Explanation of the Transition Effect for "Stars" in the Stratosphere

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An explanation based on cascade multiplication of nucleons is given for the transition curves of nuclear "stars". It is shown that significant transition effects for stars should be observed at heights exceeding 15 km above sea level even when heavy element absorbers are used; this prediction is in agreement with experimental results. In the case of absorbers consisting of heavy elements, the neutron excess in the nucleus must be taken into account.

THE basic experimental data, obtained (the first results were published in the papers of Ref. 1) from investigations of transition effects for nuclear "stars" during the period from 1950 to 1954, lead to the following conclusions:

1. The transition effect for stars with three or more prongs (i.e., the increase in number of stars in an emulsion exposed under an absorber above the number of the same type of star in an emulsion exposed in air without absorber) even for π-mesons depends essentially on the atomic number of the absorber and on the altitude above sea level at which the experiment is done.

2. In the experiments on transition effects for stars, lead, graphite and paraffin absorbers were used down to thicknesses of 10, 30 and 50 cm, respectively. In all experiments at altitudes of 22-29 km, we observed an increase in the number of stars in photographic plates under lead (compared with the number of stars in plates without absorber) by 30-50% (depending on the altitude of observation, as well as on the thickness and the arrangement of the lead blocks).

3. In the case of light element absorbers (graphite and paraffin), at an altitude of 22 km no significant increase was observed in the number of stars in plates under absorber compared with the number of stars in plates exposed in air, without absorber. Only at an altitude of $\sim$27-28 km did we observe an increase in number of stars (by approximately 10-20%) upon entering a graphite absorber 30 cm in thickness.

4. It was shown experimentally that the transition effect both in heavy element absorbers (lead) and in absorbers consisting of light elements (graphite and paraffin) is due mainly to stars with 3-7 prongs. This indicates that a considerable number of star-producing particles of low energy (from 60 to 500 mev) are generated in the absorbers.

5. The altitude dependence of number of stars in the range 20-27 km was explained.

6. We obtained the altitude dependence of slow π-mesons stopping in emulsions. The transition effect for π-mesons in lead, graphite, paraffin and copper absorbers was explained. We explained the energy spectrum of slow π-mesons in air, and the spectrum of π-mesons generated in the various absorbers.

While omitting all the details of the various experiments and of the processing of plates exposed in the stratosphere, we shall here try to explain the experimental results, which have been briefly cited in points 1-5, starting from the general picture of the passage of the nucleonic component of cosmic rays through matter.

We shall assume that the investigation of star transition effects is carried out at some definite depth ϑ_B in the atmosphere. We set ourselves the problem of finding the total number N of particles with energy greater than E, at depth ϑ_A in the absorber, which are capable of producing stars. We assume, in accordance with accepted notions, that the star-producing particles are neutrons, protons and π-mesons of sufficient energy. Consequently, each of the particles which enters the absorber, can produce a definite number of neutrons, protrons and π-mesons via cascade multiplication in the course of reaching the depth ϑ_A. We shall denote by $N_{1n}(E_n, E, \vartheta_A)$, $N_{1p}(E_n, E, \vartheta_A)$, and $N_{1\pi}(E_n, E, \vartheta_A)$, respectively, the numbers of neutrons, protons and π-mesons with energy greater than E at depth ϑ_A, due to a single neutron which enters the absorber from air, with energy E_n. Then

$$N_1(E, \vartheta_A) = \sum_{i,k} N_{1i}(E_k, E, \vartheta_A)$$

denotes the total number of particles of all three types, with energy greater than E at depth ϑ_A, due to a neutron, proton and π-meson incident on the surface of the absorber with energies E_n, E_p and E_π, respectively. The total number of all such particles, due to neutrons, protons and π-mesons with energies greater, respectively, than E_n, E_p and E_π, will be

$$N(E,\ \vartheta_A,\ E_n,\ E_p,\ E_\pi,\ \vartheta_B) \quad (1)$$

$$= \sum_{i,k} \int_{E_k}^{\infty} N_{1i}(E'_k,\ E,\ \vartheta_A)\, K(E'_k,\ \vartheta_B)\, dE'_k,$$

where the subscripts i and k take on three values for neutron, proton and π-meson, while $K(E'_k, \vartheta_B)\, dE'_k$ denote the differential spectrum of the particles at depth ϑ_B.

