with mean stability $\overline{\Delta e}_\alpha$ = 0.56 ±0.16 mev and N_m = 66 with $\overline{\Delta e}_\alpha$ = 0.58 ±0.30 mev and confirmed the existence of the shell $N_m = 82$ with $\overline{\Delta e}_\alpha$ = 3.42 ±1.50 mev. The following proton subshells were found or confirmed: $\mathbf{Z_m = 16}$ with $\overline{\Delta e}_\alpha$ = 0.92 ±0.28 mev, $Z_m = 40$ with $\overline{\Delta e}_\alpha$ = 1.54 ±0.6 mev, $Z_m = 58$ with $\overline{\Delta e}_\alpha$ = 1.30 ±1.20 mev and $Z_m = 64$ with $\overline{\Delta e}_\alpha$ = 0.36 ±0.24 mev (newly discovered subshells are in boldface). The new subshells $N_m = Z_m = 16$ are excellently confirmed by the elevation of the first excited level of the even-even nuclei[11].

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Estimate of the $\pi^+ - p$ Scattering Cross Section from the $\pi^- - d$ Scattering Cross Section near Resonance

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A PHASE analysis of experiments on the scattering of pions by protons[1-3] reveals a strong interaction in the $p^{3/2}_{3/2}$ state (p-state with total spin $j = 3/2$ and isotopic spin $I = 3/2$. In connection with this we have undertaken a theoretical explanation of $\pi - p$ scattering based on a hypothesis regarding the resonance nature of the $\pi - p$ interaction in the $p^{3/2}_{3/2}$ state[4,5]. The thoeretical scattering curves obtained in this manner are in good agreement with experiment, so that the existence of the resonance as an important qualitative characteristic of the $\pi - p$ interaction is very probable.

We shall consider a few quantitative consequences of resonance in the $p^{3/2}_{3/2}$ state. From the phase analysis the contribution of this state to the total $\pi^+ - p$ scattering cross section is

$$\sigma^+_{3/2} = 8\pi k^{-2} \sin^2\delta, \tag{1}$$

where k is the momentum of the meson in the center of mass system and δ is the scattering phase (in the system of units in which $h = c = \mu = 1$). At resonance $\sin^2\delta = 1$, $k \approx 2.65$, which gives $\sigma^+_{3/2}$ = 185 mb. Hence for the maximum of the total $\pi^+ - p$ scattering cross section we obtain the estimate

$$\sigma^+_{\max} > \sigma^+_{3/2\,\max} = 185 \text{ mb}. \tag{2}$$

The lower limit in (2) cannot be lowered essentially by increasing k^2 since this would result in a contradiction with other experimental data. Hence, if it should be definitely established by experiment that $\sigma^+_{\max} < 185$ mb this would be decisive evidence against the resonance nature of the $\pi - p$ interaction, and in particular, against the isobar theory[5].

Unfortunately the available experimental data on $\pi^+ - p$ scattering close to the cross section maximum are not exact and are at times contradictory. In this connection great interest attaches to the extremely careful measurements reported in Ref. 6 on the scattering of π^-- mesons by hydrogen and deuterium in the energy range from 140 to 400 mev. From these data, by using the relationship

$$\sigma^+ = \sigma(\pi^- d) - \sigma(\pi^- p), \tag{3}$$

indirect information can be obtained about the magnitude of σ^+, giving $\sigma^+_{\max}$ = 152.4 ±5.5 mb, that is, a value which is below the lower limit in Eq. (2). Since this result contradicts the "resonance" hypothesis it is necessary to investigate the error in Eq. (3). As will be shown, the actual value of σ^+ is somewhat higher than the value obtained from (3).

In order to estimate the error in Eq. (3) it is, strictly speaking, necessary to solve the problem of meson scattering on the deuteron, which cannot be done for a number of reasons. It is only possible to give a rough estimate of the following factors: (1) the effect of interference of the waves scattered by the proton and neutron; (2) an allowance for the internal motion of the nucleons in the deuteron. The interference effect was examined by Brueckner[7] who showed that at scattering phases exceeding 45° interference results in a decrease of the deuteron cross section compared with the sum of the cross sections for free nucleons, with this decrease reaching 20% at $\delta = 90°$. Although this estimate is in our opinion somewhat too high there is no doubt that interference reduces σ^+.

