Motion of a Charged Particle in an Optically Active Anisotropic Medium, I

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The Fourier method is used to obtain expressions for the components of the electromagnetic field and for the total energy losses of a charged particle moving in an optically-active anisotropic medium.

1. THE passage of a charged particle through an optically-active anisotropic medium is characterized by many features. First, to satisfy the conditions for the Cerenkov radiation the moving charge need have a considerably lower velocity in an anisotropic medium than in an isotropic medium, because the index of refraction of the waves becomes large at certain definite frequencies. Second, if a charge moves in an active anisotropic medium with a uniform velocity greater than the phase velocity of light in the same medium, the light emitted by the charge is more complicated in nature as compared with the isotropic case. Instead of the single circular cone of rays observed for the isotropic body, we have in the active anisotropic case two noncircular cones of rays, with the radiation intensity varying on different generatrices of these conical surfaces.

The electrodynamics of anisotropic media was developed in the investigations of Ginzburg[1] who examined in particular the emission from an electron moving in a uniaxial crystal and from an oscillator placed in such a crystal. The problem of the energy losses of a charged particle moving in an anisotropic medium was subsequently treated in several investigations[2-5]. The work of Ref. 1 was generalized by one of the authors for the case when the medium is optically-active (gyrotropic) in addition to being anisotropic.

This article employs a method different from that used in Ref. 3 to determine the components of the electromagnetic field and the total energy losses of a charged particle moving in an optically-active anisotropic medium. The question of the singularities of the expressions obtained in Refs. 1-5 for the losses is discussed, and a computation method that does not lead to singularities is given.

2. The electromagnetic field produced in a medium in which a point charge q moves at a velocity $\mathbf{v}$ is given by Maxwell's equations:

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}; \qquad (1)$$

$$\operatorname{rot}\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}q\mathbf{v}\,\delta(\mathbf{r}-\mathbf{v}t);$$

$$\operatorname{div}\mathbf{H} = 0; \quad \operatorname{div}\mathbf{D} = 4\pi q\delta(\mathbf{r}-\mathbf{v}t).$$

We shall solve this system using the Fourier method

$$\mathbf{E}(\mathbf{r},t) = \iint \mathbf{E}(\mathbf{k},\omega)\,e^{i\mathbf{k}\mathbf{r}-i\omega t}d\mathbf{k}\,d\omega \qquad (2)$$

etc. Using the connection between the Fourier components of the induction and of the field intensity

$$D_i(\mathbf{k},\omega) = \varepsilon_{ik}(\omega)E_k(\mathbf{k},\omega); \quad (\varepsilon_{ik} = \varepsilon^*_{ki}), \qquad (3)$$

we obtain the following equation for the Fourier components of the electric field intensity $\mathbf{E}(\mathbf{k},\omega)$

$$T_{ik}E_k = -i\frac{q}{2\pi^2}\frac{v_i}{\omega^2}\delta\left(\frac{n}{c}\varkappa_j v_j - 1\right), \qquad (4)$$

where

$$T_{ik} = n^2(\varkappa_i\varkappa_k - \delta_{ik}) + \varepsilon_{ik}; \qquad (5)$$

$$n^2 = k^2c^2/\omega^2; \quad \varkappa_i = k_i/k.$$

Using the inverse tensor T^{-1}_{ik}, we can represent the solution of Eq. (4) in the following form:

[1] V. L. Ginzburg, J. Exptl. Theoret. Phys. (U.S.S.R.) 10, 601, 608 (1940).

[2] A. A. Kolomenskii, Dokl. Akad. Nauk SSSR 86, 1097 (1952).

[3] A. A. Kolomenskii, J. Exptl. Theoret. Phys. (U.S.S.R.) 24, 167 (1953).

[4] M. I. Kaganov, J. Tech. Phys. (U.S.S.R.) 23, 507 (1953).