Treatment of the problem in this general form requires the solution of a system of three integro-differential equations (including energy loss by ionization). Janossy, Messel and others[2], using the method of Bhabha and Chakrabarty[3], have solved the equation, not including mesons, i.e., they treated the nucleon cascade in the atmosphere. They assumed that the effective nucleon-nucleon interaction cross section is a uniform function of the energy of the particles after collision. Our initial attempt[1] at explaining the transition effects for stars was phenomenological in character, since the energy spectrum of the particles was essentially taken from experimental data. In the present paper we present a general scheme for solving the problem of transition curves, based on a treatment of the cascade multiplication of a flux of nucleons passing through matter, using the methods developed in Ref. 2. Since our experiments as well as those of other observers show that the number of stars due to π-mesons is small, the calculation will be made only for the nucleonic component.

If we neglect ionization losses of the proton, then neutrons and protons will behave in the same way, and the number of nucleons (i.e., neutrons plus protons) with energy greater than E, at depth ϑ_A, due to a nucleon incident on the absorber surface with energy greater than E_0, will be

$$N_1(E_0,\ E,\ \vartheta_A) \quad (2)$$

$$= \frac{1}{2\pi i} \int_{s'_0 - i\infty}^{s'_0 + i\infty} \left(\frac{E}{E_0}\right)^{-s'} \exp\{-\ \vartheta_A \cdot f(D_A \alpha_{s'})\}\, \frac{ds'}{s'},$$

where $f(x) = 1 - 2x^{-2}[1 - (1+x)e^{-x}]$, D_A is the number of collisions of the initial nucleon with nucleons in the nucleus, when it traverses the diameter of a nucleus of atomic weight A. The function α_s depends on the nature of the nucleon-nucleon interaction, i.e.,

$$\alpha_s = \int_0^1 \int_0^{1-\varepsilon_2} (1 - \varepsilon_1^s - \varepsilon_2^s)\, W(\varepsilon_1, \varepsilon_2)\, d\varepsilon_1 d\varepsilon_2 \quad (3)$$

$$\text{and } \epsilon_1 = \frac{E_1}{E_0},\quad \varepsilon_2 = \frac{E_2}{E_0},$$

where E_1 and E_2 are the energies, after collision, of the nucleons, one of which was at rest while the other had energy E_0 before the collision, $W(\epsilon_1, \epsilon_2)\, d\epsilon_1 d\epsilon_2$ is the probability that after the collision of the incident nucleon with energy E_0 and the nucleon at rest, they have energies E_1 and E_2, respectively.

We shall assume that $K(E_0, \vartheta_B)\, dE_0$ is the number of nucleons with energy in dE_0 at E_0 at the depth ϑ_B in the atmosphere. Then the number of nucleons with energy greater than E at a depth of absorber equal to ϑ_B, which were formed by initial particles with energies in dE_0 at E_0, is $N_1(E_0, \vartheta_A, E)\, K(E_0, \vartheta_B)\, dE_0$. The total number of nucleons with energy greater than E at depth ϑ_A, due to initial nucleons with energy greater than E_0, is given by

$$N(E_0,\ \vartheta_A,\ E,\ \vartheta_B) \quad (4)$$

$$= \int_{E_0}^{\infty} N_1(E_0,\ \vartheta_A,\ E)\, K(E_0, \vartheta_B)\, dE_0$$

$$= \frac{1}{2\pi i} \int_{E_0}^{\infty} \int_{s_0 - i\infty}^{s_0 + i\infty} K(E_0, \vartheta_B) \left(\frac{E}{E_0}\right)^{-s}$$

$$\times \exp\{-\vartheta_A f(D_A \alpha_s) \vartheta_A\}\, \frac{ds}{s}\, dE_0.$$

If we use the initial conditions

$$K(E_0,\ 0) = \begin{cases} \gamma E_c^\gamma / E_0^{\gamma+1} & \text{for } E_0 > E_c \\ 0 & \text{for } E_0 < E_c. \end{cases}$$

then

$$N(E_0, E_c, \vartheta_B) \tag{5}$$

$$= \frac{\gamma}{2\pi i}\int_{s_0-i\infty}^{s_0+i\infty}\left(\frac{E_0}{E_c}\right)^{-s} e^{-\vartheta_B f(D_B\alpha_s)}\frac{ds}{s(\gamma-s)}$$

$$= \int_{E_0}^{\infty} K(E_1, E_c, \vartheta_B)\, dE_1;$$

$$K(E_0, E_c, \vartheta_B) = -\frac{\partial N(E_0, E_c, \vartheta_B)}{\partial E_0} \tag{6}$$

$$= \frac{\gamma}{2\pi i}\int_{s_0-i\infty}^{s_0+i\infty}\left(\frac{E_0}{E_c}\right)^{-s} e^{-\vartheta_B f(D_B\alpha_s)}\frac{ds}{E_0(\gamma-s)}.$$

