The Hodoscopic Delayed Coincidence Method and Its Application to the Investigation of Slow Meson Formation in Cosmic Radiation.

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There is presented the basis of the hodoscopic delayed coincidence method of studying the processes of formation of slow π^+-mesons by the nuclear-active component of the cosmic radiation. Aside from describing the apparatus, an analysis of the various instrumental effects is carried out. An estimate is made of the effective energy of formation of the particles and also of the instrumental corrections, which must be taken into account in order to determine the energy spectrum and absolute intensity of formation of slow $\pi^{\pm}$-mesons.

FOR the study of the elementary acts of interaction of high energy nucleons ($10^9 - 10^{10}$ ev) with atomic nuclei, the most reliable and detailed information which characterizes this interaction has been obtained by the method of the thick photographic emulsions. However, along with its undoubted values, this method has a set of disadvantages which in certain cases limit considerably the possibilities of the experimenter. The most serious disadvantage is the laborious processing of the material and the difficulty of connecting the observed processes to nuclei with a definite atomic number, and also of determining the signs of charged particles. All of the specified difficulties can be overcome to a considerable degree by working with a controlled Wilson chamber, located in a magnetic field and provided with a plate of a given composition. However the latter method suffers from its own specific disadvantages, which combine with the usual large distortions, and the controlling system introduces errors into the observations of the numbers of particles and the energies and scattering angles of the particles.

We therefore go to indirect methods of investigating nuclear interactions which permit the detection of certain special phenomena which escape direct observation. The clearest example of this type was given by Grigorov[1], where the method was used for an analysis of the latitude and altitude dependence of the primary component of the cosmic rays and it was shown that in the interaction of nucleons of high energy with light nuclei the nucleons retain, on the average, 70% of their original energy. The ionization method can be applied to the investigation of the integral properties of the radiation and may be used in certain cases for direct, although not entirely well-defined, studies of the characteristics of elementary processes. One of the possible methods of this type is offered by the hodoscope, where, under the proper conditions, one is permitted not only to observe the trajectories of individual particles, but also to measure the energy and to identify the nature of the particles, as is given in particular by use of the well known mass spectrometer system of Alikhanov and Alikhanian. In spite of its weaknesses, the hodoscope has advantages over the Wilson chamber or photographic emulsion, combining both a large "luminosity" and a small dead time of the apparatus. The latter permits use of the simplest control system of counters which will essentially only reduce the discrimination and introduces a statistical regularity into the study of the phenomena.

As examples of various successful uses of the method of counters for investigating individual aspects of the elementary acts of nuclear interactions it is possible to cite experiments carried out in the course of a series of flights, on an extensive atmospheric scale, by the group of Zatsepin and Nikol'skov by the method of the "correlated hodoscope" (see Ref. 2) and also the first experiments on the study of the decay electrons from μ-mesons[3] and the investigation of the spectrum of formation of π-mesons in the mass spectrometer of Alikhanian.[4] In the case of the decay of μ-mesons, a simple application of the method of delayed coincidences (without a hodoscope) permitted one to make a satisfactory reliable identification of the pheno-

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[1] N. L. Grigorov, Dokl. Akad. Nauk SSSR 94, 835 (1954).

[2] N. A. Dobrotin, G. T. Zatsepin et al, Usp. Fiz. Nauk 49, 185 (1953).

[3] V. Kamalian and A. Alikhanian, Dokl. Akad. Nauk SSSR 97, 425 (1954).

[4] G. B. Zhdanov and A. A. Khaidarov, Dokl. Akad. Nauk SSSR 65, 287 (1949).

menon above the background of a considerable number of possible instrumental effects.

The decay of μ-mesons is by no means the only phenomenon in the study of which it is possible to use the method of delayed coincidences. Beginning in 1947, this method was used by us to study the processes of formation of slow mesons in the depths of the atmosphere (for example, see Ref. 5). The clarification of the nature of these processes, in particular, the determination of the number and average energy of formation of these particles and also the energy of the mesons in the backwards stream (from below upwards) is of great interest from the point of understanding the mechanism of interaction of fast nucleons and nuclei. So, for example, if they emerge from the interaction of two virtually free nucleons, then, in view of the relatively high speed of the combined center of gravity, the appearance of slow mesons in electron-nuclear showers can be principally the result of secondary processes, depending to a great degree on the atomic number of the nucleus. In order to study directly the processes of formation of slow mesons it is natural to supplement the usual method of delayed coincidences with a special counter hodoscope. Such a hodoscope was constructed in 1950-1951 by Korablev, and was used by us in work done in 1951-1952[6].

Unlike other hodoscopes, which use the combination of the delayed coincidence scheme (see Ref. 7), in the hodoscope system of Korablev (which we call GK-5) the delay of the coincidences is accomplished in every hodoscope cell, whereupon the output of the cell in the form of a single cascade is collected on the negative electrode of the thyratron MTX-90, allowing us to operate the system with a very high "luminosity" and to detect the processes of slow meson formation with a frequency of the order of 10 per hour.