Consideration of the internal motion of the nucleons also lowers the value of σ^+ obtained from the deuteron data by comparison with the scattering cross section for free nucleons. In order to explain this qualitatively we shall assume that the nucleon is at rest in the laboratory system and that the energy of the incident meson is such that the total energy of the meson and nucleon in the c.m. system has the resonance value. If under these conditions the nucleon is in motion in the laboratory system the energy in the c.m. system is shifted with respect to the resonance point and a smaller cross section results. As a qualitative estimate of this effect let us consider the scattering cross section of a π^--meson on the neutron of the deuteron abstracting from the $\pi^- - p$ interaction and interference. This will enable us to make a rough estimate of the effect in which we are interested since, according to Eq. (3), only the difference of the cross sections is then important.

In the impulse approximation the scattering cross section on a bound neutron is

$$\sigma_{\text{bound}} = \int \sigma_0(\varepsilon) \, |\psi_D(\mathbf{p})|^2 d\mathbf{p}, \tag{4}$$

where $\varepsilon = \omega + E - M$, and ω and E are the energies of the meson and neutron in the c.m. system; p is the momentum in the system in which the deuteron as a whole is at rest. We express ϵ in terms of the momentum of the incident meson k_1 (the subscript 1 refers to quantities in the laboratory system):

$$\varepsilon = [M^2 + \mu^2 + 2(E_1\omega_1 - \mathbf{p}_1\mathbf{k}_1)]^{1/2} - M = \varepsilon_0 - \mathbf{p}_1\mathbf{k}_1/(\varepsilon_0 + M), \tag{5}$$

(ϵ_0 is the value of ϵ at $p_1 = 0$; M and μ are the nucleon and meson masses). For σ_0 we may take the value in (1) which is the main term at resonance. Removing the slowly varying function from under the integral sign we obtain $\sigma_{\text{CB}} = 8\pi k_0^{-2}\,\overline{\sin^2\delta}$ and, since at resonance $\sin^2\delta = 1$,

$$\sigma_{\text{CB}}/\sigma_0 = \overline{\sin^2\delta} = \int \sin^2\delta \, |\psi_D(\mathbf{p})|^2 d\mathbf{p}. \tag{6}$$

In the vicinity of resonance $\sin^2\delta = \Gamma^2/[(\varepsilon - \Delta)^2 + \Gamma^2]$. Substituting this quantity in (6) and taking (5) into consideration we obtain

$$\frac{\sigma_{\text{bound}}}{\sigma_0} = 1 - \frac{1}{3}\left(1 + \frac{\varepsilon_0}{M}\right)^{-2}\left(\frac{k_1}{\Gamma}\right)^2 \frac{\overline{E}_{\text{kin}}}{M}, \tag{7}$$

where $\overline{E}_{\text{kin}} = \int \frac{p^2}{M} |\psi_D(\mathbf{p})|^2 d\mathbf{p}$ — is the average kinetic energy of the deuteron.

In order to estimate (7) we use the value for Γ obtained from Ref. 5 and express all values in terms of the isobar excitation energy Δ which gives

$$\frac{\sigma_{\text{bound}}}{\sigma_0} = 1 - \frac{(1 + \Delta/M)^4}{3g_1^4(\Delta/\mu^2 - 1)^2} \frac{\overline{E}_{\text{kin}}}{M}. \tag{8}$$

Using for our constants values which give good agreement with experiment[5] $\Delta = 2.1\ \mu$, $g_1^2 = 0.12$, and setting $E_{\text{kin}}/M = 0.02$ we obtain $\sigma_{\text{bound}}/\sigma_0 \approx 0.88$, i.e., the cross section is reduced by approximately 10%.

Both of the effects which we have considered act in the same direction and, since there is no reason for assuming that there are other effects which could change the result qualitatively, we may conclude that the data given in Ref. 6 do not contradict the hypothesis of the resonance character of pion-nucleon interaction.

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