Then

$$N(E_0, \vartheta_A, E, E_c, \vartheta_B) \tag{7}$$

$$= \frac{\gamma}{4\pi^2 i^2}\int_{E_0}^{\infty}\int_{s_0-i\infty}^{s_0+i\infty}\int_{s'_0-i\infty}^{s'_0+i\infty}\frac{(E_0/E_c)^{-s}}{E_0(\gamma-s)}$$

$$\times\, e^{-\vartheta_B f(D_B\alpha_s)}\frac{(E/E_0)^{-s'}}{s'}e^{-\vartheta_A f(D_A\alpha_{s'})}\, ds\, dE_0\, ds',$$

where $s > s'$. After integrating with respect to E_0 and setting the lower limit equal to E, and using the relation

$$\frac{1}{2\pi i}\int_{s'_0-i\infty}^{s'_0+i\infty}\frac{e^{-\vartheta_A f(D_A\alpha_{s'})}}{s'(s-s)}\, ds' \tag{8}$$

$$= \frac{1}{s}\exp\{-\vartheta_A f(D_A\alpha_s)\},$$

we get

$$N = \frac{\gamma}{2\pi i}\int_{s_0-i\infty}^{s_0+i\infty}\frac{(E/E_c)^{-s}}{(\gamma-s)s} \tag{9}$$

$$\times \exp\{-\vartheta_B f(D_B\alpha_s) - \vartheta_A f(D_A\alpha_s)\}\, ds,$$

where $\gamma > s_0$. The location of the maximum of the transition curve is found from the conditions

$$\partial N(\vartheta_A, E, E_c, \vartheta_B)/\partial\vartheta_A = 0 \quad \text{or}$$

$$f(D_A\alpha_s) = 0, \text{ i.e., } \alpha_s = 0.$$

We calculate the values of the function N by the saddle point method:

$$N = \frac{\gamma}{2\pi i}\int_{s_0-i\infty}^{s_0+i\infty} e^{S(s)}ds \tag{10}$$

$$= \gamma(2\pi)^{-1/2}[-S''(s_0)]^{-1/2}e^{-S(s_0)},$$

where

$$-S(s) = -s\ln\varepsilon \tag{11}$$

$$-\ln s(\gamma-s) - \vartheta_B f(D_B\alpha_s) - \vartheta_A f(D_A\alpha_s),$$

$$-S'(s) = -\frac{\partial S}{\partial s}\bigg|_{s=s_0} = -\ln\varepsilon - \frac{\gamma-2s_0}{s_0(\gamma-s_0)}$$

$$-\vartheta_B f'(D_B\alpha_s) - \vartheta_A f'(D_A\alpha_s) = 0.$$

Then

$$\vartheta_A\big|_{\max} = \frac{-\ln\varepsilon - \dfrac{\gamma-2s_0}{s_0(\gamma-s_0)} - \vartheta_B\dfrac{2}{3}\alpha'_{s_0}D_B}{{}^2/_3\,\alpha'_{s_0}D_A}, \tag{12}$$

where $\alpha' = (\partial\alpha_s/\partial S)_{s=s_0}$. Using $E = 50$ mev, $E_c = 2\times10^3$ mev, $\gamma \approx 1.3$, $D_{Pb} = 8.7$, $D_B \approx 3.6$, and the α_s from the most recent work of Messel, we obtained

$$\vartheta_{Pb} = 0.578 - 0.414\,\vartheta_B, \tag{13}$$

where ϑ_B is measure in units of 65 gm/cm^2, and ϑ_{Pb} in units of 160 gm/cm^2. The dependence of $X_{Pb} \approx 160\,\vartheta'_{Pb}$ on X_B is shown in Fig. 1.

