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Cerenkov Radiation in Anisotropic Ferrites

V. E. PAFOMOV
P. N. Lebedev Physical Institute, Academy of Sciences, USSR
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This work considers the radiation of electromagnetic waves by a charge moving in an anisotropic ferrite with a velocity greater than the phase velocity of light. Such radiation differs from that occurring in the case of an anisotropic dielectric, both with respect to the intensity distribution in the generatrices of the cones and with respect to the energy of the radiation.

SOME years ago Ginzburg[1] considered the question of the field and the radiated energy of an electron in an anisotropic dielectric (see also the works of Kolomenskii[2,3] and Kaganov[4]). As compared with the case of an anisotropic dielectric, the Cerenkov effect in the case of an anisotropic magnetic material possesses certain special effects, the consideration of which will, we believe, prove interesting. Such consideration might even be considered essential in connection with the possible use of the Cerenkov effect in isotropic and anisotropic ferrites for the generation of microwaves. The method used here for the investigation of the Cerenkov radiation in anisotropic ferrites is also applicable to the solution of other electrodynamic problems in ferrites.

In the present article we also derive formulas for the Cerenkov effect in an anisotropic dielectric which are more general than those which have previously been known.

1. THE HAMILTONIAN METHOD IN THE ELECTRODYNAMICS OF AN ANISOTROPIC FERRITE

In order to solve the problem under consideration we shall use the Hamiltonian method in a manner similar to that in which Ginzburg used it in the solution of the problem of the Cerenkov radiation in an anisotropic dielectric[1-5].

We shall assume that

$$\mathbf{D} = \varepsilon \mathbf{E}; \quad B_x = \mu_x H_x, \quad B_y = \mu_y H_y, \quad B_z = \mu_z H_z. \tag{1}$$

We shall set the conductivity of the medium equal to zero. The extent to which this approximation is justified for ferrites, which are semiconductors, is well known. Using the additional condition

$$\operatorname{div} \mathbf{A} = 0, \tag{2}$$

introduced by Ginzburg[6], and resolving the vector potential **A** into a Fourier series

$$\mathbf{A}(\mathbf{x}, t) = \sum_{\lambda, i} q_{\lambda i}(t) \mathbf{A}_{\lambda i}(\mathbf{x}), \quad \mathbf{A}_{\lambda i} = \sqrt{4\pi} c \mathbf{a}_{\lambda i} e^{i \mathbf{k}_\lambda \mathbf{x}}, \tag{3}$$

we obtain, by a method analogous to that used in Ref. 5,

$$\ddot{q}_{\lambda i} + \omega_{\lambda i}^2 q_{\lambda i} = \sqrt{4\pi} e (\mathbf{v a}_{\lambda i}) e^{-i \mathbf{k}_\lambda \mathbf{x}(e)}, \tag{4}$$

provided the conditions

$$(\mathbf{a}_{\lambda i} \mathbf{a}_{\lambda i}) = 1; \quad (\mathbf{a}_{\lambda i} \mathbf{k}_\lambda) = 0; \quad (\mathbf{a}_{\lambda i} \mathbf{a}_{\lambda j}) = 0; \quad (\mathbf{h}_{\lambda i} \mathbf{b}_{\lambda j}) = 0. \tag{5}$$

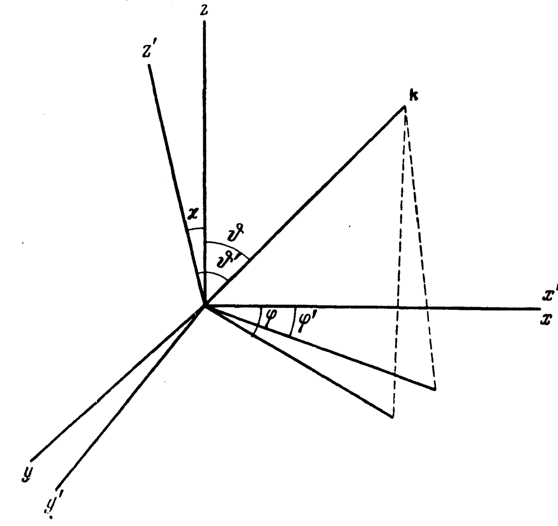
are fulfilled. The indices i and j take on the values 1 and 2 corresponding to the different polarizations. In the last two equations of (5) $i \neq j$. Here $\mathbf{a}_{\lambda i}$ is the normalized electric field intensity vector of the polarization wave of type i; $\mathbf{a}_{\lambda i}$ is the electric induction vector; $\mathbf{h}_{\lambda i}$ is the magnetic field vector ; $\mathbf{b}_{\lambda i}$ is the magnetic induction vector.