2. DESCRIPTION OF THE APPARATUS

The hodoscope of the type GK-5 makes it possible to investigate the formation of slow π^+-mesons in thick substances from the ensuing π-μ-e decays within the confines of the apparatus. For this purpose the apparatus can be set up in one of two ways: on the one hand, with the help of certain detectors we select nuclear interactions of a definite class (in particular, electron-nuclear showers) where it is possible to determine how often and with what energy or angular distribution slow π^+-mesons are generated in a given interaction; on the other hand, we have a detector which operates only in the presence of any μ-e decay, and it is possible to investigate how often a stopped meson is formed within the confines of the apparatus, and what kinds are characteristic of nuclear interactions which are accompanied by such generation. Consequently, there are two specific problems to be treated and two types of experimental arrangements (GK-5A and GK-5B), characteristic types of which are shown in Fig. 1*a* and 1*b*. In both cases the arrangement consists of the following principle parts: a) a control counter system (row VU and NU) in Fig. 1*a* and 1*b*, designed to detect mesons and other particles which accompany their generation; b) another series of counters, a stratified filter *d*, which, combined with the rows of the GK-5 hodoscope, are so designed as to detect directly the stopping of mesons as well as electronic decays; c) a side group of counters (BG not shown in Fig. 1b) designed for selecting wide angle showers; d) the hodoscope GK-3, which, combined with the control and side counters, is intended to specify the character of the processes of meson formation; e) an electronic amplifier which accomplishes the selection and detection of the necessary delays from the control system and also controls the hodoscope.

The thickness of the filter *d* usually conforms to the range of the decay electron so that the efficiency of any layer of the hodoscope is small and depends on the presence of other filters in the arrangement.

The principle of operation of the apparatus in each of the variations *A* and *B* is easy to explain with the help of the block diagrams shown in Fig. 2a and 2b respectively. The pulse from the control counters (VU and NU) enters the corresponding cell of the hodoscope GK-3 and simultaneously enters the delay block through the preliminary shaping block (arrangement *A*) or cathode follower (arrangement *B*). In the first case, the amplitude of the pulse from the control group is proportional to the number of counters that were fired, which permits one to assign a definite lower limit to the number of particles in the shower being studied. The operation of the delay block involves the formation of three square master-pulses (M_1, M_2, and M_3) whose amplitude and duration determine which hodoscope section is allowed to function. The pulse M_1 (duration $T_1 \sim 0.7\mu$ sec) feeds the upper series of cells of the GK-5 hodoscope which

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[5] A. Abdullaev, G. Zhdanov et al, J. Exptl. Theoret. Phys. (U.S.S.R.) **20**, 673 (1950).

[6] A. A. Abdullaev, G. B. Zhdanov, L. N. Korablev and A. A. Khidarov, J. Exptl. Theoret. Phys. (U.S.S.R.) 21, 1078 (1951).

[7] A. O. Vaisenberg, J. Exptl. Theoret. Phys. (U.S.S.R.) 24, 545 (1953).

serves to detect non-delayed particles, i.e., slow mesons and accompanying particles. The pulse $M_2(T_2 = 1.3\mu$ sec) is formed with a delay ($\Delta t = 1.5$ sec) and goes to the lower series of cells of GK-5, which can detect only particles which are delayed for a time relative to the original meson, comparable to their own lifetime. A sufficiently high resolving power of the neon lamps of the hodoscope is achieved by means of proper shaping of not only the master pulses but also the pulses entering from the counters. The details of the scheme of both (upper and lower) of the cells of the hodoscope GK-5 for detecting either non-delayed or delayed discharges in one of the counters are shown in Fig. 3. Finally the pulse $M_3 (T_3 = 3\mu$ sec) goes to the neon lamp hodoscope of the usual type GK-3, having a low resolving power.

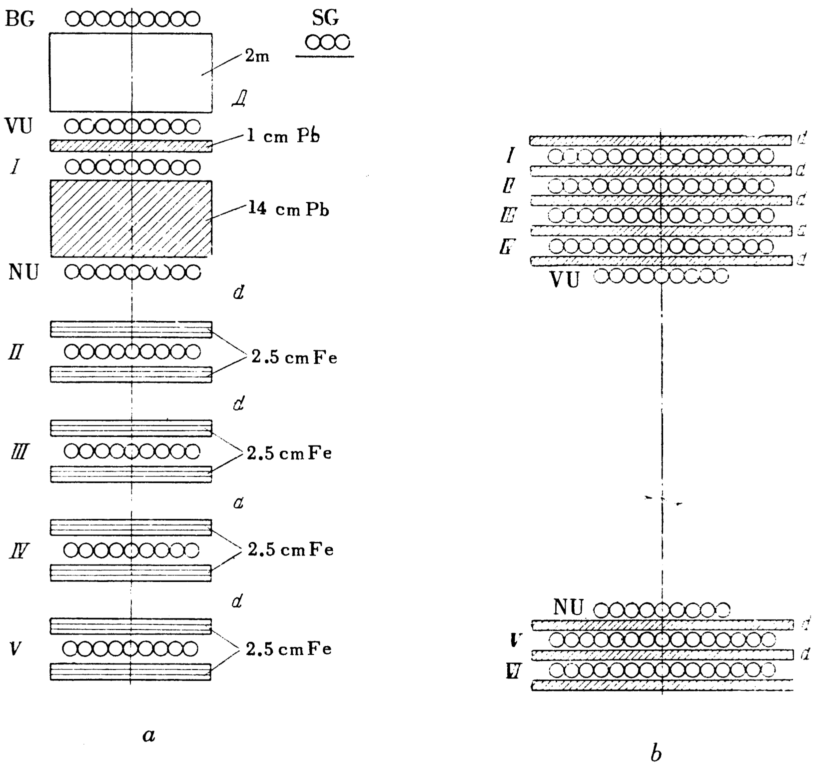

FIG. 1. Two variations of the arrangement of the hodoscope for studying the process of slow meson formation by the delayed coincidence method. a — arrangement of type GK-5A, b — arrangement of type GK-5B; VU and NU — control group of counters (upper and lower), BG and SG — supplementary group of counters (upper and side) for examining the role of air showers, I — VI — series of counters combined with the double cell of the GK-5 hodoscope, d — filter, designed to slow down mesons and act as a source of decay electrons.