One is not able experimentally to establish accurately the position of the maximum of the transition curve. Despite this, the general behavior of the curve and its dependence on the altitude of observation, as obtained from examination of plates exposed in the stratosphere at various altitudes under absorbers (of lead, graphite and paraffin) of different thicknesses, lead to the following conclusions: 1) the maximum transition effect is observed at the highest altitude (27-29 km); 2) the effect decreases with decreasing altitude, and below 15 km is not observed (within the limits of experimental error);

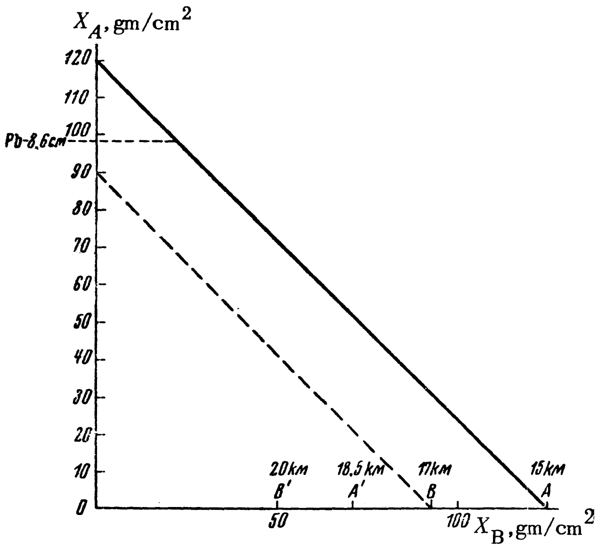

FIG. 1. Dependence of the position X_A on the maximum of the transition curve for stars on depth X_B in the atmosphere.

3) the position of the maximum apparently also depends on altitude.

The experimental data mentioned are entirely in accord with the general theoretical picture of the passage of the nucleonic component through matter. In fact, from the graph shown in Fig. 1, it follows that below 15 km there should be no transition effect (i.e., no increase in number of stars) under a lead absorber, and in addition, the position of the maximum should depend on the altitude of observation, which is also in accord with our data obtained from flights during 1953; according to Fig. 1, the position of the maximum shifts to smaller thicknesses with increasing depth in the atmosphere.

We should mention that the general conclusions are not essentially changed if, instead of $W(\epsilon_1\epsilon_2) = 15\,(1-\epsilon_1)\,\epsilon_2^2$ (the solid line), we use $W(\epsilon_1\epsilon_2) = 120\,\epsilon_1\epsilon_2\,(1-\epsilon_1-\epsilon_2)$ (the dotted line). Therefore, these conclusions are probably also changed only slightly for the actual W, which cannot be predicted at present.

We should point out that the points A and B are shifted to the left (to points A' and B') if we consider the total intensity of nucleons by using Gross's transformation.

The calculations carried out above explain the altitude dependence of the transition effect for stars, as well as the dependence of the maximum of the transition curve on the altitude of observation, but they do not explain the magnitude of the transition effect (this can be shown from estimates using the saddle point method). For example, the transition effect for a graphite absorber is calculated to be somewhat greater (5%) than for a lead absorber, whereas experiment gives the opposite result (cf. the beginning of the paper).

It appears that to explain the magnitude of the transition effect for a lead absorber we must take into account the neutron excess in the nucleus of a heavy element (lead) as compared to nuclei of light elements where the numbers of neutrons and protons are the same; the point is that the majority of small stars, for which the transition effect is a maximum, are formed precisely by neutrons. To get this result, we solve the equations

$$\frac{\partial S^{ik}_{(E,\,\theta)}}{\partial\theta} + S^{ik}_{(E,\,\theta)} = F(A,k) \tag{14}$$

$$\times \int_0^\infty \{S^{i,k}_{(E',\,\theta)} + S^{i,3-k}_{(E',\,\theta)}\}\, V\left(\frac{E}{E'}\right)\frac{dE'}{E'} + \delta_{ik}\beta\frac{\partial}{\partial E} S^{i,k}_{(E,\,\theta)},$$

which reduce to the usual equations of cascade multiplication of nucleons passing through the atomsphere if we set $F(A,k) = \frac{1}{2}$ (cf. Messel[3]), since for air (and in general for light elements),

$$F(A,k) = Z/A \approx (A-Z)/A = {}^1/_2.$$

In the case of passage of nucleons through a heavy element absorber:

$$k = 1:\ \frac{\partial S^{i,1}_{(E,\,\theta)}}{\partial\theta} + S^{i,1}_{(E,\,\theta)} \tag{15}$$

$$= \frac{Z}{A}\int_0^\infty \{S^{i,1}_{(E',\theta)} + S^{i,2}_{(E',\theta)}\}\, V\left(\frac{E}{E'}\right)\frac{dE'}{E'} + \beta\frac{\partial}{\partial E} S^{i,1}_{(E,\,\theta)},$$

$$k = 2:\ \frac{\partial S^{i,2}_{(E,\,\theta)}}{\partial\theta} + S^{i,2}_{(E,\,\theta)}$$

$$= \frac{A-Z}{A}\int_0^\infty \{S^{i,2}_{(E',\,\theta)} + S^{i,1}_{(E',\,\theta)}\}\, V\left(\frac{E}{E'}\right)\frac{dE'}{E'},$$

where i refers to the initial nucleon ($i = 1$ for a proton, $i = 2$ for a neutron). $S^{i,k}_{(E,\theta)}$ is the average number of protons or neutrons (depending on the value of the index k) in dE at energy