[5] K. Tanaka, Phys. Rev. 93, 459 (1954).

$$E_k = -i\frac{q}{2\pi^2\omega^2}T_{ki}^{-1}v_i\delta\left(\frac{n}{c}\varkappa_j v_j - 1\right). \qquad (6)$$

To calculate the energy losses of a moving charge we employ the relationship

$$-d\mathcal{E}/dl = -(q/v)(\mathbf{vE})_{\mathbf{r}=\mathbf{v}t}, \qquad (7)$$

where the value of the field is taken at the point where the charge is located. Using (6) and (2) we obtain the following value per unit path for the total energy losses due to the remote collisions:

$$-\frac{d\mathcal{E}}{dl} = i\frac{q^2}{2\pi^2 v} \qquad (8)$$

$$\times \int_{-\infty}^{\infty}\int_{0}^{k_m}\int_{4\pi} T_{ki}^{-1}v_k v_i\delta\left(\frac{n}{c}\varkappa_j v_j - 1\right)\frac{d\omega}{\omega^2}k^2dk\,do.$$

The integration with respect to k must be carried out up to a certain maximum value k_m of the order of magnitude of $1/b$, where b is the minimum parameter of the remote collisions.

3. Let us apply the equation obtained to the motion of a charged particle in an optically-active uniaxial crystal having a dielectric-constant tensor of the following form:

$$\varepsilon_{ik} = \begin{pmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}. \qquad (9)$$

In the case under consideration the tensor T_{ik} is defined by the matrix

$$T_{ik} = \begin{pmatrix} n^2(\varkappa_1^2 - 1) + \varepsilon_1 & n^2\varkappa_1\varkappa_2 - i\varepsilon_2 & n^2\varkappa_1\varkappa_3 \\ n^2\varkappa_1\varkappa_2 + i\varepsilon_2 & n^2(\varkappa_2^2 - 1) + \varepsilon_1 & n^2\varkappa_2\varkappa_3 \\ n^2\varkappa_1\varkappa_3 & n^2\varkappa_2\varkappa_3 & n^2(\varkappa_3^2 - 1) + \varepsilon_3 \end{pmatrix}. \qquad (10)$$

To obtain the components of the inverse tensor it is necessary to divide the minors of the corresponding elements of tensor T_{ik} by the determinant consisting of its components. This determinant equals

$$T = (\varepsilon_1\sin^2\theta + \varepsilon_3\cos^2\theta)\,[n^2 - n_1^2(\theta)]\,[n^2 - n_2^2(\theta)], \qquad (11)$$

where

$$n_{1,2}^2(\theta) = \frac{(\varepsilon_1^2 - \varepsilon_2^2)\sin^2\theta + \varepsilon_1\varepsilon_3(1+\cos^2\theta) \pm \sqrt{(\varepsilon_1^2 - \varepsilon_2^2 - \varepsilon_1\varepsilon_3)^2\sin^4\theta + 4\varepsilon_2^2\varepsilon_3^2\cos^2\theta}}{2(\varepsilon_1\sin^2\theta + \varepsilon_3\cos^2\theta)} \qquad (12)$$

are the ordinary and extraordinary index of refraction and θ is the angle between the optical axis of the crystal and the propagation direction **k** of the wave.

Equation (8) is a general expression for the energy losses of a moving particle. Let us apply this equation to the two simplest motions of a particle, along and perpendicular to the optical axis of a crystal.

4. In the case of a particle moving along the optical axis, we orient the coordinates as in Fig. 1. In this case Eq. (8) takes the following form:

$$-\frac{d\mathcal{E}}{dz} = i\frac{q^2 v}{\pi c^3}\int_{-\infty}^{\infty}\int_{0}^{n_m}\int_{0}^{\pi}\frac{n^4\cos^2\theta - n^2\varepsilon_1(1+\cos^2\theta) + \varepsilon_1^2 - \varepsilon_2^2}{(\varepsilon_1\sin^2\theta + \varepsilon_3\cos^2\theta)\,[n^2 - n_1^2(\theta)]\,[n^2 - n_2^2(\theta)]} \qquad (13)$$

$$\times\,\delta(n\beta\cos\theta - 1)\sin\theta\,d\theta n^2 dn\omega d\omega,$$

where k is replaced by a new variable $n = kc/\omega$.

