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Scattering of High-Energy Electrons and Positrons by Electrons

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The scattering of charged particles is considered in the energy region $V(\pi)/e \ll \ln(E/m) \ll \pi/e^2$. It is shown that the cross section for electron-electron scattering with arbitrary radiation dees not differ from that given by zero-order approximation. For the positron-electron scattering the cross section is changed in the region of small angles. This change is determined by solving an integral equation.

IN two previous papers[1,2] we have discussed the scattering of an electron by an external field and the Compton scattering at energies in the region $\sqrt{\pi}/e \ll \ln(E/m) \ll \pi/e^2$. This revealed the following situation. The alteration in the cross section for the scattering by the external field and for Compton scattering is such that the reduction in the cross section for the basic process, without the emission of additional quanta, is compensated by an increase in the cross section for processes with multiple emission of additional hard quanta. The total cross section for processes with arbitrary emission is still given by the zero-order approximation. In the case of the Compton effect[2] however, it was discovered that, in addition to the effect just described, there is a change in the angular distribution for small angles, which is not compensated by the inclusion of processes with additional radiation. The angular region in which this happens is so small that the change does not affect the total cross section; however, the forward scattering may change appreciably.

In the present paper we examine the scattering of charged particles by each other. We shall not be interested in the amount of radiation emitted in the course of the scattering, i.e., we shall determine in practice the sum of the cross sections for the processes involving the emission of one, two, &c, real quanta. As we know from the previous papers[1,2], such emission processes are compensated in a generalized graph by integrals over the momenta of the virtual quanta, taken over the region $k^2 = 0$, Here we consider only the possible change in the cross section not relating to any real emission.

1. BASIC DIAGRAMS

The basic diagrams for single electron-electron scattering in zero approximation are shown in Fig.

[1] A. A. Abrikosov, J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 96, (1956); Soviet Phys JETP 3, 71,(1956).

[2] A. A. Abrikosov, J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 386 (1956);

1. Consider first the question of virtual quanta with $k^2 = 0$. We shall show that these are compensated by the effect of corresponding real quanta. For this purpose we consider one of the generalized diagrams for the process in question, e.g., the one shown in Fig. 2, *a*. In Fig. 2, *b* we show the different types of virtual lines, and the real ones which cancel them. Each type is labeled by a number, and the corresponding real line by the same number with a prime. The proof of this cancellation to order e^2, i.e., for only one line, is extremely simple. For the general case the proof does not differ materially from that given in Ref. 2.

There is only one point worth noting. The electron-electron scattering differs from the Compton effect in particular by the presence of two fermions, and this can, in general, affect the calculation of the spinor numerators. (An example of this will occur below). However, here this fact makes no difference at all. Indeed, consider, for example, line 2 in Fig. 2, *b*. Its spinor numerator (multiplied on the left by $\hat{p}_1\hat{p}_2$ and on the right by $\hat{q}_1\hat{q}_2$, where $\hat{p}_1$ and $\hat{q}_1$ relate to the first, and $\hat{p}_2$ and $\hat{q}_2$ to the second electron, i.e., $\hat{p}_1$, $\hat{q}_1$ commute with $\hat{p}_2$, $\hat{q}_2$) is of the form

$$(\hat{p}_1\gamma_\mu(\hat{p}_1 - \hat{k})\gamma_\nu\hat{q}_1)(\hat{p}_2\gamma_\mu(\hat{p}_2 + \hat{k})\gamma_\nu\hat{q}_2)$$
$$\approx (\hat{p}_1\gamma_\mu\hat{p}_1\gamma_\nu\hat{q}_1)(\hat{p}_2\gamma_\mu\hat{p}_2\gamma_\nu\hat{q}_2)$$
$$\approx 2(p_1p_2)(\hat{p}_1\gamma_\nu\hat{q}_1)(\hat{p}_2\gamma_\nu\hat{q}_2),$$

i.e., of the same form as in zero-order approximation. The same can be shown for any number of virtual lines with $k^2 = 0$.