Following the master pulse block is the automaton unit whose task it is to start the camera and later to extinguish the neon lamps. In arrangement *B* (Fig. 2b) the automation has a double stage, — it starts the camara and later permits the admission of the auxiliary pulse from any of the lower hodoscope cells of GK-5 i.e., from the delayed discharge of any of the counters of the hodoscope.

In connection with the usual scheme of operating with neon lamps we have the characteristic obstacles to achieving high resolving power, i.e., the lag of the gas discharge and the instability of the lamps, so it is interesting to point out here the main parameter of our apparatus, which characterizes its resolving power and (in part) the stability of operation. The resolving power of the GK-5 hodoscope for non-delayed particles is $\tau_1 = 1$ sec, for delayed particles $\tau_2 = 3$ sec ±10% (we can choose a time interval of 1-4 sec for non-delayed particles).

The resolving power of the GK-3 hodoscope is 30μsec. The range of operation of the cells of GK-5 for feeding the control anodes of the neon lamps is 60-80 volts (i.e., ±15%) and for feeding the main anodes $\sim$ 5 volts (±2.5%).

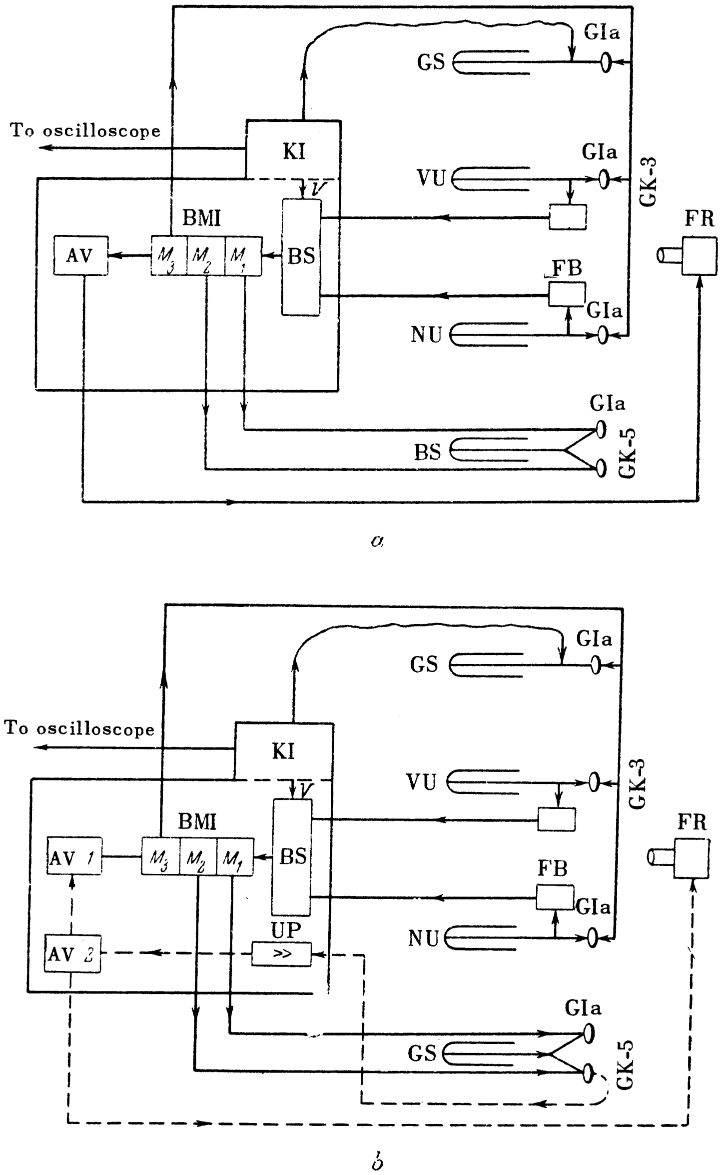

FIG. 2. Block diagram of the electronics of the experimental arrangement: a) GK-5A, b) GK-5B, VU, NU — control counters, GS,GIa— hodoscope counters and cells, FB — shaping block, BS — delay block, BMI — Master pulse block, AV — automaton, FR — camera, KI — control pulser, UP — constant current amplifier.

We will conclude by saying a few words concerning the control and regulation of the resolution of the GK-5 hodoscope. The adjustment of each of the tubes was checked once a day and in case it was necessary, it was changed by means of the special trimmers (see Fig. 3) at the entrance to the condenser which separates the circuit from the master pulses M_1 and M_2. The control of the resolution of the cells of GK-5 was done weekly, for which special pulses from the control pulser K' (which satisfactorily regulated the delay from one unit to the next) were supplied to the coincidence block and the entrance to the appropriate tube of the hodoscope.

