Contribution to the Theory of Production and Annihilation of Antiprotons

L. M. AFRIKIAN
P. N. Lebedev Physical Institute, Academy of Sciences, USSR
and
Physical Institute, Academy of Sciences, Armenian SSR

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The processes leading to production of antiprotons by π-mesons on nucleons are considered in the framework of charge-independent pseudoscalar meson theory. The cross sections for these processes are calculated near the threshold. It is shown that the interaction between the particles in the final state may lead to a capture of the antiproton produced by a nucleon into a deuteron-like orbit. This last fact may make it difficult to detect antiprotons near the threshold because of the high capture probability, and the short life ($\tau \approx 10^{-21}$ sec) of the antiproton in the bound state with respect to annihilation.

Processes leading to antiproton production in nucleon-nucleon collisions are also considered. The cross sections for these processes are calculated near the threshold, taking into account the interaction between the created particles.

THE question of the existence and the properties of antiprotons (i.e., particles of proton mass and negative charge) has recently attracted a good deal of attention in connection with the possibility of obtaining such particles from new powerful accelerators.

It is well known that the hypothesis of the existence of an antiproton (and an antineutron) arose by analogy with the conception of the positron, which was the logical consequence of the relativistic quantum theory of the electron as a particle of spin ½. Indeed, from the description of the nucleon as an elementary particle of spin ½, which is therefore described by the corresponding Dirac equation

$$\{c(\vec{\alpha}\mathbf{P}) + \beta M\}\Psi_N = E\Psi_N,$$

the requirement that there should exist a complete set of linearly independent solutions of this equation leads one in a similar way to assume the existence of antiparticles, i.e., antinucleons, which have antiprotons and antineutrons as different charge states.

It should be stressed, however, that, in spite of its origin, the antiproton hypothesis is not an exclusive feature of the Dirac theory, but is based directly on the idea of charge conjugation symmetry, which is fundamental to the whole modern theory of elementary particles. Although one cannot rule out the possibility that the nucleon has to be described by an equation of a type other than Dirac's, the only rational approach to the antiproton problem at this time is from the point of view of the Dirac theory; however, we must then allow for the strong interaction of the nucleons with the meson field. A theoretical analysis of all experimental facts from the point of view indicated above gives us indirect evidence in favor of the existence of antiprotons.

We are thinking here, in the first place, of the well-known decay of the neutral π-meson into two γ-quanta which, apparently, can proceed only through the formation and annihilation of a virtual proton-antiproton pair: $\pi^0 \to p + \tilde{p} \to 2\gamma$; second, of the appearance of an appreciable S-wave contribution in the scattering and photoproduction of charged mesons on nucleons, in conditions when the nucleon may be taken as nonrelativistic. The last fact is also due to the contribution from virtual antiproton pairs (under the assumptions of the pseudoscalar coupling theory). Third, we have the fact that the masses of the charged π-mesons are equal in good approximation, and exceed that of the π^0 by about 9 electron masses.

This difference can be deduced in order of magnitude from the Coulomb mass defect in the virtual proton-antiproton system*, a state in which the π^0-meson spends part of its time.

But what are the possibilities of finding real antiprotons under laboratory conditions? The purpose of the present work is to obtain a theoretical estimate of the cross sections for the following effects: (1) the production of antiprotons by π-mesons on nucleons, (2) the production of antiprotons in nucleon-nucleon collisions.

The theoretical study of such problems meets at present with insuperable difficulties because of the lack of a consistent theory of nuclear forces, and of adequate methods for solving the quantum mechanical problems. For this reason we were forced to obtain preliminary estimates by means of perturbation theory, and consequently, our results cannot claim to have more than qualitative value, even within the limits of our assumptions.

* For this one has to use the known data on the electromagnetic dimensions of the proton.

Nevertheless, a comparison of the results of such calculations with experimental data may turn out to be of some use for constructing a more complete theory of these phenomena.

In the future we hope to treat the same problems more rigorously, on the basis of a semi-phenomenological method, and using the theory of the spin 3/2 state[1], i.e., allowing for excited ("isobaric") states of the nucleons.

I. PRODUCTION OF ANTIPROTONS IN π-MESON-NUCLEON COLLISIONS

Consider the following antiproton production processes:

$$\text{a)}\ \pi^- + p \to n + p + \tilde{p};$$

$$\text{b)}\ \pi^+ + n \to p + p + \tilde{p};$$

$$\text{c)}\ \pi^- + n \to n + n + \tilde{p}.$$

The threshold value of the kinetic energy of a π-meson for the production of a nucleon-antinucleon pair on a nucleon at rest is

$$T_\pi^{\min} = 4M - \mu(1 + \mu/2M) = 3{,}45 \text{ bev}.$$

π-mesons of this energy begin to be emitted in a nucleon-nucleon collision when the energy of the incident nucleon (in the system in which the other nucleon is at rest) reaches the value

$$T_{N1}^{\min} = 4.15 \text{ bev}.$$

The matrix elements for the processes a), b) and c) were calculated in the first nonvanishing approximation, i.e., in the third order of perturbation theory. In this approximation the reaction of the self-field of the nucleon is neglected, and therefore such an approximation can have some kind of validity only near the threshold of the reaction, where the energy of the final nucleons (and of the antiproton) is much less than the excitation energy, $\Delta \approx 250$ mev, of the first isobaric state, i.e., as long as*

