Asymptotic Meson-Meson Scattering Theory

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WHEN the method of the asymptotic theory[1,2] is applied to meson-meson scattering, a difficulty arises in that there is an infinite number of complex diagrams with four entering meson lines, each of which makes a contribution of the same order as the simplest diagrams shown in Fig. 1. The complex diagrams are "reducible" to one of the diagrams in Fig. 1, i.e., to diagrams which consist only of squares, connected by meson lines, which go over into one of the diagrams in Fig. 1 if, succesively, two squares of the complex diagram, connected by two meson lines, are substituted by one (for instance, the diagrams in Fig. 2 are "reducible" and those in Fig. 3 are "irreducible"). To any "reducible" diagram there corresponds a logarithmically divergent quantity of the order of g_0^2 and of the type $(g_0^2/4\pi i) R_n [g_0^2(L-\xi)]$, where g_0 is the initiating charge, R_h = real numerical function, $L = \lg(\Lambda^2/m^2)$, $\xi = \lg(-k^2/m^2)$, k = momentum, intrinsic in the problem, Λ = cut off limit of the momentum[1].

"Irreducible" diagrams need not be considered since their contribution is very small – of higher order in g_0^2. In such a way the scattering amplitude $(g_0^2/4\pi i) P(k_1k_2k_3k_4)$ (k_i is the meson momentum) is determined by the infinite sum $P(k_1k_2k_3k_4) = \Sigma_n R_n(k_1k_2k_3k_4)$, extended over all "reducible" diagrams.

We considered the problem of the calculation of this sum (the so-called "parquet" problem), and discovered it was possible to construct an integral equation which determined P through the quantity R_0, corresponding to the contribution due to the simplest diagrams in Fig. 1. This possibility is connected with the fact that any complex "reducible" diagram consists of two parts, to each of which approach two external meson lines and which are connected with each other only by two meson lines. The latter becomes obvious if one considers that when one makes a "reduction" of the complex diagram, one can always reduce it to the form of Fig. 4 a, b, or c. If $F(k_1k_2, k_3k_4)$ is the sum of the contributions from all "reducible" diagrams, which are "reduced" to Fig. 4, a, and, similarly, $F(k_1k_3, k_2k_4)$ and $F(k_1k_4, k_2k_3)$ for the diagrams reduced to Figs. 4 b and c, then

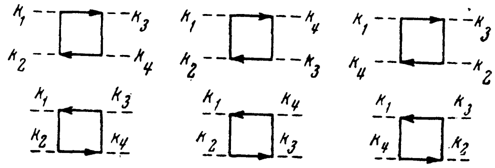

FIG. 1

it is clear that

$$P(k_1k_2k_3k_4) = R_0(k_1k_2k_3k_4) + F(k_1k_3, k_2k_4) + F(k_1k_4, k_2k_3) + F(k_1k_2, k_3k_4), \tag{1}$$

(note that P and R_0 are symmetric with respect to the arbitrary permutation k_i and $F(k_1k_2, k_3k_4)$ will not change by the mutual interchange of k_1 and k_2 or k_3 and k_4; nor by the substitution of k_1, k_2 by k_3, k_4.

One can obtain the following relation for the quantity $F(k_1k_2k_3k_4)$ (similar to the procedure for obtaining the equation for the Green's functions D, G, or Bethe-Salpeter's equation)

$$F(k_1k_2, k_3k_4) = -\frac{g_0^2}{2\pi i}\int [R_0(k_1k_2 ll') + F(k_1l, k_2l') + F(k_1l', k_2l)]\, D(l)\, D(l')\, P(-l-l'k_3k_4)\, d^4l, \tag{2}$$

where $-l' = l + k_1 + k_2$, $k_1 + k_2 + k_3 + k_4 = 0$. Obviously, (1) and (2) represent a closed system of equations, with the help of which one can calculate F and P, if R_0 is known.

For large momenta the sum of the contributions, R_0, from the diagrams in Fig. 1, depends only on the biggest of the four momenta k: $R_0 = \rho_0(\alpha)\delta_c$, where $\alpha = [1 + (5g_0^2/4\pi)(L-\xi)]^{3/5}$,

while $\rho_0 = {}^{16}/_3(\alpha - 1)$,

and $\delta_c = \delta_{\tau_1\tau_2}\delta_{\tau_3\tau_4} + \delta_{\tau_1\tau_3}\delta_{\tau_2\tau_4} + \delta_{\tau_1\tau_4}\delta_{\tau_2\tau_3}$

depend on the subscripts τ_i $(i = 1, 2, 3, 4)$ of the isotopic spin (for calculation, Green's functions which were found in Ref. 2 are used).