E, at depth ϑ in a lead absorber, due to an initial particle incident on the surface of the absorber with energy greater than E_0. Then

$$N^{i,k}(E_0, E, \vartheta_A) \approx \frac{1}{2\pi i} \tag{16}$$

$$\times \int_{s_0'-i\infty}^{s_0'+i\infty} \left(\frac{E_0}{\beta_A}\right)^{s'} \left(\frac{\beta}{E+\beta_A g^{i,k}_{(s'+1,\,\theta A)}}\right)^{s'} f_0^{i,k}(s'+1, \theta_A)\frac{ds'}{s'},$$

where

$$f_0^{i,k}(s'+1, \theta_A) = \frac{\delta_{ik}}{V_{s'}}[(V_{s'}-1$$

$$+ A_{3-k}(s')\,e^{-\theta(1-V_{s'})} - (A_{3-k}(s')-1)\,e^{-\theta}]$$

$$-\frac{(1-\delta_{ik})\,B_k(s')}{V_{s'}}[e^{-\theta(1-V_s)} - e^{-\theta}];$$

$$A_1(s') = 1-(Z/A)\,V_{s'};$$

$$A_2(s') = 1 - V_{s'}(A-Z)/A;$$

$$B_1(s') = -(Z/A)\,V_{s'};$$

$$B_2(s') = V_{s'}(A-Z)/A.$$

These expressions reduce to those given by Messel if $Z/A \sim (A-Z)/A \sim ½$. If we now apply to (16) all the considerations made for the case of equal numbers of protons and neutrons, and make use of the initial conditions, then the number of neutrons is given by the formula

$$N_n(E, \vartheta_A, \vartheta_B) \tag{17}$$

$$= \frac{\gamma}{4\pi^2 i^2}\int_E^{\infty}\int_{s_0'-i\infty}^{s_0'+i\infty}\int_{s_0-i\infty}^{s_0+i\infty}\left(\frac{E_0}{E}\right)^{s'} f_0^{j,2}(s'+1, \vartheta_A)$$

$$\times \frac{1}{s'}\left(\frac{E_c}{E_0}\right)^{s}\frac{1}{\gamma-s} f_0^{1,j}(s+1, \vartheta_B)\frac{dE, ds'\,ds}{E_0}.$$

Here we have neglected ionization loss, i.e., $\beta_A = \beta_B \to 0$. Since almost all the low-energy stars are formed by neutrons, we correctly explain only the increase in number of neutrons under lead, so we shall calculate only $N_n(E, \vartheta_A, \vartheta_B)$. As we see from Fig. 2, such a calculation, for altitudes $\sim$ 28-30 km, gives an increase in number of stars under lead of 20%, compared to the case where the neutron excess in heavy nuclei is not taken into account. This number agrees approximately with the experimental data obtained in our laboratory.

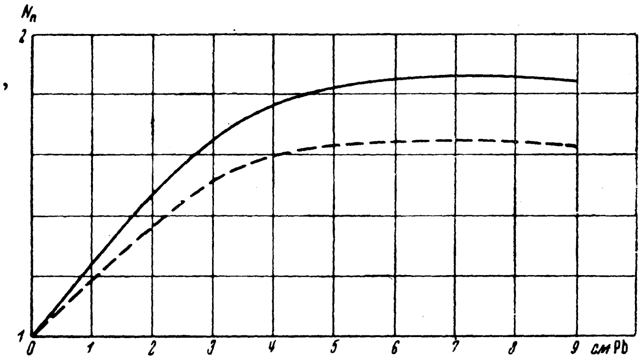

FIG. 2. Dependence of number N_n of star-producing neutrons with energy greater than 50 mev on thickness of lead absorber at altitude 27 km above sea level. The solid curve shows the intensity calculated, including the effect of neutron excess, the dotted curve is calculated without including this effect.

Thus, on the basis of the generally accepted picture of the passage of the nucleonic component through matter we are able to explain (at least approximately) the experimental data presented in brief summary at the beginning of this paper.

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