$$E - E_0 \ll 2\Delta. \qquad \text{(I.1)}$$

Near the threshold the modulus of the matrix element is a slowly varying function of the momenta of the particles in their final states, and may be represented by a series in powers of v/c. Thus, the energy dependence of the cross section is, in the indicated region, essentially determined by the statistical factor, together with general symmetry requirements. On the other hand it is easy to see that in the region limited above by (I.1) the kinetic energies of the particles in the final state do not exceed appreciably the depth of the potential well of the nuclear forces; hence, it is in general not legitimate to neglect the interactions in the final state. Therefore, the expressions for the antiproton production cross section which neglect the interactions in the final state can be right at best in a narrow energy interval, near the upper end of the threshold region. In the rest of the region, closer to the threshold, the strong interactions between the particles must substantially affect the magnitude and the energy dependence of the cross sections.

* E_0 is the threshold energy in the laboratory system.

One of the above processes, namely $\pi^- + p \to n + p + \tilde{p}$, was considered in a number of papers by various authors. This process was first studied by McConnell[2] using a generalized Weizsäcker-Williams method in the theory of radiation damping, but without allowing for exchange, or for the nucleon recoil, which are very important near the threshold. Later the same process was studied[3] by the same method as in Ref. 2, but with a phenomenological treatment of the damping. In addition, these authors[3] started from incorrect values for the threshold energy for pair creation*. In a short communication[5] calculations of cross section of the process under discussion were reported, using third-order perturbation theory, for the special case of pseudoscalar coupling. It must be pointed out that all these papers neglect the interaction of the particles in the final state since these authors[2,3,5] did not concentrate their attention on the threshold region, the only region which is in any sense accessible to the perturbation theory they were using.

1. *General statement of the problem and calculation of the cross sections without allowing for interactions in the final state.*

The basic equation of our problem is the Schwinger-Tomonaga equation

$$i\delta F[\zeta(r)]/\delta\zeta(r) = \hat{H}'(r)F[\zeta(r)], \qquad \text{(I.2)}$$

where $\hat{H}'$ is the Hamiltonian density of the interaction between the nucleon field and a pseudoscalar meson field.

$$\hat{H}' = ig\overline{\Psi}\tau_\alpha\gamma_5\Psi\tilde{\varphi}_\alpha + \frac{f}{\mu}\overline{\Psi}\tau_\alpha\gamma_5\gamma_\nu\Psi\frac{\partial\tilde{\varphi}_\alpha}{\partial x_\nu}; \qquad \text{(I.3)}$$

$$(\nu = 1, 2, 3, 4).$$

Here

* The authors[3] quote a paper[4] which, however, gives the threshold energy only for pair production in an inelastic collision between particles, without any of them disappearing.

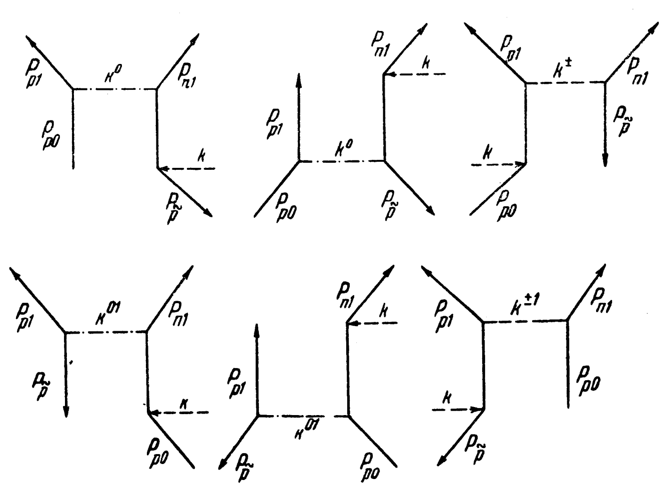

FIG. 1

$$\gamma_k = \beta\alpha_k \quad (k = 1, 2, 3);$$
$$\gamma_4 = \beta; \quad \gamma_5 = i\gamma_1\gamma_2\gamma_3\gamma_4;$$

τ_α are the isotopic spin matrices; g and f the coupling constants.

The operators of the free nucleon and meson fields are obtained from the equations

$$i\gamma_\nu \partial\Psi / \partial x_\nu - M\Psi = 0; \qquad (I.4)$$
$$i\partial\overline{\Psi} / \partial x_\nu \gamma_\nu + M\overline{\Psi} = 0;$$
$$\partial^2\widetilde{\varphi}_\alpha / \partial x_\nu^2 - \mu^2 \widetilde{\varphi}_\alpha = 0.$$

The solution of Eq. (I.2) amounts to the determination of the operator $\hat{S}$ which generates the final state function F_f of the whole system from the initial state function F_i: $F_f = \hat{S}F_i$. The probability of the transitions in the nucleon-meson field in which we are interested is then expressed in terms of the S matrix as

$$W_{if} = | S_{if}^{(n)} |^2, \qquad (I.5)$$

and we may write

$$S_{if}^{(n)} = R_{if}^{(n)} \delta(\Delta\overline{P}), \qquad (I.6)$$

where $R_{if}^{(n)}$ is the matrix element of the transition, $\delta(\Delta\overline{P})$ the four-dimensional delta-function, and $\overline{P}$ the four-momentum.