3. ANALYSIS OF INSTRUMENTAL EFFECTS AND CORRECTIONS

In order to study the processes of formation of slow π-mesons by the method of the delayed coincidence hodoscope it is necessary to take into account the following instrumental effects and

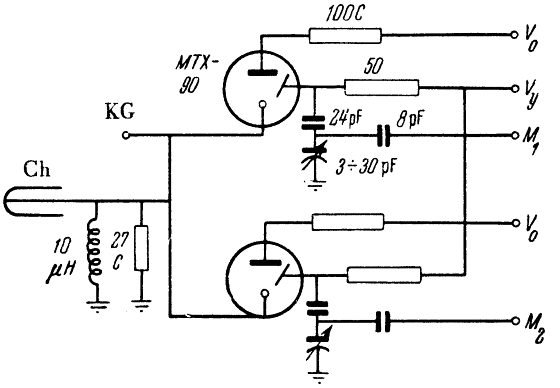

FIG. 3. Principle of the double cell scheme of the GK-5 hodoscope. Ch — counter; V_0, V_y — input supply to the main and control anodes of the lamp MTX-90; input for master pulses: M_1 — delayed; M_2 — non-delayed; KG — control connection

corrections: a) apparent decays, connected with the appearance of delayed coincidences due to accidental particles and delayed discharges of the counters (we will call these events accidentals and false coincidences); b) decay of stopped mesons which are not connected with the formation of these mesons by nuclear action of particles inside the apparatus; c) correction for the efficiency of detection of various processes, connected with the formation, stopping and finally the decay of mesons within the limits of the apparatus; d) errors in determining the point of formation of the mesons, connected with imperfection in the hodoscopic method, in particular, secondary interaction of particles in the electronic-nuclear shower; e) different values for the relation between the range and original energy of π-mesons, connected with the processes of nuclear absorption and scattering of these particles, and the circumstance whereby we detect only the projection of the range onto the plane of the hodoscopic photograph.

The relative importance of accidental coincidences can be estimated with the aid of the relation

$$N_{acc} = (N_{stop} + N'_{stop})\tau_2 N_3, \qquad (1)$$

where N_{stop} is the frequency of events connected with the stopping of the mesons, N'_{stop} is the frequency of events which imitate the stopping events, N_3 is the total of all pulses from counters which surround the stopping point, $\tau_2 = 3 \times 10^{-6}$ sec is the resolving time of the hodoscope for delayed particles, N_{acc} is the frequency of accidental coincidences that is being sought.

We compare the derived frequency N_{acc} with the frequency of true decays that are detected by the apparatus N_{det}:

$$N_{det} = N_{stop}\eta_0, \qquad (2)$$

where η_0 is the efficiency for detecting decays.

As will be shown below, the efficiency under our conditions was usually not less than 0.1. If we take into account the fact that the point at which the meson stops is in the immediate neighborhood of 5-7 counters, each having a cross section* of 100-160 cm², then the total load is $N_3 \leq 50$ sec^{-1} and in the absence of the "background" phenomenon, which imitates stopped mesons ($N'_{stop} = 0$), the accidental coincidences constitute a small part of the true decays. Unfortunately, such ideal conditions are practically impossible for two reasons: first, because of the presence in the shower of non-mesonic particles, which are connected with the stopping of particles which do not decay and only increase the "background" of accidental coincidences (this factor applies only in arrangements of class *B*, which have a sufficiently "soft" control system); second, because of the imperfect geometric conditions of the experiment (particularly in the arrangements of class *A*), when there are fast particles which are moving laterally (in the direction perpendicular to the plane of the hodoscope photograph); these give the appearance of particles which are stopped between two rows of the hodoscope. Owing to the influence of the factors just mentioned the total background of accidental coincidences in certain experiments with the arrangements of class *B* turns out to be comparable to the frequency of the decays being studied. This circumstance deprives us of the possibility of making a simple interpretation of each hodoscope photograph; however, for the statistical treatment of the data (in particular, for the study of the energy spectrum of the mesons) it is sufficient to determine the number of accidental coincidences with the aid of a special controlled experiment. For this purpose we made use of the usual experiment with the additional time displacement of the delayed master-pulse M_2, in which the apparatus did not detect any decays, but detected accidental coincidences at the same rate as in the primary experiment.

In contrast to the accidental coincidences, the dangers of false coincidences (caused by delayed discharges of the counters) did not enter into the work. This is explained by the fact that the experimentally selected displacement of our delayed master-

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* Experiment showed that the passage of decay electrons through the more remote counters did not, as a rule, have to be taken into account.

pulse was already sufficiently larger than the time for the delayed discharge of the counters. Actually, if the delayed discharges were essential, then one would need not only the end of the "track", but also the middle, which is practically not observable.

We investigate the effects of processes which may imitate the local formation of slow mesons by nuclear action of particles. Here we are concerned with various types of electromagnetic processes, and also nuclear splittings without emission of mesons. A share of these processes (electron-photon showers, δ-showers, from fast mesons and particles of non-mesonic nature) may imitate the formation and decay of mesons only because of the appearance of accidentally delayed particles, and also when one takes account of the controlled accidental delayed coincidences. In all cases, we have the point of the actual decay of the meson in the apparatus, but this meson either comes into the apparatus from the air (δ-showers from slow mesons, air showers) or is generated by nuclear-inactive particles (photogeneration)**.