The delta function yields the integral with respect to the angle variable, while the integration with respect to n must be restricted to a region from $1/\beta$ to $n_m = k_m c/\omega$. The integration yields

$$-\frac{d\mathcal{E}}{dz} = \frac{q^2}{\pi c^2}\,\mathrm{Re}\, i\int_0^\infty \frac{(1-\varepsilon_1\beta^2)(n_1^2-\varepsilon_1)-\beta^2\varepsilon_2^2}{\varepsilon_1\beta^2(n_1^2-n_2^2)} \ln\frac{n_m^2\beta^2-n_1^2\beta^2}{1-n_1^2\beta^2}\,\omega\, d\omega$$
$$+\frac{q^2}{\pi c^2}\,\mathrm{Re}\, i\int_0^\infty \frac{(1-\varepsilon_1\beta^2)(n_2^2-\varepsilon_1)-\beta^2\varepsilon_2^2}{\varepsilon_1\beta^2(n_2^2-n_1^2)} \ln\frac{n_m^2\beta^2-n_2^2\beta^2}{1-n_2^2\beta^2}\,\omega\, d\omega, \quad (14)$$

where

$$n_{1,2}^2 = \{(\varepsilon_1^2-\varepsilon_2^2+\varepsilon_1\varepsilon_3)\beta^2-(\varepsilon_3-\varepsilon_1) \pm [(\varepsilon_1^2-\varepsilon_2^2-\varepsilon_1\varepsilon_3)^2\beta^4$$
$$-2\varepsilon_1(\varepsilon_1^2-\varepsilon_2^2+\varepsilon_3^2)\beta^2+2\varepsilon_3(2\varepsilon_1^2+\varepsilon_2^2)\beta^2+(\varepsilon_3-\varepsilon_1)^2]^{1/2}\}/2\varepsilon_1\beta^2, \quad (15)$$

are the values of the indices of refraction in the direction of the emission maxima, determined by the following equations:

$$\cos^2\theta_{1,2} = 1/\beta^2 n_{1,2}^2(\theta_{1,2}).$$

It is evident that the energy losses of the moving charge will be caused by the frequency regions in which the arguments of the logarithms become negative, and also by frequencies at which the integrands have poles, for it is only in these cases that the real part of (14) differs from zero. We thus have

$$-\frac{d\mathcal{E}}{dz} = -\frac{q^2}{c^2}\int \frac{(1-\varepsilon_1\beta^2)(n_1^2-\varepsilon_1)-\beta^2\varepsilon_2^2}{\varepsilon_1\beta^2(n_1^2-n_2^2)}\,\omega d\omega - \frac{q^2}{c^2}\int \frac{(1-\varepsilon_1\beta^2)(n_2^2-\varepsilon_1)-\beta^2\varepsilon_2^2}{\varepsilon_1\beta^2(n_2^2-n_1^2)}\,\omega\, d\omega$$
$$+\frac{q^2}{v^2}\sum_i \frac{\omega_i}{|d\varepsilon_1/d\omega|_i}\ln\left\{1+\frac{k_m^2 v^2}{\omega_1^2}\frac{\varepsilon_1}{\varepsilon_3}\right\},$$

where the integration is carried out in the first two terms over the frequency regions defined respectively by the following inequalities

$$n_m^2\beta^2 > n_1^2\beta^2 > 1; \quad n_m^2\beta^2 > n_2^2\beta^2 > 1, \quad (16)$$

and in the third term the summation is over those frequencies ω_i, at which ϵ_1, ϵ_2, and ϵ_3 vanish simultaneously. Substituting the explicit expressions for n_1^2 and n_2^2 we obtain finally

$$-\frac{d\mathcal{E}}{dz} = \frac{q^2}{2c^2}\int\left(1-\frac{1}{\beta^2\varepsilon_1}\right)$$
$$\times\left\{1 \pm \frac{\varepsilon_1(\varepsilon_1^2-\varepsilon_2^2-\varepsilon_1\varepsilon_3)\beta^4-(2\varepsilon_1^2-2\varepsilon_1\varepsilon_3+\varepsilon_2^2)\beta^2-\varepsilon_3+\varepsilon_1}{(1-\varepsilon_1\beta^2)[(\varepsilon_1^2-\varepsilon_2^2-\varepsilon_1\varepsilon_3)^2\beta^4-2\varepsilon_1(\varepsilon_1^2-\varepsilon_2^2+\varepsilon_3^2)\beta^2+2\varepsilon_3(2\varepsilon_1^2+\varepsilon_2^2)\beta^2+(\varepsilon_3-\varepsilon_1)^2]^{1/2}}\right\}\omega d\omega$$
$$+\frac{q^2}{v^2}\sum_i \frac{\omega_i}{|d\varepsilon_1/d\omega|_i}\ln\left\{1+\frac{k_m^2 v^2}{\omega_i^2}\left(\frac{\varepsilon_1}{\varepsilon_3}\right)_i\right\} \quad (17)$$

(integration over the regions defined by Eq. 16).

