = 58$, the stabilities of which are relatively high, were discovered first. The confirmation of existence of the subshell $Z_m = 24$ may be seen in the slight elevation of the first excited state of Cr isotopes[8]. The same subshell is partly substantiated by Fig. 1 in Groshev's work[9], where the point corresponding to Cr lies above the curve. The existence of the subshell $N_m = 58$ is confirmed by the existence of 5 stable nuclides with 58 neutrons and by the fact, that Cd^{106} ($N = 58$) is more abundant (1,215%) than Cd^{108}(0.815%)[10]. There are as yet no additional indications for the existence of the subshell $Z_m = 46$. Since the experimental data for plotting the curves pertaining to the subshell Z_m=46 are very crude, its existence is still doubtful.

The shells and subshells are satisfactorily explained by the scheme of the nucleonic levels under

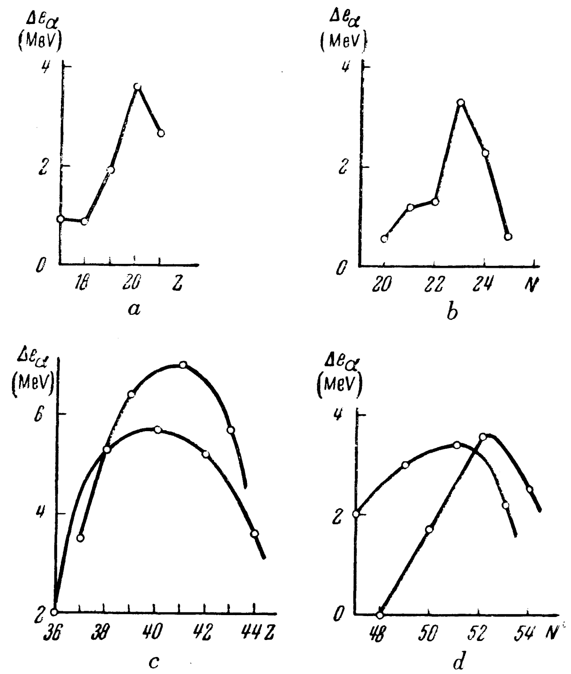

FIG. 2. a — $N_m = 20$, b — $N_m = 50$, c — $Z_m = 20$, d — $Z_m = 38$

the assumption of the spin-orbit coupling. The subshell $Z_m = 24$ is then explained as follows: the subshell $Z_m = 20$ is closed by filling the $3d_{3/2}$ level, but since the levels $3d_{3/2}$ and $4f_{7/2}$ are for protons very close to each other, the appearance of four additional protons makes it convenient, from energy considerations, to let four other protons leave the level $3d_{3/2}$ and occupy the $4f_{7/2}$ level. Additional protons fill up the $3d_{3/2}$ level once more. This is confirmed by the equal stability of subshells consisting of 20 and 24 protons. The subshell $N_m = 58$ is explained by the filling of the $5g_{7/2}$ level, coming next after the $5g_{9/2}$ level ($N_m = 50$).

The different subshells found here for neutrons and protons indicate that the sequence of level filling is different for the two types of particles. In the case of neutrons, the $4f_{7/2}$ level is followed by $3p_{3/2}$, $3p_{1/2}$, $4f_{5/2}$ and $5g_{9/2}$, while for protons the sequence is $3p_{3/2}$, $4f_{5/2}$ $3p_{1/2}$ and $5g_{9/2}$.

The previous confusion with regard to nuclides of higher stability in the region of Sr and Zr[7] can be removed here. The center of this region is formed by the nuclide Sr^{88} with double proton-neutron shell ($N_m = 50$, $Z_m = 38$).

Mean Stability of Nuclear Shells and Subshells

Neutron shells and subshells			Proton shells and subshells		
Number of neutrons N_m forming the shell or subshell	Mean Stability $\overline{\Delta e_\alpha}$ (MeV)	Even number of protons Z, for which stability is greatest	Number of protons Z_m, forming the shell or subshell	Mean Stability $\overline{\Delta e_\alpha}$ (MeV)	Even number of neutrons N for which stability is greatest
8	7,4±0,4	8	**8**	3,8±0,2	8
14	1,8±0,2	16	14	2,5±0,1	16
20	2,0±0.1	20	20	1,8±0,2	24
28	2,5±0,1	28	24	1,8±0,2	28
32	3,1±0,8	29	28	4,0±0,4	34
34	1,3±0.2	28	32	2,0±0,2	40
40	0,7±0,2	32	38	2,3±0,3	52
50	4,9±0,4	40	46	1,6±0,9	56?
58	1,8±0.6	48	**50**	4,2±0,7	70
64	0,6±0,4	48	—	—	—
70	1,1±0.4	>54	—	—	—
—	—	—	**82**	>4,1 ±0,4	126
—	—	—	88	0.47±0,05	136
126	>5.5 ±0,1	82	92	0.39±0,05	144
152	0.76±0,04	?	96	0,45±0,05	146

A fuller account of the results of the present work will be published separately.