The differential cross section for nucleon-antiproton production processes in a π-meson nucleon collision neglecting the interactions in the final state becomes

$$d\sigma = \frac{1}{2J_0} \sum_i^s \sum_f^s | R_{if} |^2 \rho_f \frac{1}{(2\pi)^4} \delta(\Delta E)\, \delta(\Delta \mathbf{P}); \qquad (I.7)$$

$$\rho_f = \frac{1}{(2\pi)^9} \prod_{k=1}^{3} d\mathbf{P}_k; \quad \Delta E = E_i - E_f; \Delta\mathbf{P} = \mathbf{P}_i - \mathbf{P}_f;$$

Here J_0 is the incident particle flux. The square modulus of the matrix element is to be averaged over the spin in the initial state, and summed over the spins in the final state, with the aid of the projection operators:

$$\Lambda_+ \to (\hat{P} + M) / 2E \text{ for a nucleon;}$$
$$\Lambda_- \to (\hat{P}_\sim - M) / 2E_\sim \text{ for an antiproton*.}$$

It is seen from (I.6) and (I.7) that the determination of the differential cross sections requires the solution of Eq. (I.3). The only Lorentz-invariant method for solving this equation which is known at present is the approximate representation of the state function F as a series in increasing powers of the coupling constant. For constructing this series one can immediately obtain the required operator $\hat{S}$ from the given operator $\hat{H}'$ using the well-known Feynman-Dyson prescriptions[6,7]. Thus, in the case of the process $\pi^- + p \to n + p + \tilde{p}$ we have for the transition matrix element in the first nonvanishing approximation six Feynman graphs (Fig. 1). Three of these graphs correspondto the possibility of an exchange between the recoil proton and the vacuum proton; they are a consequence of the antisymmetry of the wave amp-

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* We use throughout the notation of Feynman[6], using for an arbitrary four vector a the convention $\hat{a} = \gamma_\nu a_\nu$ $(\nu = 1, 2, 3, 4)$, with $(\overline{a}\overline{b}) \equiv a_4 b_4 - \mathbf{ab}$, so that $\hat{P} \equiv i\beta(\partial/\partial t) + i\beta(\vec{\alpha}\nabla)$.

litude of the initial state in the nucleon variables ξ:

$$\chi_i(\xi_1, \xi_2, \eta) = 2^{-1/2}\{\Psi_{p_-}(\xi_1)\Psi_{p0}(\xi_2) - \Psi_{p_-}(\xi_2)\Psi_{p0}(\xi_1)\}\varphi(\eta).$$

Each of these graphs is equivalent to two chains of the ordinary perturbation theory, and contains as elements both the interaction of the nucleons with the external field (charged mesons) and the interaction of the nucleons with each other, which is transmitted by charged as well as neutral mesons.

The complete matrix element for the transition is

$$R_{if} = \sum_{n=1}^{6} R_{if}^{(n)}, \qquad (I.8)$$

where

$$R_{if}^{(1)} = iA\left(\bar{u}_{p1}\left\{ig\gamma_5 + \frac{f}{\mu}\gamma_5\hat{k}^0\right\}\tau_0 u_{p0}\right)[k^{02} - \mu^2]^{-1} \times \left(\bar{u}_{n1}\tau_0\left\{\left(ig\gamma_5 - \frac{f}{\mu}\gamma_5\hat{k}^0\right)[-\hat{P}_\sim + \hat{k} - M]^{-1}\left(ig\gamma_5 + \frac{f}{\mu}\gamma_5\hat{k}\right)\right\}\tau_- u_{\tilde{p}}\right);$$

$$R_{if}^{(2)} = iA\left(\bar{u}_{p1}\left\{ig\gamma_5 + \frac{f}{\mu}\gamma_5\hat{k}^0\right\}\tau_0 u_{p0}\right)[k^{02} - \mu^2]^{-1} \times \left(\bar{u}_{n1}\tau_-\left\{\left(ig\gamma_5 + \frac{f}{\mu}\gamma_5\hat{k}\right)[\hat{P}_{n1} - \hat{k} - M]^{-1}\left(ig\gamma_5 - \frac{f}{\mu}\gamma_5\hat{k}^0\right)\right\}\tau_0 u_{\tilde{p}};$$

$$R_{if}^{(3)} = iA\sum_{k';k=+,-}\left(\bar{u}_{p1}\tau_k\left\{ig\gamma_5 - \frac{f}{\mu}\gamma_5\hat{k}^\pm\right\}[\hat{P}_{p0} + \hat{k} - M]^{-1} \times \left\{ig\gamma_5 + \frac{f}{\mu}\gamma_5\hat{k}\right\}\tau_- u_{p0}\right)[k^{\pm 2} - \mu^2]^{-1}\left(\bar{u}_{n1}\left\{ig\gamma_5 + \frac{f}{\mu}\hat{k}^\pm\right\}\tau_{k'} u_{\tilde{p}}\right);$$