Now consider the possibility of a change in the cross section which is not compensated by the emission of real quanta. From Ref. 2 it is evident that such a contribution can come only from lines of type 2 or 3 (Fig. 2, *b*). Assume there is one virtual line of type 2. In that case we have in the denominator of the matrix element the expression

$(p_1-k)^2(p_2+k)^2k^2(l-k)^2$. In the numerator, on the other hand, we have the spinors $\hat{p}_1-\hat{k}$ and $\hat{p}_2+\hat{k}$. By comparison with Ref. 2 we conclude that the appearance of the terms for which we are looking requires that $|(p_1, q_1)| >> |(p_1, p_2)|$. It is easy to see that this is impossible. Indeed, consider the scattering in the centre-of-mass system (Fig. 3). We see that

$$(p_1p_2)=E^2+\mathbf{p}^2\approx 2E^2,$$

$$(p_1q_1)=E^2-\mathbf{p}^2\cos\theta\leqslant E^2+\mathbf{p}^2.$$

Now assume there is one line of type 3. Then we require that $|(p_1q_1)|\gg|(p_1q_2)|$. This may happen, since in the center-of-mass system $(p_1q_2) = E^2-\mathbf{p}^2\cos\theta'$, which may be much less than $p_1\cdot q_1$, if the angle θ' is small enough, i.e., if in the center-of-mass system the scattering is backward, Nevertheless this effect is not observable. Indeed, besides the diagram 1, a which we have considered, there is also a contribution from diagram 1, b. In view of the fact that the contribution from any diagram is essentially of the order l^{-2}, where l is the momentum of the photon which is transmitted from one electron to the other, the diagram 1, b gives the dominant contribution when $|(p_1q_2)|\ll|(p_1q_1)|$, and this diagram does not give the term we require. It would do so in the opposite case, but then the diagram 1, a is dominant. We conclude, therefore, that owing to the possibility of electron exchange the expression for the electron-electron cross section is given correctly by zero-order approximation in the approximation considered here.

The situation is different in the case of positron-electron scattering. There there is no exchange, because the two particles are distinguishable, and the effect we are looking for may take place.

The basic diagrams for this process are shown in Fig. 4. In this case, as in the previous one, a term of the required kind will appear in Fig. 4 from a virtual line of type 3 (Fig. 2, b) if $p_1\cdot q_1 >> p_1\cdot q_2$. But, contrary to the previous situation, diagram 4, b cannot mask this effect, since, according to Fig. 3, $\hat{p}_1$, $\hat{p}_2$ is the largest of all the scalar products of momenta. Therefore diagram 4, b will, first, be of the same order as 4, a, and second, will itself give a contribution of the required kind.

The conservation of four-momentum requires that

$$p_1-q_1=q_2-p_2. \tag{1}$$

Hence, we may obtain the following equality for the scalar products (remembering that the squares of all vectors p and q are negligible)

$$(p_1q_1)=(p_2q_2)=(p_2p_1)-(p_2q_1) = (p_2p_1)-(p_1q_2). \tag{2}$$

It follows from this that if $(p_1q_2)\ll(p_1q_1)$ $(p_1q_1)\approx(p_1p_2)$, i.e., $l^2\approx -l'^2$.

It should be pointed out here that in the diagram 4, b it is possible, in principle, to find a contribution of the required type from a line of type 2, provided $p_1\cdot q_1 << p_1\cdot q_2$. However, in that case diagram 4, a will dominate, and this does not give any such contribution. Thus an interesting contribution can come only from virtual lines of type 3. With the help of this result we shall now determine the matrix element for positron-electron scattering.

2. THE MATRIX ELEMENT FOR POSITRON-ELECTRON SCATTERING

In analogy with Ref. 2, we can determine the matrix element by setting up an equation which gives the sum of all diagrams shown in Fig. 5. Whereas in the previous case[2] the equation could immediately be replaced by a direct summation, here it gives much the simplest way of finding the matrix element. This is connected with the fact that we are now dealing with two electrons. Consider first the diagrams derived from 4,a. Taking account of the results of the preceding section, we find the equation for the matrix element in the form

$$M^{ab,cd}(p_1, p_2; q_1, q_2; l)=\gamma_\mu^{ab}\gamma_\mu^{cd}l^{-2} + \frac{e^2}{\pi i}\int\left[\gamma_\mu\frac{1}{\hat{p}_1-\hat{k}-m}\right]^{ax}M^{xb,yd}(p_1-k, p_2; q_2, q_1-k; l-k)\left[\gamma_\mu\frac{1}{\hat{k}-\hat{q}_2-m}\right]^{cy}\frac{d^4k}{k^2}. \tag{3}$$

In this formula a and b refer to the initial and final states of the electron, and d, c , the initial and final states of the positron. (For the latter we take the four-momentum with the opposite sign, and the lines have to be followed in the reverse direction).