This expression determines the total energy losses of the particle, including both the polarization and the Cerenkov losses. Let us note that Eq. (4.5) of Ref. 3 leads to the same expression for the integral of (17), which determines the Cerenkov losses.

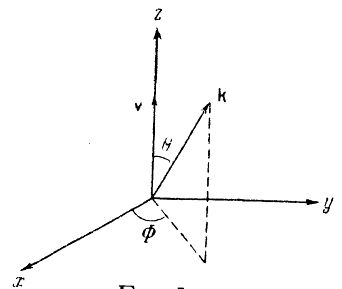

FIG. 1

5. To clarify the character of the losses given by (17), let us compute the energy flux through a cylindrical surface surrounding the trajectroy of the charge. For this purpose we shall first determine the field produced in an optically-active anisotropic medium by the motion of a point charge. Inserting Eq. (6) into Eq. (2), and using the known relationships

$$\int_0^{2\pi} e^{i\varkappa r\cos\vartheta}\,d\vartheta = 2\pi J_0(\varkappa r);$$

$$\int_0^{\infty}\frac{J_0(\varkappa r)\,\varkappa d\varkappa}{\varkappa^2+k^2} = K_0(kr);\quad \mathrm{Re}\,k>0,$$

we obtain the components of the electric field intensity in cylindrical coordinates as follows:

$$E_z(\mathbf{r},t) = \frac{-iq}{\pi v^2}\int_{-\infty}^{\infty}\left\{\frac{(1-\varepsilon_1\beta^2)(n_1^2-\varepsilon_1)-\beta^2\varepsilon_2^2}{n_1^2-n_2^2}K_0(rk_1) + \frac{(1-\varepsilon_1\beta^2)(n_2^2-\varepsilon_1)-\beta^2\varepsilon_2^2}{n_2^2-n_1^2}K_0(rk_2)\right\}e^{i\omega(z/v-t)}\frac{\omega d\omega}{\varepsilon_1}; \tag{18}$$

$$E_r(\mathbf{r},t) = \frac{q}{\pi v}\int_{-\infty}^{\infty}\frac{1}{\varepsilon_1}\left\{\frac{n_1^2-\varepsilon_1}{n_1^2-n_2^2}k_1K_1(rk_1) + \frac{n_2^2-\varepsilon_1}{n_2^2-n_1^2}k_2K_1(rk_2)\right\}e^{i\omega(z/v-t)}d\omega;$$

$$E_\varphi(\mathbf{r},t) = \frac{iq}{\pi v}\int_{-\infty}^{\infty}\frac{\varepsilon_2}{\varepsilon_1(n_1^2-n_2^2)}\{k_1K_1(rk_1)-k_2K_1(rk_2)\}\,e^{i\omega(z/v-t)}d\omega,$$

where $k_{1,2}^2 = (\omega/v)^2(1-\beta^2 n_{1,2}^2)$, and $n_{1,2}^2$ are given by (15).

For the magnetic field intensity we obtain analogously:

$$H_z(\mathbf{r},t) = -\frac{q}{\pi\beta v^2}\int_{-\infty}^{\infty}\frac{\varepsilon_2}{\varepsilon_1}\left\{\frac{1-\beta^2n_1^2}{n_1^2-n_2^2}K_0(rk_1) + \frac{1-\beta^2n_2^2}{n_2^2-n_1^2}K_0(rk_2)\right\}e^{i\omega(z/v-t)}\omega d\omega;$$

$$H_r(\mathbf{r},t) = -\frac{iqc}{\pi v^2}\int_{-\infty}^{\infty}\frac{\varepsilon_2}{\varepsilon_1(n_1^2-n_2^2)}\{k_1K(rk_1)-k_2K_1(rk_2)\}e^{i\omega(z/v-t)}d\omega; \tag{19}$$