$$R_{if}^{(4)} = -iA\left(\bar{u}_{p1}\left\{ig\gamma_5 + \frac{f}{\mu}\gamma_5\hat{k}^{01}\right\}\tau_0 u_{\tilde{p}}\right)[k^{02} - \mu^2]^{-1} \times \left(\bar{u}_{n1}\tau_0\left\{\left(ig\gamma_5 - \frac{f}{\mu}\gamma_5\hat{k}^{01}\right)[\hat{P}_{p0} + \hat{k} - M]^{-1}\left(ig\gamma_5 + \frac{f}{\mu}\gamma_5\hat{k}\right)\right\}\tau_- u_{p0}\right);$$

$$R_{if}^{(5)} = -iA\left(\bar{u}_{p1}\left\{ig\gamma_5 + \frac{f}{\mu}\gamma_5\hat{k}^{01}\right\}\tau_0 u_{\tilde{p}}\right)[k^{012} - \mu^2]^{-1} \times \left(\bar{u}_{n1}\tau_-\left\{\left(ig\gamma_5 + \frac{f}{\mu}\gamma_5\hat{k}\right)[\hat{P}_{n1} - \hat{k} - M]^{-1}\left(ig\gamma_5 - \frac{f}{\mu}\gamma_5\hat{k}^{01}\right)\right\}\tau_0 u_{p0}\right);$$

$$R_{if}^{(6)} = -iA\sum_{k:k=+,-}\left(\bar{u}_{p1}\tau_k\left\{ig\gamma_5 - \frac{f}{\mu}\gamma_5\hat{k}^{\pm 1}\right\}[-\hat{P}_\sim + \hat{k} - M]^{-1} \times \left\{ig\gamma_5 + \frac{f}{\mu}\gamma_5\hat{k}\right\}\tau_- u_{\tilde{p}}\right)[k^{\pm 12} - \mu^2]^{-1}\left(\bar{u}_{n1}\left\{ig\gamma_5 + \frac{f}{\mu}\gamma_5\hat{k}^{\pm 1}\right\}\tau_{k'} u_{p0}\right).$$

Here $(2\pi)^{-6}A = (2/\pi E_\mu)^{1/2}$; u_{p1}, u_{p0}, u_{n1}, $u_{\tilde{p}}$ are bispinors corresponding to the final and initial states of the recoil proton and to the states of the neutron-antiproton pair; $\hat{P}_{p1}$, $\hat{P}_{p0}$, $\hat{P}_{n1}$, $\hat{P}_{\tilde{p}}$ are the operators of the energy-momentum vectors of the particles in the states indicated by the subscript; $\hat{k}_1$, $\hat{k}_1^0$; $\hat{k}_1^{01}$, $\hat{k}_1^\pm$, $\hat{k}^{\pm 1}$ are the energy-momentum vectors of the real and virtual mesons (both neutral and charged).

By using the Dirac equation for the nucleons in their initial and final states:

$$(\hat{P}_{p0} - M)u_{p0} = 0; (-\hat{P}_{\tilde{p}} - M)u_{\tilde{p}} = 0; \qquad (I.9)$$
$$\bar{u}_{n1}(\hat{P}_{n1} - M) = 0; \quad \bar{u}_{p1}(\hat{P}_{p1} - M) = 0,$$

and remembering the relations $\gamma_5\gamma_\nu = -\gamma_\nu\gamma_5$; $\gamma_5^2 = -1$; $\hat{a}\hat{b} + \hat{b}\hat{a} = 2(\overline{ab})$, it is easy to simplify the expression (I.8). We then form the square modulus of the matrix element and note that

$$\hat{P}^2 = M^2; \quad \hat{k}^2 = \mu^2; \quad \bar{\gamma}_5 = \gamma_4\overset{+}{\gamma_5}\gamma_4 = -\gamma_5,$$

We then obtain for $1/2\sum_{i}^{s}\sum_{f}^{s}|R_{if}|^2$ a very complicated experssion, which is valid in any coordinate system.

In the region of interest to us, in which the final particle energies are small, this expression takes a particularly simple form in the center-of-mass system in which the energy-momentum conservation equation $\bar{k} + \bar{P}_{p0} = \bar{P}_{n1} + \bar{P}_{\tilde{p}}$ becomes, in three-dimensional notation

$$\sqrt{k^2+\mu^2}+\sqrt{k^2+M^2} \qquad (I.10)$$
$$= E_{n1}+E_{p1}+E_{\tilde{p}};$$
$$0 = \mathbf{P}_{n1}+\mathbf{P}_{p1}+\mathbf{P}_{\tilde{p}}.$$

Then we have near the threshold, where

$$|\mathbf{P}_{n1}| \approx |\mathbf{P}_{p1}| \approx |\mathbf{P}_{\tilde{p}}| \ll M$$

for *PS-PS* theory

$$\frac{1}{2}\sum_i^s\sum_f^s |R_{if}|^2 \approx 6.12\, g^6 \frac{\pi^3}{M^7}(2\pi)^8$$

and for *PS-PV* theory

$$\frac{1}{2}\sum_i^s\sum_f^s |R_{if}|^2 \approx 19.2\, f^6 \frac{\pi^3}{M^7}\left(\frac{M}{\mu}\right)^6 (2\pi)^8.$$

Thus, to the approximation used here, the differential cross section of the process depends on the momenta and directions of the final particles only through the element of phase space of the system in its final state:

$$(2\pi)^{-9}\, d\mathbf{P}_{n1}\, d\mathbf{P}_{p1}\, d\mathbf{P}_{\tilde{p}}\, \delta(\Delta E)\, \delta(\Delta \mathbf{P}).$$

The cross section for the production of an antiproton in a π-meson-proton collision becomes, in terms of the laboratory energy:

$$\sigma_1 = K_1 (E_\mu - 4M)^2 / M^2 c\mathcal{M}^2, \qquad (I.11)$$

where $K_1 = 6.2 \times 10^{-30} g^6$ for *PS-PS*, and $K_1 = 2 \times 10^{-24} f^6$ for *PS-PV* theory.