Since the behavior of the matrix element M is in zero order given essentially by the factor l^{-2}, we assume that $M=M'l^{-2}$, where M' is some slowly

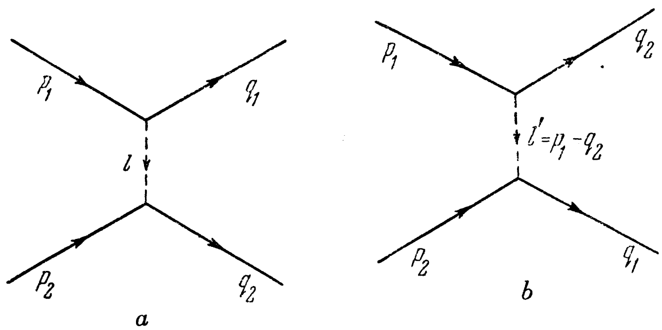

FIG. 1

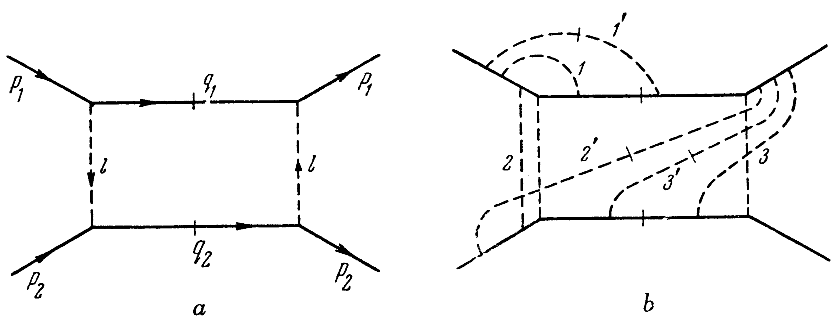

FIG. 2

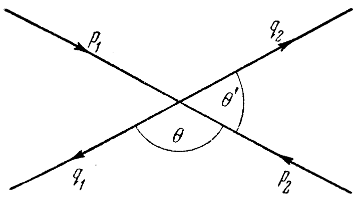

FIG. 3

varying spinor. In addition we carry out a transformation similar to the one used for the corresponding lines in reference[2]. For this purpose we replace $\hat{p}_1 - \hat{k}$ by $\hat{k}$, average the quantities $\hat{k}_{ktrans} \ldots \hat{k}_{ktrans}$ in the numerator over the polarizations, and cancel $\hat{k}^2_{ktrans}$ in the numerator against the same factor in the denominator, remembering the region $l^2 uv \gg (p_1 - q_2)^2$, in which this is permissible. In the end, the equation takes the form

$$M'^{ab,\,cd} = \gamma^{ab}_{\mu}\gamma^{cd}_{\mu} + \frac{e^2}{2\pi i}[\gamma_\mu\gamma_\nu]^{ax}\int \frac{M'^{xb,\,yd}(k,\,p_2;\,q_1,q_2 - p_1 + k + q_2;\,-q_1 + k)\,d^4k}{(k^2 - m^2)(p_1 - k)^2(q_1 - k)^2}[\gamma_\mu\gamma_\nu]^{cy}. \quad (4)$$

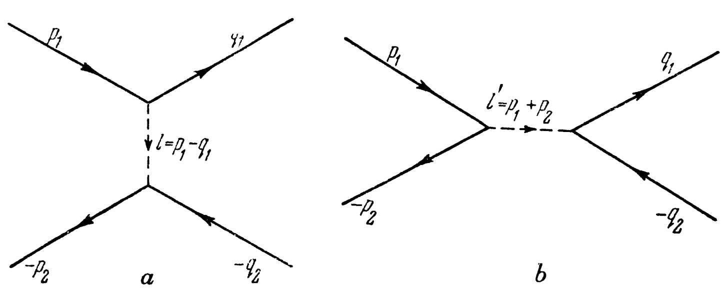

FIG. 4

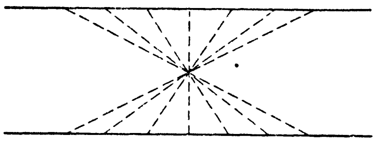

FIG. 5

From this equation it is evident that M' must be a scalar product of two tensors, of which one relates to the electron and the other to the positron. These tensors are products of the γ_μ; their rank therefore cannot exceed four. This means that any tensor of rank higher than four can be expressed in terms of tensors of lower rank, Since the only tensor which can in this case dispose of surplus subscripts is $\delta_{\mu\nu}$, the reduction in rank must always go in even steps. Examination of Eq. (4) shows then that M' may contain scalar products of tensors of first and third rank. Indeed, the tensors of fifth rank in the term with the integral can be expressed in terms of tensors of first and third rank.

