It is clear that the relativistic attraction effect in a quasi-coulombic repulsive potential with potential g^2/r can give rise to metastable levels only if the constant g is more than 15 times larger than the elementary charge e.

The calculations shown above confirm that in the relativistic quantum theory for a spinless particle in a scalar purely attractive field, appears a repulsion effect, and in a static vector, purely repulsive field, an attractive effect takes place.

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The Kinetics of Paramagnetic Phenomena

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IN the kinetic theory of electrons in metals, it usually suffices to limit oneself to the use of the distribution function $f(\mathbf{p},\mathbf{r})$ which gives the number of electrons per cell of phase space. However, for processes related to the change of spin states of the electrons, this appears to be insufficient and one has to introduce the vector phase space magnetization density $\vec{\sigma}(\mathbf{p},\mathbf{r})$, which is a generalization of the density matrix in the mixed representation for the case of a system of particles with spin ½. One can write the following equation* for $\vec{\sigma}(\mathbf{p},\mathbf{r})$:

$$\frac{\partial\vec{\sigma}}{\partial t}+\left(\frac{\mathbf{p}\partial}{m\partial\mathbf{r}}\right)\vec{\sigma}+e\left(\mathbf{E}+\frac{[\mathbf{pH}]}{mc},\frac{\partial}{\partial\mathbf{p}}\right)\vec{\sigma}+\frac{\beta}{\hbar}[\vec{\sigma}\mathbf{H}]=-\mathbf{J}_U-\mathbf{J}_\tau \tag{1}$$

where $\beta/2$ is the effective magnetic moment of the particle and $\mathbf{J}_\tau$ and $\mathbf{J}_U$ are integral operators, taking into account collisions without and with spin change, respectively. Usually the corresponding relaxation times τ and U satisfy the inequality $U \gg \tau$, hence the introduced separation between the integral operators does not give rise to any problems. Note that $\mathbf{J}_\tau$ and $\mathbf{J}_U$, generally speaking, depend on $\vec{\sigma}$ as well as on f. We shall not write their exact expression. We can approximate $\mathbf{J}_U$ by:

$$\mathbf{J}_U=U^{-1}(\vec{\sigma}-\vec{\sigma}_{00}), \tag{2}$$

where U is the spin relaxation time, and $\vec{\sigma}_{00}$ is the equilibrium value of the phase density of magnetization, which differs from zero in the case of a permanent magnetic field. In general, one cannot use for $\mathbf{J}_\tau$ an approximation similar to (2), because

$$\int \mathbf{J}_\tau d\mathbf{p}=0, \tag{3}$$

which is inconsistent with the equation analogous to (2). Equation (3) is an obvious consequence of the fact that a collision without spin change does not change the magnetization.

Being interested in the equation for the space magnetization density $\mathbf{M}(\mathbf{r},t)$ we assume that

$$\vec{\sigma}=\vec{\sigma}_0+\vec{\Sigma}=\mathbf{M}(\mathbf{r},t)F(p^2)+\vec{\Sigma}, \tag{4}$$

where

$$\int d\mathbf{p}F(p^2)=1,\quad \int d\mathbf{p}\vec{\Sigma}=0. \tag{5}$$

Then, from (1), we get the following system of equations:

$$\frac{\partial\mathbf{M}}{\partial t}+\int d\mathbf{p}\left(\frac{\mathbf{p}}{m}\frac{\partial}{\partial\mathbf{r}}\right)\vec{\Sigma}+\frac{\beta}{\hbar}[\mathbf{M}\mathrm{x}\mathbf{H}]=-\frac{1}{U}(\mathbf{M}-\mathbf{M}_0), \tag{6}$$

$$F(p^2)\left(\frac{\mathbf{p}}{m}\frac{\partial}{\partial\mathbf{r}}\right)\mathbf{M}+\mathbf{M}e\left(\mathbf{E}\frac{\partial}{\partial\mathbf{p}}\right)F+\frac{\partial\vec{\Sigma}}{\partial t}+\frac{\vec{\Sigma}}{U}+\left(\frac{\mathbf{p}}{m}\frac{\partial}{\partial\mathbf{r}}\right)\vec{\Sigma}-F\int d\mathbf{p}'\left(\frac{\mathbf{p}'}{m}\frac{\partial}{\partial\mathbf{r}}\right)\vec{\Sigma}(\mathbf{p}')+e\left(\mathbf{E}+\frac{[\mathbf{p}\mathrm{x}\mathbf{H}]}{mc},\frac{\partial}{\partial\mathbf{p}}\right)\vec{\Sigma}+\frac{\beta}{\hbar}[\vec{\Sigma}\mathrm{x}\mathbf{H}]=-\mathbf{J}_\tau(\vec{\sigma}_0+\vec{\Sigma}) \tag{7}$$

to get the equation for $\mathbf{M}$ it is sufficient, with the help of (7) to express $\vec{\Sigma}$ in terms of M. This is not difficult to do in the case $\Sigma \ll \sigma$ i.e., when the part of the phase magnetization density, which is assymetric in p-space is small.** In first approximation equation (7) can be written in the following form:

$$F(p^2)\left(\frac{\mathbf{p}}{m}\frac{\partial}{\partial\mathbf{r}}\right)\mathbf{M} + \mathbf{M}e\left(\mathbf{E}\frac{\partial}{\partial\mathbf{p}}\right)F = -\mathbf{J}_\tau(\vec{\sigma}_0) - \mathbf{J}_\tau(\vec{\Sigma}). \tag{8}$$

