In the region $\alpha \sim 1$ or $\beta \sim 1$ the functions P_c and P_{12} can be found only by the numerical integration of the system of asymptotic equations, which follow from (1) and (2).

V. V. Sudakov succeeded in obtaining a simple equation directly for $P(\alpha)$ and in completely constructing this function. The solution found by him,

$$P(\alpha) = \frac{16}{11} \alpha \frac{1 - \alpha^{-19/3}}{1 + {}^{8}/_{11} \alpha^{-19/3}} \quad (4)$$

for the cases $\alpha - 1 < 1$ and $\alpha > 1$ is identical with (3).

It follows from (3) or (4) that the entire sum $P(\alpha)$ is a quantity of the same magnitude as the contribution $\rho_0(\alpha)$ from the simplest diagrams in Fig. 1 (for $\alpha > 1$, $P(\alpha)$ differs from $\rho_0(\alpha)$ only by the factor 11/3).

The latter is of importance for the conclusion[4] of the pseudo-scalar theory that the meson charge of a nucleon is zero, since this conclusion is based on the results of the theory[2], in which (in the equation for the operator of the peak part) the diagram of Fig. 5 and also the infinite set of analogous diagrams obtained from Fig. 5 by sub-

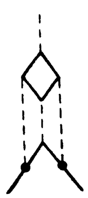

FIG. 5

stitution of a square for any "reducible" diagram was not considered. Since $P(\alpha)$ and $\rho_0(\alpha)$ are quantities of the same order, the contribution of the infinite set of all diagrams of the type in Fig. 5 is a quantity of the same magnitude as that from the one diagram in Fig. 5, i.e., they can be neglected, (the contribution from the diagram in Fig. 5 is a quantity of the order of g_0^2 in comparison with the terms taken into consideration in Ref. 2).

The authors express their thanks to V. V. Sudakov for important remarks and to I. Ia. Pomeranchuk who stimulated interest in the above problem.

* Here the case when $g_0^2 \ll 1$ and $L \gg 1$ is considered, so that $g_0^2 L$ is an arbitrary quantity. When two cut off limits are introduced[3] the quantity $\tilde{g}_0^2 = g_0^2 \left[1 + \frac{g_0^2}{\pi}(L_p - L_k)\right]^{-1}$ enters everywhere instead of g_0^2; $\tilde{g}_0^2$ is small for an arbitrary g_0^2 if $\frac{1}{\pi}(L_p - L_k) = \frac{1}{\pi} \lg \frac{\Lambda_p^2}{\Lambda_k^2}$ is sufficiently large.

[1] L. D. Landau, A. A. Abrikosov and I. M. Khalatnikov, Dokl. Akad. Nauk SSSR 95, 497, 773, 1177 (1954).

[2] A. A. Abrikosov, A. D. Galanin and I. M. Khalatnikov 97, 793 (1954).

[3] A. A. Abrikosov and I. M. Khalatnikov, 103, 993 (1955).

[4] I. Ia. Pomeranchuk, 103, 1005 (1955).

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Relativistic Deuteron Disintegration in the Electric Field of the Nucleus

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THE disintegration of deuterons with energy of the order of 200 mev in the electric field of the nucleus was investigated by Dancoff[1] in the non-relativistic approximation. In the present communication the problem is solved, taking into account relativistic effects. It will be shown below that the relativistic corrections to the cross section for "electric" disintegration $\sigma_{1\to 1}$, found in Ref. 1, are small if $v^2 \sim 0.2$ ($E_d \sim 200$ mev)*, but are important if $v^2 \sim 1$, and that the "magnetic" disintegration investigated in Ref. 1, in which the proton-neutron system undergoes transition from the triplet to the singlet state, does not take place; the corresponding cross-section $\sigma_{1\to 0}$ is of an order of magnitude smaller than $\sigma_{1\to 1}$ in the extremely relativistic case.

As in Ref. 1, we shall make the following approximations: a) the motion of the center of mass of the proton-neutron system in the field of the nucleus is investigated in the Born approximation, which is permissible for $Z/137v \ll 1$; b) $n-p$ forces are assumed central with a zero radius of action; c) the nuclear electric field is "cut off" at $r = R_0$, the sum of the radii of the nucleus and the deuteron**; d) the deuteron radius R_d is considered small in comparison with R_0.