Conditions (5) differ from the corresponding conditions in Ref. 5, the physical meaning of which does not require explanation.

The method considered can be used for the solution of various electrodynamic problems. For example, we may consider the radiation from an oscillator in an anisotropic ferroelectric.

2. CERENKOV RADIATION IN A UNIAXIAL FERRITE CRYSTAL

We restrict ourselves in the present work to uniaxial crystals.



The formula for the radiation energy of the waves with type i polarization has the form*:

$$H_{\text{rad}}^{i} = \frac{e^2 vt}{2\pi c^2} \times \int \left| \frac{(a_{\lambda i})_v^2 n_{\lambda i}^3}{n_{\lambda i} - (\partial n_{\lambda i}\, \partial \vartheta) \operatorname{ctg} \vartheta} \right|_{\vartheta = \vartheta_0} \omega_{\lambda i} d\omega_{\lambda i} d\varphi, \quad (6)$$

where $(a_{\lambda i})_v$ is the projection of the normalized electric field intensity vector in the direction of the forward motion of the electron; $n_{\lambda i}$ is the index of refraction for waves with type i polarization; φ is the azimuthal angle; ϑ is the angle between the normal to the wave front and the velocity of the charged particle. In formula (6) all the magnitudes are taken for $\vartheta = \vartheta_0$, where ϑ_0 is the angle between $\mathbf{k}_\lambda$ and $\mathbf{v}$ for the radiated waves. It is found from the usual condition for the Cerenkov radiation, $(\mathbf{k}_\lambda \mathbf{v}) = \omega_{\lambda i}$, and the integration is carried out over the region where this condition is fulfilled.

For convenience in what is to follow, we introduce two systems of coordinates. We choose our basic system, that in which we shall write our final formulas, such that the z axis coincides with the direction of motion of the electron. We call it system Σ. We will let the z' axis of the auxiliary coordinate system Σ' be directed along the optic axis of the uniaxial crystal. The yz and $y'z'$ surfaces of these two systems coincide, as do the x and x' axes. In other words system Σ' is obtained by rotating system Σ, like an hour hand on a clock, by an angle $\varkappa$ about the x axis, where $\varkappa$ is the angle between the crystal axis and the direction of motion of the electron. The equations for the transformation of coordinates have the form

$$x' = x; \quad y' = y \cos\varkappa + z \sin\varkappa; \quad z' = -y \sin\varkappa + z\cos\varkappa. \quad (7)$$

The transformation equations for the trigonometric functions which will be needed below may be found from (7). They are

$$\cos^2\vartheta' = (\cos\vartheta \cos\varkappa - \sin\vartheta \sin\varphi \sin\varkappa)^2; \quad (8)$$

$$\cos^2\varphi' = \frac{\cos^2\varphi \sin^2\vartheta}{\cos^2\varphi \sin^2\vartheta + (\sin\varphi \sin\vartheta \cos\varkappa + \cos\vartheta \sin\varkappa)^2}. \quad (9)$$

We shall suppose that the dielectric constant is isotropic and different from unity. In this case the equations for the indices of refraction for the ordinary (o) and extraordinary (e) waves will be, respectively, as follows:

$$n_o^2 = \mu_o \varepsilon; \quad \frac{\varepsilon}{n_e^2} = \frac{\cos^2\vartheta'}{\mu_o} + \frac{\sin^2\vartheta'}{\mu_e}, \quad (10)$$

where $\mu_{x'} = \mu_{y'} = \mu_o$ and $\mu_{z'} = \mu_e$.

The projections of the normalized electric field vectors along $\mathbf{v}$ can be found from conditions (5):

$$\varepsilon (a_o)_v^2 = (\cos\vartheta' \sin\varphi' \sin\varkappa - \sin\vartheta' \cos\varkappa)^2; \quad (11)$$

$$\varepsilon (a_e)_v^2 = \sin^2\varkappa \cos^2\varphi'.$$

The conditions for radiation have the form

$$n_o^2 \beta^2 \cos^2\vartheta = 1; \quad n_e^2 \beta^2 \cos^2\vartheta = 1, \quad (12)$$

$$\beta^2 = v^2/c^2.$$

where $\beta^2 = v^2/c^2$.