An estimate of the possible portion of the mesons which come from the air was made by us by a computation made with the help of a special control experiment. The calculation of this estimate starts with the data on the flux from, on the one hand, slow mesons and mesons from wide air showers of a given density (several particles over the area of the apparatus), and on the other hand, nuclear-active particles with energies that are sufficient for efficient meson formation. The control experiment was designed to determine the role of the δ-showers and was based, first, on the height of passage through the apparatus at which the shower was detected, and second, on the study of the penetrating power and spatial distribution of the shower particles. The role of the air showers was determined both with the use of the special group of side counters, as well as by the method of analyzing the penetrating power of the shower particles.

The result of all of the mentioned estimates and control experiments consisted of the following. The contribution of δ-showers comprised no more than 5% in set-ups of class *A* (in this case the apparatus can only operate from twofold δ-showers), in set-ups of class *B* – 10% of the frequency of occurrence of local meson forming showers. The contribution of air showers in arrangements of class *A* was ~ 10% of the number of showers which generated mesons in the apparatus, whereupon the main part of these processes is dependent on nuclear-active particles, which go along with low density air showers, and they are distinguished from showers of the local type only by the very large average energy of the particles which form slow mesons in the apparatus***. The air showers in the arrangement GK-5B cause 25% of the total number of delayed coincidences that are detected from showers. However the majority of similar events are attributed to accidental coincidences, which is easy to understand. Because of the "softness" of the control system of counters the apparatus separates efficiently nuclear-active particles which have comparatively low energies, and hence may not have much air accompaniment.

The possible contribution of processes of photogeneration of mesons may only be estimated by calculation. We proceed from the flux of photons with energy > 200 mev in the atmosphere where as is well known (from accelerator data) we have the maximum effective coss section for photogeneration, and obtain a value of the order of 5% (in comparison with the frequency of generation by the nuclear-active component) as an upper limit for the relative frequency of photogeneration of mesons in our apparatus.

We pass now to the estimate of the efficiency of detection of slow mesons which are generated in the apparatus. This estimate is of interest from various points of view: for example, it permits one to make a comparison between the true decays and the "background" of accidental events, and also between the intensities of slow meson formation from various nuclei; by means of a comparison of the well known flux and energy spectrum of nuclear-active particles, it permits one to determine the efficiency of formation of the mesons under study over a range of energies. Aside from this, an analysis of the question of the efficiency of detection permits one to go into the corrections that are necessary to determine the true form of the spectrum for meson formation.

The problem of the efficiency of the apparatus, i.e., the relation between the number of generated and detected slow mesons, can be broken down into two independent parts: first, the determination of the probability with which the nuclear-active particles of a fixed energy create in the filter of

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** In both cases, the particles which accompany the meson may sometimes imitate local showers and appear as a source of formation and decay of mesons in the apparatus.

*** As was shown in the experiments of Liubimov et al[8], an increase in the energy of the particles which generate electron-nuclear showers, is accompanied by an increase in the probability of concomitant air showers.

[8] N. L. Korablev, A. L. Liubimov and A. T. Nevraev, Dokl. Akad. Nauk SSSR 68, 273 (1949).

the apparatus a π^+-meson of a given range[****]; which is confined within the limits of the apparatus; second, the probability of detection of electrons, which originate as a result of the decay of a meson which stops in one of the filters of the apparatus. For a single meson, proceeding into the apparatus from the air, only the second possibility remains. This is the simpler part of the problem at which we begin our calculation.

If we denote by d the thickness of the filter between the rows of the counters of the GK-5 hodoscope which detect the decay electrons, and by $F[E(X)]$ – the integral energy spectrum (X is the range of these electrons), then the average efficiency for detecting isotropically emitted decay electrons is to a first approximation of the form:

$$\eta_0 = \left\{\exp\left(-\frac{t_1}{\tau_\mu}\right) - \exp\left(-\frac{t_2}{\tau_\mu}\right)\right\}\frac{1}{d}\int_0^d dX \int_0^{\theta_{\max}(X)} F\left[E\left(\frac{X}{\cos\theta}\right)\right] d(\cos\theta). \tag{3}$$

In this expression t_1 and t_2 are the limits of the time interval in which the hodoscope can detect delayed particles, $\tau_\mu = 2.15\ \mu$ sec – the mean lifetime of the μ-meson and $\theta_{\max}$ is the limiting angle at which there can emerge from the filter the electrons which originate at a depth X and which possess the maximum possible length. The scattering of the electrons in the layers of the filter and at the ends of the filter (in a direction perpendicular to its thickness) is thereby not taken into account; however, it cannot introduce any essential error into the calculation of η_0.

For a practical calculation of the efficiency it is expeditious not to use the energy spectrum of the decay electrons, which was determined with great precision by use of the Wilson chamber or photographic plates, but to go directly from the integral range spectrum, i. e., the absorption curve, determined for various substances by the method of delayed coincidences from counters (see, for example, Refs.4 and 9 and 10). The efficiency η_0 calculated from this data for several variations of our apparatus is presented below.

Thickness of filter, d	2 cm Pb	2 cm Fe	2 cm C
Efficiency for detecting decays, η_0	0.14	0.20	0.30

The derived value lacks certain corrections, which are connected with the fact that a part of the counters, which detect decay electrons, may appear to be "busy" with discharges from non-delayed particles. For a single meson, such particles may appear only from the stopping of mesons, and in this case (when only one counter near the point of decay is busy) the average correction for the effect of the busy counters is a constant factor $\eta_i = 0.8$. In order to check experimentally the efficiency for detecting the decay of such mesons, we carried out a special control experiment in an underground installation (i.e., under the conditions when the penetrating particles consist only of mesons). The experiment was arranged so that the control system of the arrangement GK-5A was "adjusted" to select single penetrating particles, going in a narrow solid angle, and arranged under one of the rows of the GK-5 hodoscope which detected all stoppings of these particles as well as the accompanying decay electrons. We determined by this experiment a value of the efficiency which agreed well with the value deduced above (from the calculation of the correction due to the counters being "busy").

