On the basis of nonuniform density of nucleons, of the type (4) and (5), we can analyze the effective empirical nuclear radii, finding the mean values of different powers of r from $\rho(r)$, on which the various effects depend**. Thus, the mean value of r^2 from $\rho_Z^{(II)}(r)$ is

$$\overline{R_Z^{2(II)}} = 3/5\left[(y_0^3 + 5y_0^2 + 10y_0 + 10)/y_0^2(y_0+3)\right] R_{0Z}^{2(II)}. \quad (7)$$

Finding the mean square radius $\{\overline{R_Z^{2(II)}}\}^{1/2}$ for the average $y_0 \approx 1.8$ and introducing the equivalent radius of constant density proton distribution that gives the same $\{\overline{r^2}\}^{1/2}$ ($R_Z = (5/3)^{1/2}\{\overline{R_Z^2}\}^{1/2}$), we find

$$R_Z^{(II)} \cong 1{,}21\times10^{-13}\ A^{1/3} \quad (8)$$

in good agreement with empirical electromagnetic nuclear radii[11].

Since for the densities (4) and (5) the nucleon density differs markedly from zero at distances larger than $R_Z \sim 1.2 \times 10^{-13} A^{1/3}$ one can understand qualitatively, on the basis of the examined $\rho(r)$, the considerably larger values of nuclear radii ($1.5 \times 10^{-13} A^{1/3}$) obtained from the cross section in processes in which nucleons (and evidently π-mesons) take part and data found from α-decay where the effective radii are connected with the region of action of nuclear forces.

If we shall assume the same $\rho(r)$[(4) and (5)] and the same level scheme (3) for neutrons, the corresponding parameters x_{0N} and y_{0N} for $\rho(r)$ will be correlated with N as x_0 and y_0 with Z.

Finally, we note that both $\{\overline{R_Z^{2(II)}}\}^{1/3}$ and the effective radii

$$\tilde{R}_Z^{(II)} = R_{0Z}^{(II)} + 1/\beta = (1 + 1/y_{0Z})\, R_{0Z}^{(II)}$$

will change nonmonotonically because of the sawtooth like change of y_{0Z}. Magic nuclei will have lower $\{\overline{R_Z^2}\}^{1/2}$ and $\tilde{R}_Z$. Therefore, the relative drop in the value of the radius should be more pronounced for the doubly-magic nuclei. The effective empirical nuclear radii show also relative drops for the magic and some sub-magic nuclei[12]. Such nonmonotonic character of the effective nuclear radii can be regarded as caused by deviations from spherical symmetry.

I wish to express my thanks to Prof. D. D. Ivanenko and N. N. Kolesnikov for the discussion of the problem and valuable remarks.

Note added in Proof: For the density of the form $\rho_z(r) = \rho_0 [1 + e^{K(r-c)}]^{-1}$ the parameter Kc calculated from $\bar{L}$ for Au ($Z = 79$) is in sufficient agreement with the value $Kc \approx 12.0$, for which best agreement between theory and experiment is observed for the cross section angular dependence for the scattering of high-energy electrons on Au_{79} nuclei[13].

* In the following, dealing with protons, we shall keep in mind that unless otherwise mentioned, the results are valid for neutrons as well.

** The parameters x_0 and y_0 were determined from the numbers of first occurrence and the r^2 also in Ref. 7.

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Some Cases of Generation of Heavy Unstable Particles on Beryllium Nuclei

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IN recent years, it has been made evident[1-3] that hyperons and K-mesons can be created by pairs in reactions of the type:

$$\pi + p \rightarrow \Lambda + K \quad \text{or} \quad N + N \rightarrow \Lambda + K + N.$$

In the case of interaction of π-mesons with free protons, a correlation of the positions of the planes of emission and of decay of the hyperons is observed. Such a correlation is not observed if the hyperons are produced as a result of irradiation of heavy nuclei (Pb) by cosmic rays[4].

Single cases of formation of hyperons and K-mesons on light nuclei (Be) irradiated by cosmic rays have been observed in our experiments performed at an altitude of 3860 m above sea level. The experimental set-up consisted of a Wilson chamber with a diameter of 30 cm and a depth of illumination of 8 cm. The chamber contained a 5 cm thick beryllium plate, and under it a 1 cm thick lead plate. The chamber was in an 8,500 oersted field of an electromagnet. The chamber was controlled by a system of counters separating electron-nuclear showers.

For 25 observed cases of generation of showers on beryllium, there were observed 3 cases of decay of heavy particles (formed on the beryllium), during their flight. The main characteristics of these cases are reported in Table I.

Figure 1 shows the photograph of case 117.63.