In the isotropic case Eq. (6) gives

$$H_{\text{rad}} = \frac{e^2 vt}{c^2} \int \mu \left(1 - \frac{1}{\beta^2 \varepsilon \mu}\right) \omega d\omega, \quad (13)$$

i.e., the result obtained by Sitenko*[7]. For $\mu = 1$ we have the well-known formula of Frank and Tamm[10]. We note that in (16), and also in the formulas derived below for the radiation from a charge moving faster than light in anisotropic media, μ enters as a factor, and hence the index of refraction of the radiated energy is μ times as great as it is in the case of nonmagnetic media. Unfortunately, in the centimeter region in the known ferrites $\mu < 2$ and the gain is small. However, the possibility of obtaining larger values of μ is not excluded, and then the use of ferrites for the generation of radio waves would be preferable to the use of dielectrics[11].

In what follows we shall designate the energy

* See Ref. 3. Equation (6) is valid for any type of anisotropy, since in its derivation Eq. (4), which is derived by means of a corresponding choice of conditions imposed on $a_{\lambda i}$, is used.

Equation (6) does not take into account the optical activity of the medium. We note that we must separate the integral sign from the modulus of the expression under the integral.

* Equation (13) for the intensity of Cerenkov radiation in an isotropic magnetic material was also obtained by Watson and Jauch[8], but in a more complicated manner than in Ref. 7. The question of the losses of energy in isotropic magnetic materials has been considered especially in Ref. 9. However, erroneous results were there obtained (see Ref. 7).

of radiation for the ordinary and extraordinary waves in an anisotropic magnetic medium (anisotropic μ, isotropic ϵ) as H^{o}_{ex} and H^{e}_{ex}, respectively, and the energy of radiation of the ordinary and extraordinary waves in an anisotropic dielectric (anisotropic ϵ, isotropic μ), as ${}_{\partial}H^{o}_{ex}$ and ${}_{\partial}H^{e}_{ex}$, respectively. For the calculations we use the formulas (6) and the corresponding expressions from (10) and (11).

When the specifications of (8) and (9) are used, the result takes the following form:

$$H^{o}_{ex}=\frac{e^2tv}{2\pi c^2}\times\int\mu_o(\cos\vartheta_0'\sin\varphi'\sin\varkappa-\sin\vartheta_0'\cos\varkappa)^2\omega d\omega d\varphi; \tag{14}$$

$$H^{e}_{ex}=\frac{e^2tv}{2\pi c^2}\times\int\left|\frac{\mu_o\mu_e\sin^2\varkappa\cos^2\varphi'}{\mu_o+(\mu_e-\mu_o)(\sin\varphi\sin\varkappa-\cos H\operatorname{ctg}\vartheta_0)\sin\varphi\sin\varkappa}\right|\times\omega d\omega d\varphi, \tag{15}$$

where ϑ_0 is found from the corresponding condition of radiation (12), and the integration is taken over the region where the condition

$$n^2_{o,e}\beta^2>1. \tag{16}$$

is fulfilled.

In order to make a comparison with the case of an anisotropic dielectric, we write two formulas which may be obtained from the equations derived in Ref. 1:

$${}_{\partial}H^{e}_{ex}=\frac{e^2tv}{2\pi c^2}\int\mu\sin^2\varkappa\cos^2\varphi'\omega d\omega d\varphi; \tag{17}$$

$${}_{\partial}H^{e}_{ex}=\frac{e^2tv}{2\pi c^2}\times\int\left|\frac{\mu(\varepsilon_e\sin\varphi'\cos\vartheta_0'\sin\varkappa-\varepsilon_o\sin\vartheta_0'\cos\varkappa)^2}{[\varepsilon_o+(\varepsilon_e-\varepsilon_o)\cos^2\vartheta_0'][\varepsilon_o+(\varepsilon_e-\varepsilon_o)(\sin\varphi\sin\varkappa-\cos\varkappa\operatorname{ctg}\vartheta_0)\sin\varphi\sin\varkappa]}\right|\omega d\omega d\varphi. \tag{18}$$

The specifications of (8) and (9) have been taken into account in these formulas. In the case of an anisotropic dielectric, $\epsilon_{x'}=\epsilon_{y'}=\epsilon_o$, $\epsilon_{z'}=\epsilon_e$ (isotropic μ), and the equations for the indices of refraction are:

$$n_o^2=\mu\varepsilon_o;\quad \frac{\mu}{n_e^2}=\frac{\cos^2\vartheta'}{\varepsilon_o}+\frac{\sin^2\vartheta'}{\varepsilon_e}. \tag{19}$$

From formulas (15) and (17) it is clear that for the motion of an electron along the optic axis of a uniaxial crystal ($\varkappa=0$), in an anisotropic magnetic material only the ordinary waves are radiated, and in an anisotropic dielectric only the extraordinary waves are radiated.