In a similar manner we find for the cross section for antiproton production in a π^+-neutron collision*

$$\sigma_2 = K_2 (E_\mu - 4M)^2/M^2\ c\mathcal{M}^2, \qquad (I.12)$$

where $K_2 = 1.5 \times 10^{-30} g^6$ for *PS-PS*, and $K_2 = 1.2 \times 10^{-24} f^6$ for *PS-PV* theory.

Turning now to the antiproton production by a negative π-meson on a neutron

$$\pi^- + n \rightarrow n + n + \tilde{p},$$

we have in the first nonvanishing approximation six Feynman graphs. Without writing down the corresponding matrix elements we note as the distinguishing feature of this process that the modulus of the matrix element vanishes if we neglect the momenta in the final state. We therefore go to the next approximation, including terms of the order $|\mathbf{P}|^2/M^2$, and find for the total cross section

$$\sigma_3 = K_3 (E_\mu - 4M)^3 / M^3 c\mathcal{M}^2, \qquad (I.13)$$

where $K_3 = 2.5 \times 10^{-34} g^6$ for *PS-PS* theory.

2. Calculation of the antiproton production cross section with allowance for the interactions between particles in their final states.

As already mentioned, the upper limit of validity of the approximation used above comes from the requirement that the energies of the final particles be small compared to the excitation energy of the first isobaric state. For antiproton production processes in π-meson-nucleon collisions this gives the condition (in the center-of-mass system):

$$E - 3M \ll \Delta \approx 2\mu \approx 300 \text{ mev} \qquad (I.14)$$

or, in other words, the share of the energy belonging to each of the final particles must satisfy $\epsilon \ll 100$ mev. For the energy of the incident π-meson we have (in the laboratory system)

$$E_\mu - 4M \ll 3\mu \approx 450 \text{ mev}.$$

Thus, the approximation used above is valid at best if the energies of the final particles do not exceed 20-30 mev (in the center-of-mass system) and this is of the same order of magnitude as the depth of the potential well of the nuclear forces. Therefore, the expressions (I.11) to (I.13) which we obtained by neglecting the interactions between the particles in the final state, can be valid only in a narrow interval near the upper limit, since in the rest of the threshold region it is essential to allow for the interactions.

It is well known that the transition matrix element including the interaction between the created particles can be expressed as

$$R'_{if} \approx R_{if}\Phi(0), \qquad (I.15)$$

where $\Phi(r)$ is the solution of the Schrödinger equation in the potential field of the interaction between the two components of the pair.

As regards the interaction between the created particles, which interact via the pseudoscalar meson field, we note that the sign of the coupling constant with this field is the same for nucleons and antinucleons*[8]. Hence, one may assume that, at any rate at large distances of the order of the

* It is easy to see that in this case, to the approximation considered here, there are only two Feynman graphs.

* We are informed by Markov that the contrary statement in his paper[8] is the result of an error.

π-mesonic forces, the nucleon-antinucleon force is similar to the nucleon-nucleon force. One may, therefore, for a crude estimate of the effect of the final-state interactions, use the phenomenological potential

$$V(r) = V_0 e^{-kr} / (1 - e^{-kr})$$

$$V_0^{\text{trip}} = 46.6 \text{ mev}; \quad V_0^{\text{sing}} = 27.2 \text{ mev}; \quad k = \mu_\pi,$$

which fits the low-energy nucleon scattering. If the pair creation occurs in the continuous spectrum we have

$$\Phi(r) = Ar^{-1} e^{-(b-1)kr/2} (1 - e^{-kr}), \qquad (I.16)$$

where

$$A = \frac{1}{k} \left[\frac{\pi b \sin h\, 2\pi\alpha}{\alpha(\cos h\, 2\pi\alpha - \cos 2\pi \sqrt{(b-\alpha^2)})} \right]^{1/2},$$

and where $b = MV_0 / k^2$; $\alpha = q/k$; q is the relative momentum of the components of the pair.

At low energies,

$$\Phi_s^2(0) \approx (qdq / 2\pi)\, bk \qquad (I.17)$$

and we find for the cross section for the production of an antiproton by a π^--meson on a proton

$$\sigma_1 = K_1' \left(\frac{E_\mu - 4M}{M} \right)^{3/2} cM^2, \qquad (I.18)$$

where $K_1' = 1.5 \times 10^{-29} g^6$ for PS-PS, and $K_1' = 4.8 \times 10^{-25} f^6$ for PS-PV theory.