For mesons which strike the filter of the hodoscope accompanied by other particles, the correction for the occupied counters is calculated and the average value depends on the scattering from the point of formation of the original local shower to the meson track. In this case the correction to the factor η_i is determined purely experimentally, by means of utilizing the hodoscope pictures and determining the average probability of counters being occupied by various scatterings from the point of formation of the shower[+].

After determining the efficiency for detecting

[****] In order to simplify the problem we will, for the present, assume that the range has a vertical component only.

[9] J. Steinberger, Phys. Rev. 75, 1136 (1949).

[10] E. P. Hincks and B. Pontecorvo, Phys. Rev. 77, 102 (1950).

[+] In contrast to the case of the single meson the position of the occupied counters relative to the point of decay is no longer essential and hence the correction factor η_i can be taken as simply equal to the number of occupied counters of the hodoscope.

decays we turn to the calculation of what is called the "luminosity" of the apparatus, i.e., the frequency $N_{\rm det}$, of detected slow mesons, produced by the nuclear-active component and stopped inside the same apparatus. The expression for the luminosity can be written in the following form:

$$N_{\rm det} = \int_{E_0}^{\infty} \left[1 - \exp\left(-\frac{\Delta R}{\lambda(E)}\right)\right] S(E)\, \overline{W}_{\rm p}(E)\, \bar{\nu}_{\pi^+}(E)\, dE$$

$$= \frac{1}{l} \sum_{ij} \eta_0 \eta_i \frac{d\Phi_{\pi}(r_{\rm B})}{dr_{\rm B}}\, \Delta_i r \Omega_{ij}(r_{\rm B}), \qquad (4)$$

where E_0 is the threshold energy for generation of slow mesons by nucleons, ΔR is the sum of the thicknesses of all of the filters of the apparatus, whose number is denoted by 1, $\Delta_i r$ is the thickness of each separate filter, $\lambda(E)$ is the average path for generating mesons by nuclear-active particles of energy E, $S(E)$ is the energy spectrum of the nuclear-active component in the atmosphere, $\overline{W}_{\rm p}(E)$ is the mean probability of detection of showers by the control system of counters, the showers being formed with a fixed energy, $\bar{\nu}_{\pi^+}(E)$ is the average multiplicity of generation of the slow π^+-mesons in such showers, $\eta_0\eta_i$ is the previously determined efficiency for detecting decays and the correction for the occupied counters, $\bar{\Phi}_{\pi}(r_{\rm B})$ is an average over the primary integral energy spectrum of the meson range (actually, the vertical component of the range), $\Omega_{ij}(r_{\rm B})$ is called the "geometrical factor", i.e., the correction taking into account the finite size of the hodoscope filters.

We note that in the average over the primary meson energy spectrum (in order to normalize the spectrum, we must determine the energy dependence of the factor $\bar{\nu}_{\pi^+}$) we must take care of the circumstance that the true dependence of the spectrum on the energy E appears in the form of the function under the integral in expression (4) which cannot be calculated compared to such functions as $\lambda(E)$, $S(E)$ and $\overline{W}_p(E)$++.

There is no possibility of examining in detail all of the quantities which enter into (4), so we limit ourselves here only to some short remarks, after which we will give, for various cases derived by us, the function under the integral, which may be called "the energy generating function" of slow mesons by the nuclear-active component.

The threshold energy "E_0" and the total range of energies for an efficient generation of mesons can be obtained by combining the data, determined with accelerators, on the generation cross section for heavy nuclei and photographic plate data on the probability of "inelastic" nuclear collisions for various energies of the nucleons in the cosmic radiation+++. The differential energy spectrum $S(E)$ of the nuclear-active component in the atmosphere, in particular at mountain altitudes (where we carried out our experiments) is known at the present time from a whole series of works, carried out by various methods[8*], and can be represented satisfactorily by a function of the form $p^{-2.5}$ (p is the momentum). The probability of detecting showers $\overline{w}_{\rm p}(E)$ for an actual system of control counters may be calculated to a satisfactory approximation only in cases when the operation of the system is determined by penetrating shower particles; then we may use the photographic plate data for multiple formation of relativistic particles (see, for example, Refs. 12 and 13). As an average energy spectrum of slow mesons we can use the spectrum of Camerini et al.[12], which was determined with an arrangement in which the average energy of the nuclear-active particles was not too different from the energies at which our apparatus is efficient. The least obvious point appears to be the question of the function $\bar{\nu}_{\pi}(E)$, i.e., the multiplicity of generation of slow mesons. The fairly meager photographic plate

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++If the multiplicity $J_{\pi+}$ is not a monotonically increasing or rapidly decreasing function of the energy E, then the three functions also determine the effective range of primary energies for the given arrangement.

+++ Suitable data were considered by us in the survey Ref. 11.

[8*] The similarity of the conditions of observation is explained by the energy estimate made below.

[11] G. B. Zhdanov, Usp. Fiz. Nauk 54, 435 (1954).

[12] U. Camerini, P. H. Fowler et al., Phil. Mag. 41, 413 (1950).