Using the relations (8)-(12), (14), (18), we obtain for $\varkappa=0$:

$$H^{o}_{ex}=\frac{e^2tv}{c^2}\int\left(\mu_o-\frac{2}{\varepsilon\beta^2}\right)\omega d\omega; \tag{20}$$

$${}_{\partial}H^{e}_{ex}=\frac{e^2tv}{c^2}\int\left|\left(\mu-\frac{1}{\varepsilon_o\beta^2}\right)\right|\omega d\omega. \tag{21}$$

The region of integration can be found from Eq. (16), which can be written in the form $\beta^2\mu_o\epsilon>1$ for Eq. (20) and in the form $(\epsilon_e/\epsilon_o)(\beta^2\mu\epsilon_o-1)>0$ for Eq. (21).

For motion of the electron perpendicular to the optic axis ($\varkappa=\pi/2$), we obtain

$$H^{o}_{ex}=\frac{e^2tv}{2\pi c^2}\int\frac{(\mu_o-1/\varepsilon\beta^2)\sin^2\varphi\,\omega d\omega d\varphi}{\beta^2\varepsilon\mu_o\cos^2\varphi+\sin^2\varphi}; \tag{22}$$

$$H^{e}_{ex}=\frac{e^2tv}{2\pi c^2}\times\int\left|\frac{\mu_o^2(\beta^2\varepsilon\mu_o-1)\cos^2\varphi}{[\beta^2\varepsilon\mu_o\cos^2\varphi+\sin^2\varphi][\mu_o+(\mu_e-\mu_o)\sin^2\varphi]}\right|\times\omega d\omega d\varphi; \tag{23}$$

$${}_{\partial}H^{o}_{ex}=\frac{e^2tv}{2\pi c^2}\int\frac{\mu(\beta^2\varepsilon_o\mu-1)\cos^2\varphi\,\omega d\omega d\varphi}{\beta^2\varepsilon_o\mu\cos^2\varphi+\sin^2\varphi}; \tag{24}$$

$${}_{\partial}H^{e}_{ex}=\frac{e^2tv}{2\pi c^2}\times\int\left|\frac{(\beta^2\varepsilon_e\mu-1)\sin^2\varphi}{\beta^2(\beta^2\varepsilon_o\mu\cos^2\varphi+\sin^2\varphi)[\varepsilon_o+(\varepsilon_e-\varepsilon_o)\sin^2\varphi]}\right|\times\omega d\omega d\varphi. \tag{25*}$$

The difference between Cerenkov radiation in media which are anisotropic in their magnetic properties from that in media which are anisotropic in their dielectric properties is clear from Eqs. (22)-(25). For example, the maximum in-

* For $\mu=1$, Eq. (24) coincides with the corresponding equation in Ref. 1. Equations (21) and (25) for $\mu=1$ do not coincide with the corresponding equations in Ref. 1 because of an error made in this reference in carrying out an integration.

tensity in a magnetically anisotropic medium corresponds to zero intensity in a medium which is anisotropic in its dielectric properties, and conversely, and this is true both for the ordinary and the extraordinary waves.

The case of double anisotropy is also interesting. For the simplest medium of this type we choose a system of coordinates in which both the dielectric constant tensor and the magnetic permeability tensor are diagonal. The conditions imposed on $a_{\lambda i}$ and leading to Eq. (4) have the form

$$(d_{\lambda i}\, a_{\lambda i}) = 1; \quad (d_{\lambda i}\, k_\lambda) = 0; \tag{26}$$

$$(d_{\lambda i}\, a_{\lambda j}) = 0; \quad (h_{\lambda i}\, b_{\lambda j}) = 0$$

and for $\mu_x = \mu_y = \mu_z = 1$ coincide with the results of Ginzburg[5].

In a uniaxial crystal with a double anisotropy, both polarizations correspond to extraordinary waves. Equation (18), and consequently (21) and (25), but with μ_0 in place of μ, holds for one polarization. The energy of radiation of the waves of the other polarization is determined by Eq. (15), but with ϵ_0 in place of ϵ. The extraordinary waves corresponding to the second polarization [Eq. (15)] are not radiated for a motion of the charge along the optic axis.

In conclusion, the author takes the opportunity of expressing his sincere gratitude to Prof. V.L. Ginzburg for proposing the problem and for valuable suggestions.

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