In the system of reference K in which the deuteron is at rest before the collision, the potentials of the field of the nucleus Ze, which has a velocity v in the z-direction, are

$$\varphi = Ze/R, \quad A_x = A_y = 0, \quad A_z = vZe/R, \quad (1)$$

$$R = R(\mathbf{r},\ t) = [(1 - v^2)\rho^2 + (z - vt)^2]^{1/2}, \qquad (2)$$
$$\rho^2 = x^2 + y^2.$$

The perturbed Hamiltonian of the deuteron in the nuclear field is given by

$$V = e\varphi_p + \frac{ie}{M}(\nabla_p \mathbf{A}_p + \mathbf{A}_p \nabla_p) + \frac{e^2}{2M}\mathbf{A}_p^2 - \mu_{\text{яд}}(\mu_p \vec{\sigma}_p \mathbf{H}_p + \mu_n \vec{\sigma}_n \mathbf{H}_n) \qquad (3)$$
$$\approx \frac{Ze^2}{R_p} + v\frac{Ze^2}{2M}\left\{\frac{2i}{R_p}\frac{\partial}{\partial z_p} - \frac{i}{R_p^3}(z_p - vt) - (1 - v^2)\mu_p [\mathbf{r}_p \vec{\sigma}_p]_z \frac{1}{R_p^3} - (1 - v^2)\mu_n [\mathbf{r}_n \vec{\sigma}_n]_z \frac{1}{R_n^3}\right\}$$

[in the Born approximation we drop the term $(e^2/2M)\mathbf{A}_p^2$], where M is the mass of the nucleon.

In the expressions for the wave functions of the initial and final states of the unperturbed system

$$\Psi_i = L^{-3/2}\psi_d(r)\chi_1^{m_0} e^{-iE_0 t} \quad (m_0 = -1,\ 0,\ 1), \qquad (4)$$

$$\Psi_f = L^{-3/2}\exp\{i\mathbf{q}\mathbf{r}_d\}\,\psi_{\mathbf{k}S}^{(-)}(\mathbf{r})\,\chi_S^m e^{-iEt} \qquad (5)$$
$$\times (S = 0,\ 1;\ -S \leqslant m \leqslant S)$$

the basic states of the deuteron and the states of the continuous spectrum of the $n - p$ system which contain the scattered wave for $r \to \infty$ can (in the light of approximation b) be written in the form

$$\psi_d(r) = \sqrt{\alpha_1 / 2\pi}\, e^{-\alpha_1 r} / r, \qquad (6)$$

$$\psi_{\mathbf{k}S}^{(-)}(\mathbf{r}) = \frac{1}{L^{3/2}}\left(e^{i\mathbf{k}\mathbf{r}} - \frac{1}{\alpha_S - ik}\frac{e^{-ikr}}{r}\right) \qquad (7)$$
$$(S = 0,\ 1).$$

We note that the orthogonality condition $\int \psi_d(r)\psi_{\mathbf{k}1}^{(-)}(\mathbf{r})\,d^3r = 0$ is satisfied only for $S = 1$.

For the transition that interests us, the probability amplitude is given by

$$a_{i\to f} = -i\int_{-\infty}^{\infty}(f \mid V(t) \mid i)\,e^{i\omega t}dt, \quad \omega = E - E_0. \qquad (8)$$

Introducing Eq. (3), we get, after some obvious transformations,

$$\frac{iL^3}{Ze^2}a_{i\to f} = \delta_{S1}\delta_{mm_0}A_0\left[\left(1 - \frac{vq_z}{2M}\right)B_{\mathbf{q}1} + i\frac{v}{M}C\right] \qquad (9)$$
$$- v(1 - v^2)\frac{A_1}{2M}\Big[\delta_{S1}(\mu_p B_{\mathbf{q}1} + \mu_n B_{-\mathbf{q}1})(\chi_1^m,\ [\mathbf{q}\mathbf{S}]_z\chi_1^{m_0})$$
$$+ \delta_{S0}\delta_{m0}(\mu_p B_{-\mathbf{q}0} - \mu_n B_{-\mathbf{q}0})\left(\chi_0^0,\ \left[\mathbf{q},\ \tfrac{1}{2}(\vec{\sigma}_p - \vec{\sigma}_n)\right]_z \chi_1^{m_0}\right)\Big]$$