$$H_\varphi(\mathbf{r},t) = \frac{q}{\pi c}\int_{-\infty}^{\infty}\frac{1}{\varepsilon_1}\left\{\frac{\varepsilon_1n_1^2-\varepsilon_1^2+\varepsilon_2^2}{n_1^2-n_2^2}k_1K_1(rk_1) + \frac{\varepsilon_1n_2^2-\varepsilon_1^2+\varepsilon_2^2}{n_2^2-n_1^2}k_2K_1(rk_2)\right\}e^{i\omega(z/v-t)}d\omega.$$

The optical activity of the medium results in an electric field intensity component E_φ and in magnetic field-intensity components H_z and H_r which are lacking when a charge moves through an inactive medium.

Using the Poynting theorem, we now determine the quantity of energy radiated by the charge per unit path

$$-\frac{d\mathcal{E}}{dz}=\frac{c}{4\pi}\int_{-\infty}^{\infty}(E_\varphi H_z-E_zH_\varphi)\,2\pi r dt. \quad (20)$$

Inserting Eqs. (18) and (19) into Eq. (20) we obtain through simple transformation

$$-\frac{d\mathcal{E}}{dz}=\frac{q^2r}{\pi v^2}2\mathrm{Re}\int_0^\infty\Big\{\frac{(1-\varepsilon_1\beta^2)(n_1^2-\varepsilon_1)-\beta^2\varepsilon_2^2}{\varepsilon_1(n_1^2-n_2^2)}K_0(rk_1)\,k_1^*K_1(rk_1^*)$$
$$+\frac{(1-\varepsilon_1\beta^2)(n_2^2-\varepsilon_1)-\beta^2\varepsilon_2^2}{\varepsilon_1(n_2^2-n_1^2)}K_0(rk_2)k_2^*K_1(rk_2^*)\Big\}\,i\,\omega\,d\omega. \quad (21)$$

If ϵ_1, ϵ_2, and ϵ_3 have no zeros in common, the real part of the expression just obtained will be made up of contributions from only those frequencies at which k_1 and k_2 are imaginary. Noting that in the case of imaginary k we have

$$k^*K_1(rk^*)\,K_0(rk)-kK_1(rk)\,K_0(rk^*)=i\pi/2,$$

the losses due to Cerenkov radiation assume the following form [compare with Eqs. (16) and (17)]:

$$-\frac{d\mathcal{E}}{dz}=-\frac{q^2}{v^2}\int_{\beta^2n_1^2>1}\frac{(1-\varepsilon_1\beta^2)(n_1^2-\varepsilon_1)-\beta^2\varepsilon_2^2}{\varepsilon_1(n_1^2-n_2^2)}\,\omega\,d\omega$$
$$-\frac{q^2}{v^2}\int_{\beta^2n_2^2>1}\frac{(1-\varepsilon_1\beta^2)(n_2^2-\varepsilon_1)-\beta^2\varepsilon_2^2}{\varepsilon_1(n_2^2-n_1^2)}\,\omega\,d\omega. \quad (22)$$

The second term of (22) diverges logarithmically. This is because we obtained the fields $E_z(\mathbf{r},t)$ by integrating with respect to k from zero to infinity, while macroscopic electrodynamics is not valid for $k\to\infty$. It is evident that in (22) one is restricted to the frequency region $\beta^2n_m^2>\beta^2n_2^2>1$.

The total losses (taking the near collisions into account) in an isotropic medium are known to be independent of the parameter, $k_m\sim 1/b$. This parameter which enters logarithmically into the expression for the losses in the case of near collisions, and which also enters into the expression for the polarization losses that account for the interaction between the moving charge and the longitudinal field, cancels out in the final expression. In an anisotropic medium the losses in the case of near collisions are the same as in the isotropic medium. But now the parameter k_m enters logarithmically into the expression for the losses due to the radiation of the extraordinary waves, which are also longitudinal in the case of $n_2^2\to\infty$. Thus in an anisotropic medium the total losses, taking the near collisions into account, are also independent of the undetermined parameter k_m.