We therefore write

$$M'^{ab,\,cd} = \gamma^{ab}_\mu \cdot \gamma^{cd}_\mu f_1 + T^{ab}_{\mu\tau\nu} \cdot T^{cd}_{\mu\tau\nu} f_3, \qquad (5)$$

where f_1 and f_3 are scalar functions and $T_{\mu\tau\nu}$ an antisymmetric tensor of rank 3, formed from the γ_μ:

$$T_{\mu\tau\nu} = {}^1/_6\,(\gamma_\mu\gamma_\tau\gamma_\nu - \gamma_\mu\gamma_\nu\gamma_\tau - \gamma_\tau\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu\gamma_\tau + \gamma_\tau\gamma_\nu\gamma_\mu - \gamma_\nu\gamma_\tau\gamma_\mu). \qquad (6)$$

To find f_1 and f_3 we must, first of all, express the tensors $\gamma_\mu\gamma_\nu\gamma_\tau$ and $\gamma_\xi\gamma_\eta T_{\mu\tau\nu}$ which arise from the integral term in Eq. (4), in terms of γ_μ and $T_{\mu\tau\nu}$. Bringing γ_μ to the left in each term of the expression (6), we see that

$$\gamma_\mu\gamma_\tau\gamma_\nu = T_{\mu\tau\nu} + \gamma_\nu\delta_{\mu\tau} + \gamma_\mu\delta_{\nu\tau} - \gamma_\tau\delta_{\mu\nu}. \qquad (7)$$

As regards the second combination, it is easy to see that, the most general form it can have is

$$\gamma_\xi\gamma_\eta T_{\mu\tau\nu} = A\,(\delta_{\eta\mu}T_{\xi\tau\nu} - \gamma_{\eta\tau}T_{\xi\mu\nu} + \delta_{\eta\nu}T_{\xi\mu\tau} - \delta_{\xi\mu}T_{\eta\tau\nu} + \delta_{\xi\tau}T_{\eta\mu\nu} - \delta_{\xi\nu}T_{\eta\mu\tau}) \times B\delta_{\xi\eta}T_{\mu\tau\nu} + C\,[(\delta_{\xi\mu}\delta_{\eta\tau} - \delta_{\xi\tau}\delta_{\eta\mu})\gamma_\nu + (\delta_{\xi\nu}\delta_{\eta\mu} - \delta_{\xi\mu}\delta_{\eta\nu})\gamma_\tau + (\delta_{\xi\tau}\delta_{\eta\nu} - \delta_{\xi\nu}\delta_{\eta\tau})\gamma_\mu]. \qquad (8)$$

To determine the coefficient B interchange γ_ξ and γ_η on the left-hand side. This leaves

$$\gamma_\eta\gamma_\xi T_{\mu\tau\nu} = -\gamma_\xi\gamma_\eta T_{\mu\tau\nu} + 2\delta_{\xi\eta}T_{\mu\tau\nu}.$$

On the right -hand side the terms with A and C then change sign, whereas the term with B remains unchanged. Hence $B = 1$. To find A and C we take the special case $\xi = \mu$. and use Eq. (7) on the left-hand side. Displacing γ_μ and equating the coefficients of $T_{\eta\tau\nu}$ and of the combination $2(\gamma_\tau\delta_{\nu\eta} - \gamma_\nu\delta_{\tau\eta})$, we conclude that $A = 1$, $C = -1$. The next step is to multiply the expressions which we have obtained. With the help of (7) we see that

$$(\gamma_\mu\gamma_\tau\gamma_\nu)^{ab}\,(\gamma_\mu\gamma_\tau\gamma_\nu)^{cd} = T^{ab}_{\mu\tau\nu}T^{cd}_{\mu\tau\nu} + 10\gamma^{ab}_\mu\gamma^{cd}_\mu. \qquad (9)'$$