Here,

$$A_0 = \int d^3r\, e^{-i\mathbf{q}\mathbf{r}}\int_{-\infty}^{\infty}\frac{e^{i\omega t}dt}{R(\mathbf{r},\ t)} \qquad (10)$$
$$= \frac{4\pi}{vq^2(1 - v^2\cos^2\alpha)}\ \frac{2\sin[(q\cos\alpha - \omega/v)L/2]}{q\cos\alpha - \omega/v},$$

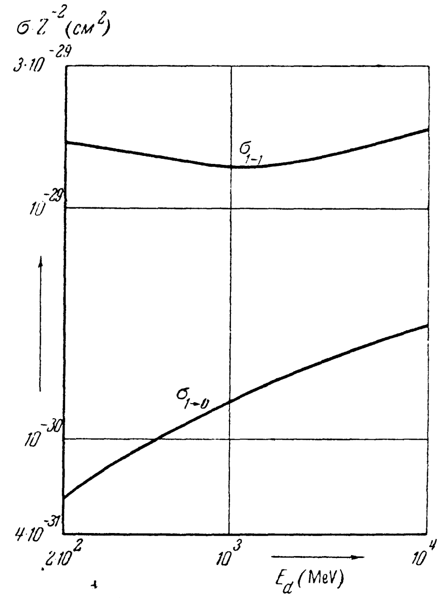
$$A_1\mathbf{q}_{\perp} = \int d^3r\,\vec{\rho}\,e^{-i\mathbf{q}\mathbf{r}}\int_{-\infty}^{\infty}\frac{e^{i\omega t}dt}{R^3(\mathbf{r};\ t)} = -\frac{i\,\mathbf{q}}{1 - v^2}A_0, \qquad (11)$$

$$B_{\mathbf{q}S} = \int d^3r\,\psi_{\mathbf{k}S}^{(-)*}(\mathbf{r})\,\psi_d(r)\,e^{i\mathbf{q}\mathbf{r}/2}, \qquad (12)$$

$$C = \int d^3r\,\psi_{\mathbf{k}1}^{(-)*}(\mathbf{r})\,\frac{\partial\psi_d(r)}{\partial z}\,e^{i\mathbf{q}\mathbf{r}/2},$$

where α is the angle between the z-axis and the vector $\mathbf{q}$. (Equations (10) and (11) are derived on the assumption that $L \to \infty$.)

Because of the "cut-off" of the electric field for $r = R_0$, the upper bound for the momentum q is



$q \lesssim q_{\max} \sim 1/R_0$; condition d) gives $q_{\max}/\alpha_1 \ll 1$, i.e., the coefficient of $e^{i\mathbf{q}\cdot\mathbf{r}/2}$ in the integrands of Eqs. (12) can be replaced by the first terms of the series expansion $1 + (1/2)\, i\,\mathbf{q}\cdot\mathbf{r}$. In view of the orthogonality of the functions $\psi_{\mathbf{k}1}^{(-)}$

and ψ_d, the basic term in the integral B_{q1} is introduced by the second term of the expansion, and in B_{q0} and C by the first term.

In Eq. (9) we will keep only the basic terms, neglecting the part containing the matrix element of the operator $[qxS]_z$ and the quantity $-vq_z/2M$ in the parentheses. It is not hard to see that the terms we have here dropped give small corrections, whose calculation would hardly make sense, since we have made the approximations a) – d).