The form of the right hand side of (8) is possible because of the linearization of the integral J_τ. The approximations made to get (8) from (7) are in any case legitimate if $J_\tau(\vec{\Sigma})$ is much larger than all the other members containing $\vec{\Sigma}$ in the left hand side of (7). This condition is realized in the region of small deviations from homogeneity, of small frequencies and weak fields. Furthermore, it is assumed that $J_\tau(\vec{\sigma}_0) \lesssim J_\tau(\Sigma)$. The latter condition is satisfied in spite of the inequality $\sigma_0 >> \Sigma$, because $J_\tau(\vec{\sigma}_0)$ is a quantity assymetric in momentum space.

The solution of (8) can be written in the form

$$\Sigma_i = \alpha_{ikl}\frac{\partial M_k}{\partial r_l} + \beta_{ikl}M_kE_l + \gamma_{ik}M_k, \tag{9}$$

where α, β and γ are tensors depending only on the vector **p**. Neglecting terms of the form $E_i\partial M_i/\partial r_k$, we get the following equation for the space density of magnetization:

$$\frac{\partial M_i}{\partial t} - c_1\Delta M_i - c_2\frac{\partial^2 M_k}{\partial r_i\partial r_k} + \frac{\beta}{\hbar}[\mathbf{M}\mathbf{x}\mathbf{H}]_i = -\frac{1}{U}(\mathbf{M}-\mathbf{M}_0)_i. \tag{10}$$

Note that the approximation $J_\tau(\vec{\Sigma}) = \vec{\Sigma}/\tau$ which satisfies (3) because of the relation (5), gives the following values for the coefficients c_1 and c_2:

$$c_1 = \frac{1}{3}\int d\mathbf{p}\,\frac{p^2}{m^2}\tau F, \quad c_2 = 0. \tag{11}$$

For metallic electrons in a degenerate state, F differs from zero only for momenta lying on the Fermi surface, hence:

$$c_1 = {}^1/_3\,(p_0/m)^2\tau(p_0) = {}^1/_3 v_0\Lambda,$$

where v_0 is the electron velocity on the Fermi surface, and $\Lambda = v_0 t$ is the mean free path. In this case Eq. (10) takes the form

$$\frac{\partial\mathbf{M}}{\partial t} - \frac{1}{3}v_0\Lambda\Delta\mathbf{M} + \frac{\beta}{\hbar}[\mathbf{M}\mathbf{x}\mathbf{H}] = -\frac{1}{U}(\mathbf{M}-\mathbf{M}_0). \tag{12}$$

Equation (12) is an equation for the magnetization density discovered by Dyson[2] in his theory of electron diffusion in metals.

To finish, let us study the limiting conditions for which one has to solve Eqs. (10) and (12). For this purpose, let us consider the limiting conditions for the phase magnetization density $\vec{\sigma}$. Let n be the normal to the surface of the body. Denote $\vec{\sigma}$ by $\vec{\sigma}^{(1)}$ for pn> 0 and by $\vec{\sigma}^{(2)}$ for pn< 0. The conservation law of the magnetic moment of the particle during the collision with the surface can then be written in the form

$$\vec{\sigma}^{(1)}(-\mathbf{pn};\,0) = \vec{\sigma}^{(2)}(\mathbf{pn};\,0). \tag{13}$$

Here, the zero value of the argument corresponds to a magnetization of the surface of the body. From condition (13) it follows that

$$\int d\mathbf{p}\,(\mathbf{pn})\,\vec{\sigma}(\mathbf{p},\,0) = 0. \tag{14}$$

In particular, for the linear approximation of $J_\tau(\vec{\Sigma})$ mentioned above, the relation (14) gives (n grad) M=0 which corresponds to the limiting condition used by Dyson[2] when he neglected the surface relaxation.

*For sake of simplicity, we shall assume a quadratic dependence of the kinetic energy on the momentum. Generalization to any case is evident. The derivation of equation (1) is absolutely identical to the derivation of the equation for the quantum distribution function-the density matrix in the mixed representation (see for instance Ref. 1)

** When Σ is neglected, Eq. (6) becomes the well known Bloch[3] equation for $T_1 = T_2 = U$.

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The Conservation of Isotopic Spin and the Scattering of Antinucleons by Nucleons

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IN recent times, evidence for the conservation of isotopic spin in the interaction of nucleons with nucleons has been obtained. With the discovery of the antiproton[1], it becomes expedient to determine the consequences of the conservation of isotopic spin in the interaction of nucleons with antinucleons In particular, such calculations can enable us to check the hypothesis itself of the