Remark added during proof: A second study of the curves $e_\alpha = f(N)$ and $e_\alpha = f(Z)$ completed for $54 < Z \leq 66$ resulted in the discovery of the neutron subshells $N_m = 16$

with mean stability $\overline{\Delta e_\alpha}$ = 0.56 ± 0.16 mev and N_m = 66 with $\overline{\Delta e_\alpha}$ = 0.58 ± 0.30 mev and confirmed the existence of the shell $N_m = 82$ with $\overline{\Delta e_\alpha}$ = 3.42 ± 1.50 mev. The following proton subshells were found or confirmed: $\mathbf{Z_m = 16}$ with $\overline{\Delta e_\alpha}$ = 0.92 ± 0.28 mev, $Z_m = 40$ with $\overline{\Delta e_\alpha}$ = 1.54 ± 0.6 mev, $Z_m = 58$ with $\overline{\Delta e_\alpha}$ = 1.30 ± 1.20 mev and $Z_m = 64$ with $\overline{\Delta e_\alpha}$ = 0.36 ± 0.24 mev (newly discovered subshells are in boldface). The new subshells $N_m = Z_m = 16$ are excellently confirmed by the elevation of the first excited level of the even-even nuclei[11].

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Estimate of the $\pi^+ - p$ Scattering Cross Section from the $\pi^- - d$ Scattering Cross Section near Resonance

IU. A. GOL'FAND

P. N. Lebedev Physical Institute

Academy of Sciences, USSR

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A PHASE analysis of experiments on the scattering of pions by protons[1-3] reveals a strong interaction in the $p^{3/2}_{3/2}$ state (p-state with total spin $j = 3/2$ and isotopic spin $I = 3/2$. In connection with this we have undertaken a theoretical explanation of $\pi - p$ scattering based on a hypothesis regarding the resonance nature of the $\pi - p$ interaction in the $p^{3/2}_{3/2}$ state[4,5]. The thoeretical scattering curves obtained in this manner are in good agreement with experiment, so that the existence of the resonance as an important qualitative characteristic of the $\pi - p$ interaction is very probable.

We shall consider a few quantitative consequences of resonance in the $p^{3/2}_{3/2}$ state. From the phase analysis the contribution of this state to the total $\pi^+ - p$ scattering cross section is

$$\sigma^+_{3/2} = 8\pi k^{-2} \sin^2\delta, \tag{1}$$

where k is the momentum of the meson in the center of mass system and δ is the scattering phase (in the system of units in which $h = c = \mu = 1$). At resonance $\sin^2\delta = 1$, $k \approx 2.65$, which gives $\sigma^+_{3/2}$ = 185 mb. Hence for the maximum of the total $\pi^+ - p$ scattering cross section we obtain the estimate

$$\sigma^+_{\max} > \sigma^+_{3/2\,\max} = 185 \text{ mb}. \tag{2}$$

The lower limit in (2) cannot be lowered essentially by increasing k^2 since this would result in a contradiction with other experimental data. Hence, if it should be definitely established by experiment that $\sigma^+_{\max} < 185$ mb this would be decisive evidence against the resonance nature of the $\pi - p$ interaction, and in particular, against the isobar theory[5].

Unfortunately the available experimental data on $\pi^+ - p$ scattering close to the cross section maximum are not exact and are at times contradictory. In this connection great interest attaches to the extremely careful measurements reported in Ref. 6 on the scattering of π^-- mesons by hydrogen and deuterium in the energy range from 140 to 400 mev. From these data, by using the relationship

$$\sigma^+ = \sigma(\pi^- d) - \sigma(\pi^- p), \tag{3}$$

indirect information can be obtained about the magnitude of σ^+, giving $\sigma^+_{\max}$ = 152.4 ± 5.5 mb, that is, a value which is below the lower limit in Eq. (2). Since this result contradicts the "resonance" hypothesis it is necessary to investigate the error in Eq. (3). As will be shown, the actual value of σ^+ is somewhat higher than the value obtained from (3).

In order to estimate the error in Eq. (3) it is, strictly speaking, necessary to solve the problem of meson scattering on the deuteron, which cannot be done for a number of reasons. It is only possible