Similarly, we find for the production cross section by a π^+-meson on a neutron

$$\sigma_2 = K_1' \left(\frac{E_\mu - 4M}{M} \right)^{3/2} cM^2, \qquad (I.19)$$

where $K_2' = 3.5 \times 10^{-30} g^6$ for PS-PS and $K_2' = 2.4 \times 10^{-25} f^6$ for PS-PV theory.

In the limiting case of low relative momenta of the created particles there exists not only the possibility of a deuteron being formed, but also of the antiproton being captured in a deuteron-like "orbit" around a nucleon. The cross section for this process can be estimated by taking as the wave function for this deuteron-like state the wave function of the bound S state in the Hulthén potential,

$$\Phi(r) = \left[\frac{kb(b^2 - 1)}{8\pi} \right]^{1/2} r^{-1} e^{-(b-1)kr/2} (1 - e^{-kr}).$$

The cross section for the formation of a bound pair by a π-meson on a proton is

$$\sigma_1 = K_1'' \left(\frac{E_\mu - 4M)}{M} \right)^{1/2} cM^2, \qquad (I.20)$$

where $K_1'' = 5.3 \times 10^{-30} g^6$ for PS-PS, and $K_1'' = 1.1 \times 10^{-24} f^6$ for PS-PV theory.

The cross section for the formation of a bound proton-antiproton pair by a π^+-meson on a neutron is

$$\sigma_2 = K_2'' \left(\frac{E_\mu - 4M}{M} \right)^{1/2} cM, \qquad (I.21)$$

where $K_2'' = 1.8 \times 10^{-30} g^6$ for PS-PS and $K_2'' = 5.4 \times 10^{-25} f^6$ for PS-PV theory.

In general, the ratio of the cross sections for pair creation in the bound state and in the continuous spectrum is equal to

$$\sigma_{\text{bound}} / \sigma_{\text{cont}} \approx \mu / (E_\mu - 4M).$$

It follows from this that if $E_\mu - 4M < \mu \approx 150$ mev, when the final kinetic energy per particle in the center-of-mass system is considerably less than the depth of the potential well of the nuclear forces, the process of formation in the bound state becomes dominant. Here in the case of process a) the probability of the creation of an antiproton in the bound state and that for the creation of a free antiproton and a deuteron are in the ratio 2:1. For processes b) and c), on the other hand, capture into bound states can arise only in the nucleon-antiproton system, because of the instability of the di-neutron and di-proton. This possibility of capture of the antiproton by a nucleon must, near the threshold, lead to a reduction of the number of observable antiprotons, in view of the practically instantaneous annihilation of a bound antiproton into π-mesons. For example, the mean life of an antiproton and a deuteron-like system due to the annihilation into two charged π-mesons is about $\tau = 10^{-21}$ sec if one assumes pseudoscalar coupling with $g^2 = 1/5$ (see Ref. 9 and part II of this paper).

By comparing these results with experiment, one may, in case the antiproton is discovered, establish the actual energy and angular dependence of the square modulus of the matrix element. Moreover, as we have indicated, a study of the energy dependence of the creation cross sections near the threshold may, to some extent, clarify the nature of the interactions in the nucleon-antiproton system and, in particular, answer the question as to the existence of a bound state in this system. We have above considered only the contribution of π-mesons to the nuclear forces. It is easy to see, however, that the pair creation processes depend to a substantial extent on the interaction at distances of the order of $\hbar / Mc$, where the fields of heavier mesons may contribute substantially.

II. PRODUCTION OF ANTIPROTONS IN NUCLEON-NUCLEON COLLISIONS

1. In addition to the processes involving the creation of nucleon-antiproton pairs in the collision of π-mesons with nucleons, there must also exist processes involving the creation of antiprotons by the virtual π-mesons in the field of the nucleons, The threshold (minimum) value of the total energy for the creation of an antiproton in a nucleon-nucleon collision is $W_0 = 8M$, corresponding to a kinetic energy of the incident nucleon (in the laboratory system) of $T_1 = 6M = 5.6$ bev. Such creation processes are

a) $p+p \rightarrow p+p+p+\tilde{p}$;

b) $n+p \rightarrow n+p+p+\tilde{p}$;

c) $n+n \rightarrow n+n+p+\tilde{p}$.

Since calculations of the cross section for processes a) and b) have already been published[10,11], we shall limit ourselves to the presentation of similar calculations for process c). It must here be pointed out that in these pspers[10,11] calculations of the creation cross sections near the threshold were carried out without allowing for the interaction between the created particles, so that these authors overlooked the possibility of creation in a bound state, which, in this region, is very important.

The differential cross section for the creation of an antiproton in a nucleon-nucleon collision is

$$d\sigma = \frac{1}{4J_0}\sum_i^s\sum_f^s |R_{if}|^2 \rho_f (2\pi)^{-4}\delta(\Delta E)\,\delta(\Delta P), \quad \text{(II.1)}$$

where, if the created particles are free,

$$\rho_f = (2\pi)^{-12} d\mathbf{P}_{N1}\, d\mathbf{P}_{N2}\, d\mathbf{P}_{N3}\, d\mathbf{P}_{\tilde{p}}.$$

The square modulus of the transition matrix element R_{if} is averaged over the spin directions in the initial, and summed over the spin directions in the final state.