6. If a charge moves along the axis of an inactive ($\epsilon_2=0$) uniaxial crystal, expression (17) for the losses is considerably simpler:

$$-\frac{d\mathcal{E}}{dz}=\frac{q^2}{c^2}\int_{\beta^2\varepsilon_1>1}\Big(1-\frac{1}{\beta^2\varepsilon_1}\Big)\omega\,d\omega+\frac{q^2}{c^2}\int\Big(\frac{1}{\beta^2\varepsilon_1}-1\Big)\omega\,d\omega+\frac{q^2}{v^2}\sum_i\frac{\omega_i}{|d\varepsilon_1/d\omega|_i}\ln\Big\{1+\frac{k_m^2v^2}{\omega_i^2}\Big(\frac{\varepsilon_1}{\varepsilon_3}\Big)_i\Big\} \quad (23)$$

(the second integral is taken over the region $n_m^2\beta^2>(\epsilon_3/\epsilon_1)(\beta^2\epsilon_1-1)>0$ and $\beta^2\epsilon_1<1$, while ω_i are the common zeros of ϵ_1 and ϵ_2). Expression (23) is identical with those obtained in Refs. 2 and 3. However, thanks to inequality $n_m^2\beta^2>(\epsilon_3/\epsilon_1)(\beta^2\epsilon_1-1)$, the boundaries of the frequency regions, over which the integration is carried out, are so shifted that expression (23) contains no singularities. If ϵ_1 and ϵ_2 have no common zeros, the third term of (23) drops out. In this case, making the transition in the limit to the isotropic medium ($\epsilon_3\to\epsilon_1$), the second term of (23) reduces to the ordinary expression for the polarization losses.

In the case of the simplest gyrotropic medium ($\epsilon_1 = \epsilon_3$) the equation for the losses becomes (cf. also Ref. 3):

$$-\frac{d\mathcal{E}}{dz} = \frac{q^2}{2c^2}\int\left(1-\frac{1}{\beta^2\varepsilon_1}\right)\left\{1 \mp \frac{\beta\varepsilon_2(1+\beta^2\varepsilon_1)}{(1-\beta^2\varepsilon_1)\sqrt{4\varepsilon_1+\beta^2\varepsilon_2^2}}\right\}\omega\, d\omega + \frac{q^2}{v^2}\sum_i \frac{\omega_i}{|d\varepsilon_1/d\omega|_i}\ln\left(1+\frac{k_m^2 v^2}{\omega_i^2}\right). \tag{24}$$

The summation extends over all frequencies at which ϵ_1 and ϵ_2 vanish simultaneously. If ϵ_1 and ϵ_2 have no common zeros, the total losses are determined by the first term of (24). The integration in (24) is carried out over the frequency regions defined by the inequalities

$$n_m^2\beta^2 > [(2\varepsilon_1^2-\varepsilon_2^2)\beta \pm \varepsilon_2\sqrt{4\varepsilon_1+\beta^2\varepsilon_2^2}]\,\beta^2/2\varepsilon_1\beta > 1.$$

Expression (24) contains no singularities in the indicated frequency region.

7. Let us consider now a charge moving perpendicularly to the optical axis of the crystal. Choosing the coordinate system shown in Fig. 2 we obtain:

$$\varkappa_1 = \sin\vartheta\cos\varphi;\quad \varkappa_2 = \cos\vartheta;$$
$$\varkappa_3 = \sin\vartheta\sin\varphi;\quad \mathbf{kv} = kv\cos\vartheta.$$

Let us determine the component of the inverse tensor T_{22}^{-1}, substitute it into Eq. (8), and integrate (8) with respect to the angle σ. As a result we get:

$$-\frac{d\mathcal{E}}{dy} = i\frac{q^2}{2\pi^2c^2}\int_{-\infty}^{\infty}\int_0^{2\pi}\int_{1/\beta}^{n_m}\frac{n^2(1-\beta^2\varepsilon_1\cos^2\varphi-\beta^2\varepsilon_3\sin^2\varphi)+\beta^2\varepsilon_1\varepsilon_3-\varepsilon_1\sin^2\varphi-\varepsilon_3\cos^2\varphi}{An^4+Bn^2+C}\times n\,dn\,d\varphi\,\omega\,d\omega,$$
$$A = \beta^2(\varepsilon_1\cos^2\varphi+\varepsilon_3\sin^2\varphi);$$
$$B = (\varepsilon_1-\varepsilon_3)\sin^2\varphi-\beta^2(\varepsilon_1^2-\varepsilon_2^2)\cos^2\varphi-\beta^2\varepsilon_1\varepsilon_3(1+\sin^2\varphi);$$
$$C = \beta^2\varepsilon_3(\varepsilon_1^2-\varepsilon_2^2)-(\varepsilon_1^2-\varepsilon_2^2-\varepsilon_1\varepsilon_3)\sin^2\varphi. \tag{25}$$

Assuming for simplicity that ϵ_1, ϵ_2, and ϵ_3 have no common zeros, we obtain finally the following equation for the total energy losses:

$$-\frac{d\mathcal{E}}{dy} = \frac{q^2}{4\pi c^2}\int_0^{2\pi}d\varphi\int\left(1-\frac{1}{\beta^2(\varepsilon_1\cos^2\varphi+\varepsilon_3\sin^2\varphi)}\right)\times\left\{1\mp\frac{B-2A\dfrac{1-\beta^2\varepsilon_1\cos^2\varphi-\beta^2\varepsilon_3\sin^2\varphi}{\beta^2\varepsilon_1\varepsilon_3-\varepsilon_1\sin^2\varphi-\varepsilon_3\cos^2\varphi}}{\sqrt{B^2-4AC}}\right\}\omega\,d\omega,$$
$$n_{1,2}^2 = (-B\pm\sqrt{B^2-4AC})/2A \tag{26}$$

(integration with respect to $d\omega$ in the regions defined by Eq. 16). The upper sign of (26) corresponds to losses due to the radiation of the ordinary waves, and the lower to those of the extraordinary ones. The conical surfaces for the ordinary and extraordinary waves are complicated in form (ex-

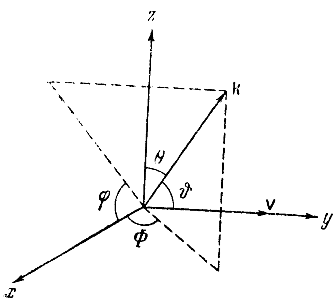

FIG. 2

hibit a dependence on the angle φ) and the intensity varies on the different generatrices of the conic surfaces.

In principle, it is possible to carry out the integration in (26) to the very end, provided the components ϵ_1, ϵ_2, and ϵ_3 of the dielectric tensor are given as the functions of the frequency.

If $\epsilon_1 = \epsilon_3$, Eq. (26) becomes identical with the result of Ref. 3, provided the integration limits are changed in that reference in the manner shown above. In the limiting case of an inactive uniaxial crystal ($\epsilon_2 = 0$) we obtain the corrected Ginzburg equation[1]:

$$-\frac{d\mathcal{E}}{dy} = \frac{q^2}{2\pi c^2}\int_0^{2\pi} d\varphi \left\{ \int\limits_{n_m^2\beta^2>\varepsilon_1\beta^2>1} \left(1-\frac{1}{\beta^2\varepsilon_1}\right)\frac{\beta^2\varepsilon_1\cos^2\varphi}{\sin^2\varphi+\beta^2\varepsilon_1\cos^2\varphi}\,\omega\, d\omega + \int\left(1-\frac{1}{\beta^2\varepsilon_3}\right)\frac{\varepsilon_3\sin^2\varphi}{(\sin^2\varphi+\beta^2\varepsilon_1\cos^2\varphi)(\varepsilon_1\cos^2\varphi+\varepsilon_3\sin^2\varphi)}\,\omega\, d\omega\right\}. \tag{27}$$

The second integral is taken over the region

$$n_m^2\beta^2 > [\beta^2\varepsilon_1\varepsilon_3 + (\varepsilon_3-\varepsilon_1)\sin^2\varphi]/(\varepsilon_1\cos^2\varphi+\varepsilon_3\sin^2\varphi) > 1.$$

In conclusion, we thank Prof. A. I. Akhiezer for interest in the work and for evaluation of the results obtained.

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95