For large momenta the scattering amplitude P also depends on only one quantity α. However, when any two of the four momenta, for instance k_1 and k_2, are very big but are almost opposite in direction so that their sum $k_1 + k_2 = -(k_3 + k_4)$

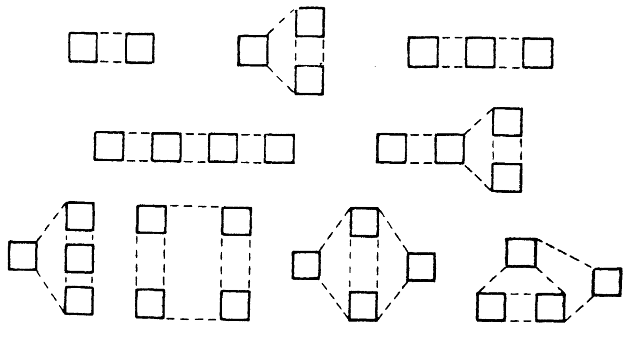

FIG. 2

is much smaller than each of them, a special case arises. (This case is essential in (2), since the integral with respect to l in (2) is logarithmically divergent, and only very big $l \approx -l'$ are of importance).

If k_3 and k_4 are the quantities of the same order as $k_3 + k_4 = -(k_1 + k_2)$, then, in this case, P depends not only on α (which corresponds to the quantities $-k_1^2$ or $-k_2{}^2$), but also on

$$\beta = [1 + (5\, g_0^2 / 4\pi)\ (L - \eta)]^{3/5},$$

$$\eta = \lg [-(k_1 + k_2)^2 / m^2] = \lg [-(k_3 + k_4)^2 / m^2],$$

while

$$P(k_1 k_2 k_3 k_4) = P_c(\alpha\beta)\, \delta_c + P_{12}(\alpha\beta)\, \delta_{\tau_1\tau_2}\, \delta_{\tau_3\tau_4}.$$

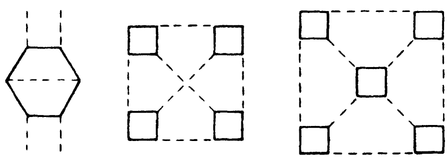

FIG. 3;

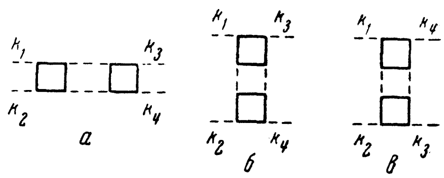

FIG. 4

If $\beta \to \alpha$, i.e., if all four momenta are of the same order of magnitude, then P_{12} vanishes. Denoting $P_c(\alpha, \alpha) = P(\alpha)$, we will have in this case

$$P(k_1 k_2 k_3 k_4) = P(\alpha)\, \delta_c.$$

The limiting values of the function P_c and P_{12} can be found with the help of the system (1), (2). They have the following form:

$$P_c(\alpha,\beta) = \begin{cases} \frac{16}{3}(\alpha - 1)\left\{1 - \frac{88}{27}(\alpha - 1)^2 - \frac{8}{3}[(\beta - 1)^2 - (\alpha - 1)^2] + \ldots\right\}, & \beta - 1 < 1 \\ \frac{16}{11}\,\alpha\left[1 + \frac{5}{3}\left(\alpha^{40/33}\,\beta^{-40/33} - \alpha^{16/33}\,\beta^{-16/33}\right)\right], & \alpha > 1 \end{cases} \tag{3}$$

$$P_{12}(\alpha, \beta) = \begin{cases} -\frac{5}{12}\left(\frac{16}{3}\right)^2(\alpha - 1)\{[(\beta - 1)^2 - (\alpha - 1)^2] + \ldots\}, & \beta - 1 < 1 \\ \frac{16}{11}\,\alpha\,(\alpha^{16/33}\,\beta^{-16/33} - 1), & \alpha > 1 \end{cases}$$

$$P_c(\alpha, \alpha) = P(\alpha) = \begin{cases} \frac{16}{3}(\alpha - 1)\left[1 - \frac{88}{27}(\alpha - 1)^2 - \ldots\right], & \beta - 1 < 1 \\ \frac{16}{11}\,\alpha, & \alpha > 1 \end{cases}$$

In the region $\alpha \sim 1$ or $\beta \sim 1$ the functions P_c and P_{12} can be found only by the numerical integration of the system of asymptotic equations, which follow from (1) and (2).