For the processes a), b) and c) the first non-vanishing term in the expansion of the matrix element R_{if} in a perturbation series is of the fourth order. The leading term for process a) contains 36 Feynman graphs, whereas for b) and c) there are 28 graphs each. To this order effect a) depends exclusively on the interaction of the nucleons with the neutral meson field, whereas processes b) and c) involve both the neutral and the charged field. Half of the quoted number of graphs arises in each case from the antisymmetrization of the initial and final state functions of the system with repsect to the neutron and proton variables. This antisymmetrization is equivalent to allowing for exchange, and reduces the cross sections for processes b) and c) in the threshold region by about 50%. For process a) the exchange effect gives an additional limitation on the final states of the system, since the Pauli principle rules out the possibility of finding three protons in the same final S state. Without writing out the transition matrix elements in full, we give the value of the square modulus of the matrix element for process c), estimated near threshold for pseudoscalar coupling:

$$\frac{1}{4}\sum_i^s\sum_f^s |R_{if}|^2 \approx 1.6\times 10^{-2}(2\pi)^{12}\frac{g^8}{M^{10}} \quad \text{(II.2)}$$

Thus, the energy dependence of the cross section near the threshold is given by the volume of phase space accessible in the final state

$$\Omega = (2\pi)^{-12}\iiiint d\mathbf{P}_{N1}d\mathbf{P}_{N2}d\mathbf{P}_{N3}d\mathbf{P}_{\tilde{p}}\,\delta(\Delta E)\,\delta(\Delta P) = \frac{1}{420}\frac{M^8}{(2\pi)^8}\left(\frac{E-7M}{M}\right)^{7/2}. \quad \text{(II.3)}$$

The total cross section for antiproton creation in a neutron-neutron collision is, in the laboratory system, without allowing for the interactions between the created particles:

$$\sigma_3 = L_3\left(\frac{E-7M}{M}\right)^{7/2} cM^2, \quad \text{(II.4)}$$

where $L_3 = 4.2\times 10^{-34} g^8$ for PS-PS, and $L_3 = 3.6\times 10^{-28} f^8$ for PS-PV theory.

The upper limit of the threshold region follows from the condition (cf. part I)

$$E-2M \ll {}^1/_2\Delta = 150 \text{ mev} \quad \text{(II.5)}$$

in the center-of-mass system, or $E-7M \ll 2\Delta = 600$ mev in the laboratory system. It follows from (II.5) that the average kinetic energy for each final particle is not large compared to the depth of the potential well of the nuclear forces, and hence over most of the threshold region the interactions in the final state must be essential. Consider first the creation of interacting pairs of particles in the continuous spectrum. Then, by a similar method to that used in part I, we estimate for the cross section of process c) including the effect of the interaction between the particles of the pair:

$$\sigma_3' = L_3'\left(\frac{E-7M}{M}\right)^{5/2} cM^2, \quad \text{(II.6)}$$

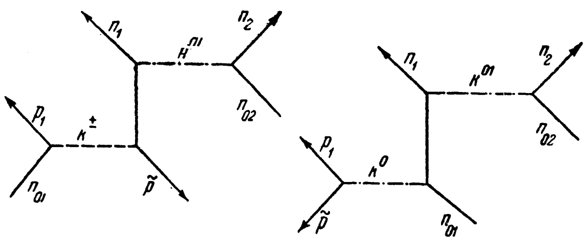

FIG. 2. Two types of Feynman graphs corresponding to process c) in the first nonvanishing order. Full lines represent nucleons, dot-dashed lines neutral π-mesons and broken lines charged π-mesons.

where $L_3' = 4.3 \times 10^{-32} g^8$ for *PS-PS*, and $L_3' = 2.8 \times 10^{-26} f^8$ for *PS-PV* theory.

In nucleon-nucleon collisions at energies close to the threshold we may find as one of the reaction products of process c) a deuteron, $n + n \to d^+ + \tilde{p} + n$. Equally, the possibility of a deuteron-like nucleon-antiproton system is not excluded, e.g., $n + n \to d^- + p + n$. The cross section for process c) with one bound pair of particles (of deuteron type) in the final state is

$$\sigma_3'' = L_3'' \left(\frac{E-7M}{M}\right)^2 cM^2, \qquad \text{(II.7)}$$

where $L_3'' = 1.8 \times 10^{-33} g^8$ for *PS-PS*, and $L_3'' = 4 \times 10^{-29} f^8$ for *PS-PV* theory.

Turning finally to the possibility of having in the final state two bound pairs of particles, $n + n \to d^+ + d^-$, we find for the total cross section of this effect

$$\sigma_3''' = L_3''' \left(\frac{E-7M}{M}\right)^{1/2} cM^2, \qquad \text{(II.8)}$$

where $L_3''' = 7 \times 10^{-34} g^8$ for *PS-PS*, and $L_3''' = 3.7 \times 10^{-27} f^8$ for *PS-PV* theory.

3. As was already mentioned, the production of antiprotons in bound deuteron-like states cannot be neglected in the energy region near the threshold. Such systems must, however, have an extremely short mean life with respect to the annihilation into π-mesons.