Using (8) and the values of the coefficients A, B, and C, we find

$$(\gamma_\xi\gamma_\eta T_{\mu\tau\nu})^{ab}\,(\gamma_\xi\gamma_\eta T_{\mu\tau\nu})^{cd} = 10T^{ab}_{\mu\tau\nu}T^{cd}_{\mu\tau\nu} + 36\gamma^{ab}_\mu\gamma^{cd}_\mu. \qquad (10)$$

We now substitute this result, as well as (5), in Eq. (4). If we assume, as in Ref. 2, that f_1 and f_3 depend only on $a = \ln[l^2/(p_1 - q_2)^2]$, and remembering the region of integration, we obtain the following equations:

$$f_1(a^2) = 1 + \frac{10e^2}{16\pi}\int_0^{a^2} f_1(x)\,dx + \frac{36e^2}{16\pi}\int_0^{a^2} f_3(x)\,dx, \qquad (11)$$

$$f_3(a^2) = \frac{e^2}{16\pi}\int_0^{a^2} f_1(x)\,dx + \frac{10e^2}{16\pi}\int_0^{a^2} f_3(x)\,dx.$$

If we now differentiate with respect to a^2, we obtain two differential equations of first order. The boundary condtions are, according to (11), $f_1(0) = 1$, $f_3(0) = 0$. The solution of the pair of equations with this boundary condition gives f_1 and f_3: (12)

$$f_1 = \frac{1}{2}\left[\exp\left(\frac{e^2}{4\pi}\ln^2\frac{l^2}{p^2}\right) + \exp\left(\frac{e^2}{\pi}\ln^2\frac{l^2}{p^2}\right)\right],$$

$$f_3 = \frac{1}{12}\left[-\exp\left(\frac{e^2}{4\pi}\ln^2\frac{l^2}{p^2}\right) + \exp\left(\frac{e^2}{\pi}\ln^2\frac{l^2}{p^2}\right)\right],$$

where $p = p_1 - q_2$.

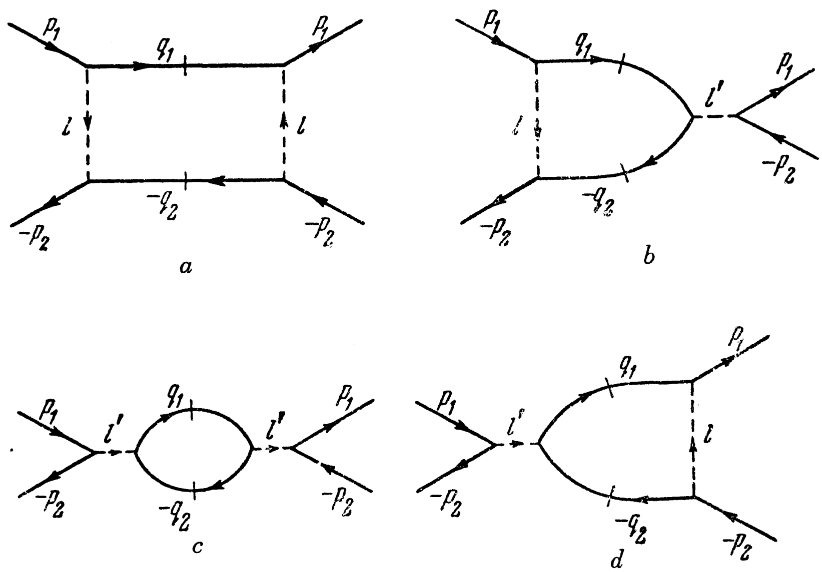

FIG. 6

If we insert these values for f_1 and f_3 in (5) we have the required expression for the sum of all graphs of the type 4, a.

Now we would have to find an analogous expression for the diagrams of type 4,b. However, this is unnecessary since the diagrams 4, b give exactly the same as 4, a. The minus sign in the denominator which comes from $l'^2 = -l^2$ is cancelled by the fact that the contribution of the diagram 4, b has to be taken with the opposite sign. The only difference is that, in the diagram 4, b the states ab, cd, are replaced by the states ad, cb.