[13] U. Camerini, J. H. Davies et al., Phil. Mag. 42, 1241 (1951).

data on the generation of slow mesons (with energies of tens of mev) in the stratosphere[14] shows that this multiplicity, does not apparently increase with the primary energy, and if it decreases, then it does so very weakly. Therefore, in carrying out our calculation, we start from two alternative assumptions about the form of the function $\bar{\nu}_{\pi^+}(E)$: a) $\bar{\nu}_{\pi^+}(E) =$ constant; b) $\bar{\nu}_{\pi^+}(E) \sim 1/n_s$ (for $E > 2$ bev), where n_s is the number of relativistic particles in the shower.

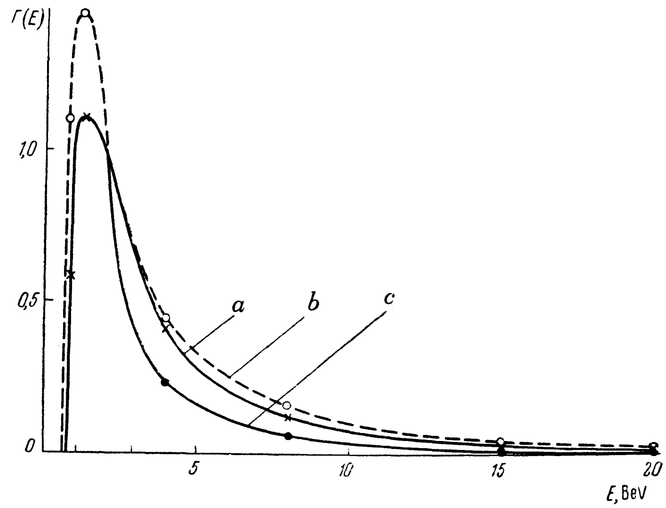

FIG. 4. Dependence of the output of slow mesons $G(E)$ on the energy of the generating particle E for various arrangements and assumptions about the multiplicity of formation: a) arrangement GK-5A, $\bar{\nu}_{\pi^+} =$ const.. b) arrangement GK-5B, $\bar{\nu}_{\pi^+} =$ const., c) arrangemnet GK-5B, $\bar{\nu}_{\pi^+} \sim {}^1/n_s$ (for $E > 2$ bev).

In Fig. 4 are shown the results of our calculation of the energy generation function of slow mesons for three different cases: a) Arrangement GK-5A, $\bar{\nu}_{\pi^+}(E) =$ constant, b) Arrangement GK-5B, $\bar{\nu}_{\pi^+}(E) =$ constant, c) arrangement GK-5B, $\bar{\nu}_{\pi^+}(E) \sim 1/n_s$ with $E > 2$ bev.

With the help of these curves, it is possible in each case to determine characteristic energies – the threshold energy E_0 and the average energy of the generated component E_{av}. These values can be determined from the energy limits to whose left there is respectively 5 and 50% of the total area, within the limits of the curves of Fig. 4. In this manner we obtain the "thresholds" and average energies (for each of the three cases):

Arrangement and multiplicity of meson generation	GK-5A $\bar{\nu}_{\pi^+} =$ const	GK-5B $\bar{\nu}_{\pi^+} =$ const	GK-5B $\bar{\nu}_{\pi^+} \sim 1/n_s$
Threshold Energy, E_0 in bev	1,1	1,0	0,9
Average Energy E_{av} in bev	2,9	3,0	2,0

By using the expression (4) we determined not only the energy dependence of the "output" of slow mesons, but the overall frequency of detecting decays by the apparatus (the "luminosity" of the apparatus). A comparison of the calculated number with the experimental observations showed that both of the initial assumptions about the dependence of the value of $\bar{\nu}_{\pi^+}$ on the initial energy are in accord with experiment (within the limits of error of the experiments and the calculations), as well as the assumption about the increase of the "output" of slow mesons with the primary energy E, for example according to the law $\nu_{\pi^+} \sim n_s(E)$ which leads directly to an es-

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[14] H. Yagoda, Phys. Rev. 85, 891 (1952).

timate of the "luminosity" of the apparatus.

We will consider methodically the question of the connection of the observed spectra and the energy of the mesons. In order to analyze the hodoscope picture of the appropriate shower, the terminus of the meson path is fixed by the appearance of the decay electron, and the average error is determined by measurement. Obviously, half the thickness of the filter separates two rows of the hodoscope counters. Errors in fixing the beginning of the track are connected with two factors: the inaccuracy in determining the point of formation of the initial shower in the filter of the hodoscope, and the possibility of generation of mesons by secondary nuclear-active particles in the shower. To explain the role of the first factor, we plot the distribution curve of the number of fired counters in the rows of the hodoscope, reading off the number of the row each time for the filter in which the shower was supposedly generated (absolute numbers are read off vertically downwards). The character of these curves plotted in Fig. 5 for two concrete arrangements (the hodoscope with graphite and lead filters each 2 cm thick), shows that the average error in determining the point of formation of the shower, apparently does not exceed half the thickness of the filter by virtue of the strong dependence of the number of counters that are fired on the number of the row. As a rule, nothing can be said about the effect of processes of secondary mesons on the shape of the hodoscopic pictures. It is possible, however, to take into account these processes with the aid of the following discussion. As is well known, the average ranges for absorption and interaction of the nuclear-active components in substances λ_{abs} and λ_{int} are connected by a simple relation (see, for example, Ref. 15):

$$\lambda_{int}/\lambda_{abs} = 1 - \bar{\nu}_{sec} \tag{5}$$

where $\bar{\nu}_{sec}$ is the average over the spectrum of the number of secondary nuclear-active particles. Taking into account the fact that, according to the data of Ref. 15, $\lambda_{int}/\lambda_{abs} = 1/3$ (independent of the atomic number of the nucleus), we obtain $\bar{\nu}_{sec} = 2/3$ and, consequently, the probability of forming secondary showers in each of the filters at a depth Δr located under the point of formation of the initial shower, is, on the average, $2/3\,\Delta r/\lambda_{int}$.