We now go over from a consideration of the probability amplitude to the cross section, sum over S and m, average over m_0, and integrate over d^3q as was done in Ref. 1. Let us write

$$\varepsilon = k^2/M, \quad \varepsilon_S = \alpha_S^2/M, \quad 1/\Gamma = v^{-1}(\varepsilon + \varepsilon_1)R_0 \ll 1.$$

Integration over the angle ϑ gives the distribution over ε $(0 \leqslant \varepsilon \leqslant \varepsilon_{\max} = v/R_0 - \varepsilon_1)$

$$\sigma_{1\to 1}(\varepsilon)\,d\varepsilon = \frac{8}{3}\left(\frac{Z}{137v}\right)^2 \frac{\sqrt{\varepsilon_1\varepsilon^3}\,d\varepsilon}{M(\varepsilon+\varepsilon_1)^4} \times\left[\ln\frac{\Gamma^2-v^2}{1-v^2} - v^2\frac{\Gamma^2-1}{\Gamma^2-v^2}\right], \quad (13)$$

$$\sigma_{1\to 0}(\varepsilon)\,d\varepsilon = \frac{2}{3}\left(\frac{Z}{137}\right)^2(\mu_p-\mu_n)^2 \times\frac{\sqrt{\varepsilon_1\varepsilon}(\sqrt{\varepsilon_1}+\sqrt{\varepsilon_0})^2\,d\varepsilon}{M^2(\varepsilon+\varepsilon_0)(\varepsilon+\varepsilon_1)^2}\left[\ln\frac{\Gamma^2-v^2}{1-v^2} - \frac{\Gamma^2-1}{\Gamma^2-v^2}\right]. \quad (14)$$

Equation (13) goes over into Dancoff's equation[1] if the expression in square brackets is designated by $\ln\Gamma^2$.

Integration over ϵ is carried out numerically. The figure shows the variation of the integral cross sections $\sigma_{1\to1}$ and $\sigma_{1\to0}$ in the energy interval $E_d = 0.2 - 10$ bev for $R_0 = 1.1 \times 10^{-13}$cm.

The authors thank Z. V. Rumiantseva, who carried out the numerical integration.

*We are employing a system of units in which $c = \hbar = 1$.

**It can be shown that in this way the diffraction disintegration of the deuteron, which is not connected with the electric field close to the nucleus (see Ref. 2), is eliminated from the consideration; the whole cross section for disintegration is given by the sum $\sigma_{\text{dif}} + \sigma_{\text{el}}$, and the interference term is absent in the approximations a) – d).

[1]S. Dancoff, Phys. Rev. 72, 1017 (1947).

[2]A. I. Akhiezer and A. G. Sitenko, Dokl. Akad. Nauk SSSR 107, 3 (1956).

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On the Paper by N. I. Steinbok "Basic Characteristics of Ionization Chambers"

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IN the paper by N. I. Steinbok[1], it is asserted that the solution of the equation of the volt-ampere characteristics of ionization chambers obtained by him are more rigorous and complete than those previously described in the literature (see, for example, Refs. 2, 3). This conclusion is obtained on the basis of the supposition that, if the ionization is relatively weak, it is possible to ignore the space charge ; however, no basis for the justification of this supposition is advanced in this paper and the error arising out of this supposition is not evaluated. What is more, it is possible to demonstrate that even where weak ionization exists, the part played by space charge in ionization chambers is quite significant, and therefore the conclusions drawn in Ref. 1 are in error.

Let us examine the problem of the variation in intensity of the electrical field between the anode and the cathode in a flat, air-type ionization chamber. Let us designate by h the distance between the anode and cathode and draw an axis x perpendicular to their surface. Seeliger[4] demonstrated that the value of the intensity of field E in the space between the anode and the cathode is completely determined by the numerical value of the ratio i/I_0, where I_0 is the strength of the saturation current and i is the strength of the current in the absence of saturation; upon variation of the strength of ionization and of the magnitude of I_0, only the absolute value of E changes at all points in the field. In the following table the values of E are given in relative units for various values of i/I_0 for cross sections of the chamber at various distances from the anode $x = 0$, $0.33h$, $0.42h$, $0.67h$, and h.

From this table and Seeliger's graph (Ref. 4, p. 348, Fig. 3; see also Ref. 3, Fig. 82) it can be seen that even in the vicinity of the zone of saturation $i/I_0 = 0.949$ the minimum value of intensity of the electrical field E at $x = 0.42\,h$ is lower by 18% than the maximum value of E at $x = h$; at $i/I_0 = 0.406$ the minimum value of the field differs from the maximum by a factor of 2.5.

From the foregoing it follows that parameter λ, which enters into the system of differential equations derived by Steinbok for ionization chambers and which contains E^2 in the denominator, cannot