3. THE CROSS SECTION

We now turn to the calculation of the cross section. The combined diagrams for this (giving as examples only the zero-order ones) are shown in Fig. 6. If we are not interested in the initial or final spin states of electron or positron, then the expression corresponding to the diagram 6, a is, apart from small terms (we omit factors which are common to all diagrams):

$$\begin{aligned} &{}^1/_4 f_1^2 \,\mathrm{Sp}\,[\hat{p}_1\gamma_\mu\hat{q}_1\gamma_\nu]\,\mathrm{Sp}\,[(-\hat{p}_2)\gamma_\nu(-\hat{q}_2)\gamma_\mu] \\ &+ {}^1/_4 f_1 f_3 \,\mathrm{Sp}\,[\hat{p}_1 T_{\mu\tau\nu}\hat{q}_1\gamma_\xi]\,\mathrm{Sp}\,[(-\hat{p}_2)\gamma_\xi(-\hat{q}_2)T_{\mu\tau\nu}] \\ &+ {}^1/_4 f_1 f_3 \,\mathrm{Sp}\,[\hat{p}_1\gamma_\xi\hat{q}_1 T_{\mu\tau\nu}]\,\mathrm{Sp}\,[(-\hat{p}_2)T_{\mu\tau\nu}(-\hat{q}_2)\gamma_\xi] \\ &+ {}^1/_4 f_3^2 \,\mathrm{Sp}\,[\hat{p}_1 T_{\mu\tau\nu}\hat{q}_1 T_{\xi\eta\lambda}]\,\mathrm{Sp}\,[(-\hat{p}_2)T_{\xi\eta\lambda}(-\hat{q}_2)T_{\mu\tau\nu}]. \end{aligned} \tag{13}$$

To evaluate this expression we first find the basic spurs. One easily finds directly:

$$\begin{aligned} {}^1/_4\mathrm{Sp}\,[\hat{p}_1\gamma_\mu\hat{q}_1\gamma_\nu] = p_{1\mu}q_{1\nu} \\ + p_{1\nu}q_{1\mu} - (p_1q_1)\,\delta_{\mu\nu}. \end{aligned} \tag{14}$$

The next spur is $\mathrm{Sp}\,[\hat{p}_1 T_{\mu\tau\nu}\hat{q}_1\gamma_\xi]$. By means of equation (7), we can write this expression in the form $\mathrm{Sp}\,[T_{\mu\tau\nu}T_{\eta\xi\lambda}]\,p_{1\lambda}q_{1\eta}$. This can be explained by the fact that $\mathrm{Sp}\,[T_{\mu\tau\nu}\gamma_\xi] = 0$, since it is impossible to represent this expression in any other way in terms of δ-functions. If we make use of the symmetry properties of the expression in question, we can write it as a corresponding sum of tensors, which contain $p_{1\mu}$, $q_{1\mu}$ and $\delta_{\mu\nu}$ with unknown coefficients. Combining terms on both sides, and comparing coefficients, we find

$$\begin{aligned} {}^1/_4\mathrm{Sp}\,[\hat{p}_1 T_{\mu\tau\nu}\hat{q}_1\gamma_\xi] = (p_{1\nu}q_{1\tau} - p_{1\tau}q_{1\nu})\,\delta_{\mu\xi} \\ + (p_{1\mu}q_{1\nu} - p_{1\nu}q_{1\mu})\,\delta_{\tau\xi} + (p_{1\tau}q_{1\mu} - p_{1\mu}q_{1\tau})\,\delta_{\nu\xi}. \end{aligned} \tag{15}$$

The last spur, $\mathrm{Sp}\,[\hat{p}_1 T_{\mu\tau\nu}\hat{q}_1 T_{\xi\eta\lambda}]$ is dealt with in the same way. As a result of this calculation we find