V. V. Sudakov succeeded in obtaining a simple equation directly for $P(\alpha)$ and in completely constructing this function. The solution found by him,

$$P(\alpha) = \frac{16}{11}\,\alpha\,\frac{1-\alpha^{-19/3}}{1+{}^{8}/_{11}\,\alpha^{-19/3}} \tag{4}$$

for the cases $\alpha - 1 < 1$ and $\alpha > 1$ is identical with (3).

It follows from (3) or (4) that the entire sum $P(\alpha)$ is a quantity of the same magnitude as the contribution $\rho_0(\alpha)$ from the simplest diagrams in Fig. 1 (for $\alpha > 1$, $P(\alpha)$ differs from $\rho_0(\alpha)$ only by the factor 11/3).

The latter is of importance for the conclusion[4] of the pseudo-scalar theory that the meson charge of a nucleon is zero, since this conclusion is based on the results of the theory[2], in which (in the equation for the operator of the peak part) the diagram of Fig. 5 and also the infinite set of analogous diagrams obtained from Fig. 5 by sub-

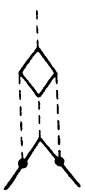

FIG. 5

stitution of a square for any "reducible" diagram was not considered. Since $P(\alpha)$ and $\rho_0(\alpha)$ are quantities of the same order, the contribution of the infinite set of all diagrams of the type in Fig. 5 is a quantity of the same magnitude as that from the one diagram in Fig. 5, i.e., they can be neglected, (the contribution from the diagram in Fig. 5 is a quantity of the order of g_0^2 in comparison with the terms taken into consideration in Ref. 2).

The authors express their thanks to V. V. Sudakov for important remarks and to I. Ia. Pomeranchuk who stimulated interest in the above problem.

* Here the case when $g_0^2 \ll 1$ and $L \gg 1$ is considered, so that $g_0^2 L$ is an arbitrary quantity. When two cut off limits are introduced[3] the quantity $\tilde{g}_0^2 = g_0^2\left[1 + \frac{g_0^2}{\pi}(L_p - L_k)\right]^{-1}$ enters everywhere instead of g_0^2; $\tilde{g}_0^2$ is small for an arbitrary g_0^2 if $\frac{1}{\pi}(L_p - L_k) = \frac{1}{\pi}\lg\frac{\Lambda_p^2}{\Lambda_k^2}$ is sufficiently large.

[1] L. D. Landau, A. A. Abrikosov and I. M. Khalatnikov, Dokl. Akad. Nauk SSSR 95, 497, 773, 1177 (1954).

[2] A. A. Abrikosov, A. D. Galanin and I. M. Khalatnikov 97, 793 (1954).

[3] A. A. Abrikosov and I. M. Khalatnikov, 103, 993 (1955).

[4] I. Ia. Pomeranchuk, 103, 1005 (1955).

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Relativistic Deuteron Disintegration in the Electric Field of the Nucleus

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THE disintegration of deuterons with energy of the order of 200 mev in the electric field of the nucleus was investigated by Dancoff[1] in the nonrelativistic approximation. In the present communication the problem is solved, taking into account relativistic effects. It will be shown below that the relativistic corrections to the cross section for "electric" disintegration $\sigma_{1\to1}$, found in Ref. 1, are small if $v^2 \sim 0.2$ ($E_d \sim 200$ mev)*, but are important if $v^2 \sim 1$, and that the "magnetic" disintegration investigated in Ref. 1, in which the proton-neutron system undergoes transition from the triplet to the singlet state, does not take place; the corresponding cross-section $\sigma_{1\to0}$ is of an order of magnitude smaller than $\sigma_{1\to1}$ in the extremely relativistic case.

As in Ref. 1, we shall make the following approximations: a) the motion of the center of mass of the proton-neutron system in the field of the nucleus is investigated in the Born approximation, which is permissible for $Z/137v \ll 1$; b) $n-p$ forces are assumed central with a zero radius of action; c) the nuclear electric field is "cut off" at $r = R_0$, the sum of the radii of the nucleus and the deuteron**; d) the deuteron radius R_d is considered small in comparison with R_0.

In the system of reference K in which the deuteron is at rest before the collision, the potentials of the field of the nucleus Ze, which has a velocity v in the z-direction, are

$$\varphi = Ze/R, \qquad A_x = A_y = 0, \qquad A_z = vZe/R, \tag{1}$$