For example, the mean life of a bound deuteron-like nucleon-antiproton system, with respect to the decay into two π-mesons, is

$$\tau = 3V_0/4(\sigma_\pi v_{12})_{v_{12}\to 0}, \qquad \text{(II.9)}$$

where σ_π is the total cross section for the annihilation of a free nucleon-antiproton pair into two π-mesons, v_{12} is the relative velocity of the particles before annihilation; $V_0 \approx |\Psi(0)|^{-2}$ is the annihilation volume of the bound system. For the case $n + \tilde{p} \to \pi^- + \pi^0$ we have

$$(\sigma_\pi v_{12})_{v_{12}\to 0} = (\pi/2)(g^2/M)^2 \text{ for } PS\text{-}PS; \qquad \text{(II.10)}$$

$$(\sigma_\pi v_{12})_{v_{12}\to 0} = 2\pi (f^2/M)^2 (M/\mu)^4 \text{ for } PS\text{-}PV.$$

For the case $p + \tilde{p} \to \pi^+ + \pi^-$

$$(\sigma_\pi v_{12})_{v_{12}\to 0} = (\pi/4)(g^2/M)^2 \text{ for } PS\text{-}PS \text{ and}$$

$$(\sigma_\pi v_{12})_{v_{12}\to 0} = \pi (f^2/M)^2 (M/\mu)^4 \text{ for } PS\text{-}PV.$$

Inserting (II.10) and (II.11) in (II.9), we find:

1) The lifetime of a bound deuteron-like neutron-antiproton system with respect to the decay $d^- \to \pi^- + \pi^0$ is

$\tau = 3.2 \times 10^{-26} g^{-4}$ sec for *PS-PS*, and

$\tau = 6 \times 10^{-26} f^{-4}$ sec for *PS-PV* theory.

2) The lifetime of a bound deuteron-like proton-antiproton system with respect to the decay $d^0 \to \pi^- + \pi^+$ is

$\tau = 6.5 \times 10^{-22} g^{-4}$ sec for *PS-PS*, and

$\tau = 1.1 \times 10^{-26} f^{-4}$ sec for *PS-PV* theory.

Thus, antiproton creation processes at energies near the threshold may in many cases manifest themselves as the creation of pairs of correlated pairs of π-mesons*. It is then easy to see that the angle between the directions of emission of the two π-mesons in the center-of-mass system of the two original nucleons will tend to 180° at the threshold.

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* Multiple creation of mesons is also not excluded.

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148

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The Conductivity of Silver Bromide in the Presence of Bromine

L. M. SHAMOVSKII, A. A. DUNINA AND M. I. GOSTEVA
All-Union Scientific Research Institute for Mineral Resources
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It was shown that silver bromide crystals, additively colored in bromine fumes, acquire hole conductivity, the magnitude of which is proportional to the square root of the partial pressure of halogen in the gas and rises with increase in temperature according to an exponential law. The magnitude of the hole conductivity in AgBr even for low pressures of bromine is comparable in value with the ionic conductivity of these crystals. In additive coloration, V_1 centers are formed. These are complex, consisting of a single vacant cation site with a hole localized on it. The thermal dissociation energy of V_1 centers in AgBr is estimated to be 0.3 ev. F-centers are not formed in silver bromide on account of lack of vacant anion sites in its structure. Neither atoms nor molecules of bromine dissolve and diffuse in the AgBr lattice.

1. INTRODUCTION

IN recent years, a large number of studies have been published concerned with clarification of the nature of the structural defects in the silver halides. On the basis of theoretical calculations[1] and experimental results[2-5] it has been established that in these crystals, Frenkel-type structural defects are formed. Seitz[6] and Mitchell[7], however, supposed that Schottky defects can exist as well in small concentrations. Such an interpretation was occasioned by the need to explain the mechanism of formation of the latent photographic image, and also the mode of development of silver dendrites in single crystals of AgBr in the process of electrolysis[8]. It was also assured that sensitizers, such as Ag_2S, aid in the formation of vacant anion sites in the crystal lattice. Hypotheses have also been expressed on the diffusion of anion vacancies pairwise with cation vacancies. Such groups do not have to reveal themselves in measurements of electrical conductivity and of transference numbers, despite the motion of anions. Such a hypothesis, however, is refuted by experimental data on self-diffusion, measured by means of radioactive tracers[9-11]. In particular, according to the data of Murina and his co-workers[10,11], the transference numbers for bromine ions in AgBr do not exceed 5.2×10^{-4} -- a value falling within the range of experimental errors*.

In subsequent works, Mitchell abandoned his initial point of view. Together with Hedges[11] he established that the internal latent image in silver bromide is formed on internal surfaces associated with the polyhedral structure of monocrystals, connected with the development of dislocations (where boundaries mesh) or mosaic blocks, owing to deformation under the influence of mechanical forces or temperature variations. Prolonged annealing of pure monocrystals near the melting point and slow cooling improved their structure, and in such specimens no latent image developed inside the crystal.

Meikliar and Putseiko[11] consider that it is unnecessary to hypothesize Schottky defects for the formation of F-centers in silver halides. As is known, on exposure of a crystal to ultraviolet light, electrons and holes arise. The latter are considered by the authors as neutral halogen atoms on lattice sites. It is assumed that in the process of photolysis, bromine atoms can pass into inter-

* Regrettably, these investigations were carried out on polycrystals. They should be repeated on monocrystals of a high degree of purity, annealed at high temperatures for elimination of their polyhedral structure.