In order to go over from the distribution in range to the initial energy spectrum of π^+-mesons it is necessary to introduce corrections of two kinds. First of all it is necessary to take into account the fact that the hodoscope photograph does not permit one to obtain a representation of the complete meson track in space, and in the best case it is possible to represent only the projection of this track on the plane of the photographic plate. However, if we neglect the effect of the finite size of the hodoscope filters and the scattering of mesons in these filters, then the distribution of the vertical component of the track r_b can be converted into the distribution of the total range without difficulty, and conversely, if we know the angular distribution of the mesons relative to the vertical. Actually, the question in this case can be resolved by the method of generalization of the well known relation of Gross between the vertical absorption curve and the global intensity of cosmic rays. For an angular distribution of the radiation, incident within the limits of a certain plane area, of the form of $\cos^n\theta$ (θ – the angle with the vertical), the relation, which connects the vertical $f_b(X)$ and the global $F_{gl}(X)$ intensity of the radiation at a depth X has the form:

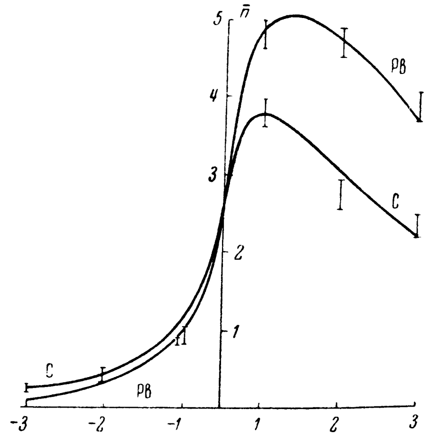

FIG. 5. Average number $\bar{n}$ of fired counters of the hodoscope through various scatterers from the point of supposed formation of the shower. The number of the hodoscope row is read from the vertical-positive downwards and negative upwards. Pb, C – are the curves for the arrangement GK-5B with lead and graphite filters respectively.

$$F_{gl}(X) = \lim_{\theta_m \to \pi/2} \int_0^{\theta_m} f_{\text{в}}\left(\frac{X}{\cos\theta}\right) \cos^n\theta \, d\cos\theta; \tag{6a}$$

$$f_b(X) = (n+1)\,F_{\text{гл}}(X) - X\frac{dF_{\text{гл}}}{dX}\,. \tag{6b}$$

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[15] S. A. Azimov, N. A. Dobrotin and A. L. Liubimov, Izv. Akad. Nauk SSSR, Ser Fiz. 17, 80 (1953).

For slow mesons we assume $n = 3$ (see 16), whereupon we can calculate the production with the help of the relation (6a) for the function $f_b(X)$, corresponding to the meson spectrum of Camerini[12], and show that the distribution from complete tracks and from the vertical components of the tracks differ very little in this case (considerably less than for isotropic radiation).

The other type of correction to the meson spectrum is connected with the calculation of the nuclear interactions with matter. All nuclear interactions, in which the meson is absorbed, not reaching the end of its ionization range, are taken into account by the simple method of multiplying the ordinate of the range spectrum with an exponential factor of the form $\exp(X/\lambda_{abs})$, where λ_{abs} is the mean range for nuclear absorption. A calculation of the nuclear scattering of π-mesons, inelastic as well as from elastic (at large angles), appears to be a considerably more complicated problem; however, over our range of energies, the effective scattering cross section is not large, and therefore to a first approximation one need take into account only pure absorption. Thus, for example, on the basis of a series of a large number of investigations, using various methods (see, for example, Refs. 17-20), it is proved that for energies up to 150 mev, the cross section for nuclear absorption of π^+-mesons constitutes 60-80% of the total (equal to the geometrical) cross section, and the cross section for processes which are accompanied by a large change of energy and angle does not exceed 15% of the geometrical cross section, although with increasing energy this cross section increases markedly.

CONCLUSIONS

We presented above an analysis of the hodoscope method with delayed coincidences and used this method to study the processes of generating slow mesons, which permits one to settle the following three questions:

1. We determined the range of the energy spectrum of nuclear-active particles, which gives the main contribution to the generation of slow mesons (under the conditions of our experiments this energy is on the average 2-3 bev).
2. We introduced all of the necessary instrumental corrections for determining the intensity of generating slow mesons from various nuclei (in above specified range of energies).
3. We introduced the instrumental corrections necessary to determine the energy spectrum of slow mesons from the point of generation.

The experimental results, which were obtained by us by use of the method considered above, were described in the second part of this paper.

In conclusion the author would like to thank for their great help L. N. Korablev, V. F. Tulinov, A. A. Abdullaev and A. A. Khidarov for adjusting the apparatus and for the time spent in making measurements.

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[17] G. Bernardini and F. Levy, Phys. Rev. 84, 610 (1951).

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[20] R. L. Martin, Phys. Rev. 87, 1052 (1952).

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