$$\tfrac{1}{4}\mathrm{Sp}\,[\hat{p}_1 T_{\mu\tau\nu}\hat{q}_1 T_{\xi\eta\lambda}] = (p_1q_1)(\delta_{\mu\xi}\delta_{\tau\eta}\delta_{\nu\lambda} - \delta_{\tau\xi}\delta_{\mu\eta}\delta_{\nu\lambda} - \delta_{\mu\xi}\delta_{\tau\lambda}\delta_{\nu\eta} + \delta_{\tau\xi}\delta_{\nu\eta}\delta_{\mu\lambda} + \delta_{\nu\xi}\delta_{\tau\lambda}\delta_{\mu\eta} - \delta_{\nu\xi}\delta_{\mu\lambda}\delta_{\tau\eta}) + (p_{1\mu}q_{1\xi} + p_{1\xi}q_{1\mu})(\delta_{\nu\eta}\delta_{\tau\lambda} - \delta_{\nu\lambda}\delta_{\tau\eta}) + (p_{1\nu}q_{1\xi} + p_{1\xi}q_{1\nu})(\delta_{\tau\eta}\delta_{\mu\lambda} - \delta_{\mu\eta}\delta_{\tau\lambda}) + (p_{1\tau}q_{1\xi} + p_{1\xi}q_{1\tau})(\delta_{\nu\lambda}\delta_{\mu\eta} - \delta_{\nu\eta}\delta_{\mu\lambda}) + (p_{1\mu}q_{1\eta} + p_{1\eta}q_{1\mu})(\delta_{\nu\lambda}\delta_{\tau\xi} - \delta_{\nu\xi}\delta_{\tau\lambda}) + (p_{1\nu}q_{1\eta} + p_{1\eta}q_{1\nu})(\delta_{\mu\xi}\delta_{\tau\lambda} - \delta_{\tau\xi}\delta_{\mu\lambda}) + (p_{1\tau}q_{1\eta} + p_{1\eta}q_{1\tau})(\delta_{\nu\xi}\delta_{\mu\lambda} - \delta_{\nu\lambda}\delta_{\mu\xi}) + (p_{1\mu}q_{1\lambda} + p_{1\lambda}q_{1\mu})(\delta_{\nu\xi}\delta_{\tau\eta} - \delta_{\nu\eta}\delta_{\tau\xi}) + (p_{1\nu}q_{1\lambda} + p_{1\lambda}q_{1\nu})(\delta_{\tau\xi}\delta_{\mu\eta} - \delta_{\mu\xi}\delta_{\tau\eta}) + (p_{1\tau}q_{1\lambda} + p_{1\lambda}q_{1\tau})(\delta_{\nu\eta}\delta_{\mu\xi} - \delta_{\nu\xi}\delta_{\mu\eta}). \quad (16)$$

Now substitute (14) to (16) in (13). After some simple calculation we obtain the expression

$$8\,(f_1^2 - 12f_1f_3 + 36f_3^2)\,(p_1q_1)^2.$$

Here we have used the fact that $p_1q_1 \approx p_2q_2 \approx p_1p_2 \gg p_1q_2$, so that we may replace p_2 by q_1 and q_2 by p_1.

Inserting now from (12), we have

$$8\exp\left(\frac{e^2}{2\pi}\ln^2\frac{l^2}{p^2}\right)(p_1q_1)^2. \quad (17)$$

We now proceed to consider the other diagrams of Fig. 6. Although the quantity l'^2, which enters into the denominator of part of the matrix element, equals $-l^2$, there is a minus sign in front of this part. We may therefore regard the matrices in all cases as differing only in the order of factors, and in the way the spurs are taken, but not in their signs or their denominators. Consider diagram 6, *b*. It is easy to see that this differs from 6, *a* only by an interchange between q_1 and $-p_2$, so that it also contributes the expression (17). On the other hand, the diagrams 6, *c* and *d* give only small contributions, as one easily verifies directly. E. g.

$$\tfrac{1}{4}\mathrm{Sp}\,[\hat{p}_1\gamma_\mu\hat{p}_2\gamma_\nu\hat{q}_2\gamma_\mu\hat{q}_1\gamma_\nu] = -\tfrac{1}{2}\,[\hat{p}_1\hat{q}_2\gamma_\nu\hat{p}_2\hat{q}_1\gamma_\nu] = -8\,(p_1q_2)(p_2q_1).$$

We now obtain the expression for the angular distribution at small angles. It is easy to see that all deviations from the well-known results are contained in the exponential factor. In the centre-of-mass system we have

$$d\sigma = \frac{e^4 d\Omega}{4E^2}\exp\left[\frac{2e^2}{\pi}\ln^2\frac{2}{(\pi-\theta)}\right] \quad (19)$$

if $m/E \ll \pi - \theta \ll \pi/2$,

and

$$d\sigma = \frac{e^4 d\Omega}{4E^2}\exp\left[\frac{2e^2}{\pi}\ln^2\frac{E}{m}\right] \quad (20)$$

if $\pi - \theta \ll m/E$.

In the second case the correction is connected with the fact that, for $(p_1 - q_2)^2 \ll m^2$ occurs in place of $(p_1 q_2)$. It is interesting that these results are very similar to those which were obtained in the investigation of the Compton effect.[2]

In conclusion I wish to express my gratitude to Academician L. L. Landau for discussions about this work.

Translated by R. Peierls
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