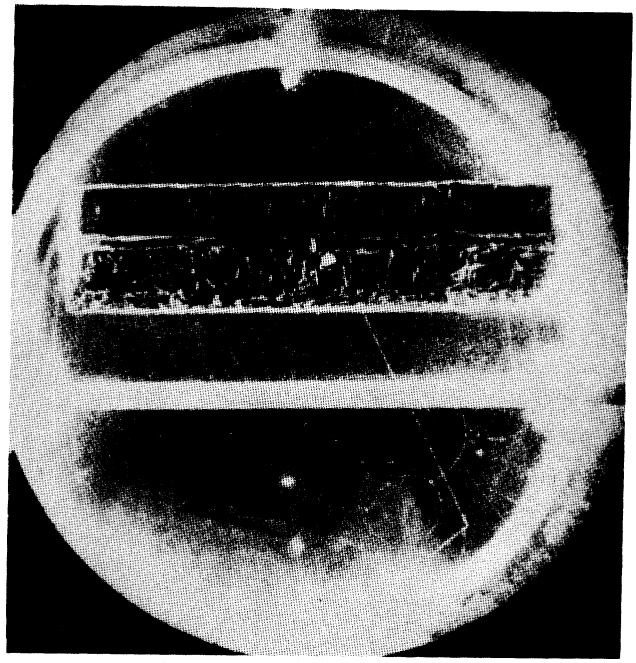

FIG. 1

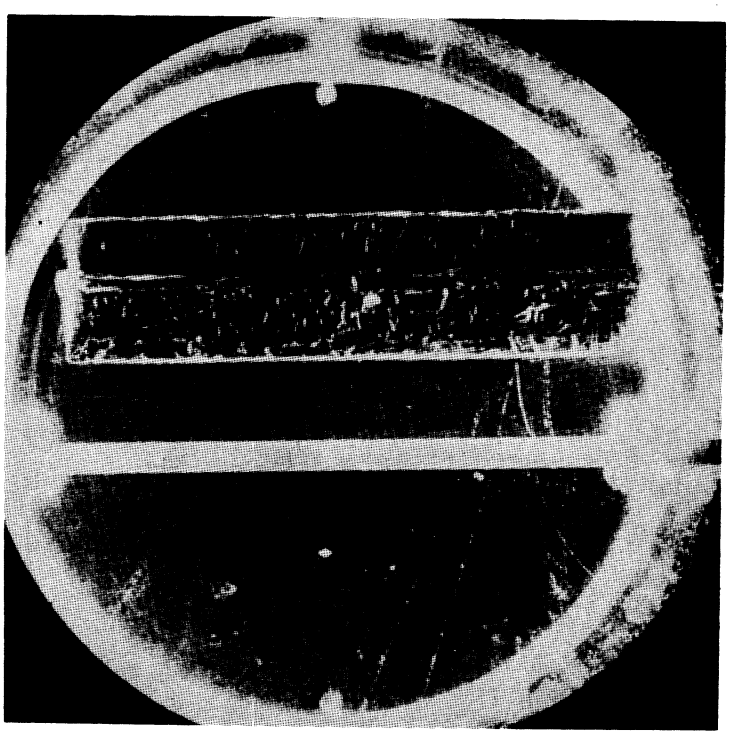

FIG. 2

We analyzed all the known schemes of decay of charged hyperons and heavy mesons with emission of a single charged secondary particle. The

TABLE I

No. of Photograph	Primary Particles: Momentum $p \times 10^8$ ev/c	Sign of Charge	Angle with the Generating Particle θ_p^{0*}	Angle[1] of Uncoplanarity δ	Assumed Nature of the Particle	Half Life[2] $T = \frac{L^{**}}{3p/M}$ sec	Secondary Particles: No. of Particle	$p \times 10^8$ ev/c	Ionization	R grams/cm^2 Pb	Angle with Primary θ	Angle Between Secondary θ_{12}	Assumed Nature of Secondary Particles
117.63	$8.6^{+15}_{-3.5}$	+	20 ± 2	---	Σ^+	7×10^{-10}	1	---	$\frac{I_{prim}}{I_{sec}} = 1.4 \pm 0.4$	---	93 ± 2	---	π-meson
120.54	---	Neutr.	9 ± 4	1 ± 2	Λ^0	4×10^{-10}	1	> 12 ---	$I_1 \sim I_{min}$	---	2 ±2	---	Proton
							2		$I_2 \geq 2.6\, I_1$	13	47 ± 2	49 ± 2	π-meson
112.66	---	Neutr.	11 ± 2	3 ± 2	θ^0	---	1	$5.3^{+3.5}_{-1.5}$	$I_1 \sim I_{min}$		14 ± 1		$\pi(\mu)$-meson
							2	---	$I_2 \sim I_{min}$	---	39 ± 1	53 ± 0.5	---

[1] Angle of uncoplanarity between the trace of the V^0-particle with the plane of the secondary particles. All angles in degrees.

[2] L-Path of the decaying particle from the point of generation to the point of decay, in centimeters.

TABLE II

No. of Case	Assumed Nature of the Particle	φ^0
117.63	Σ^+	57±10
120.54	Λ^0	74±10
112.66	θ^0	15±5

TABLE III

Nature of the Particle	Angle φ between the planes of formation and of decay, according to data from Refs. 1-3.							
Λ^0 θ^0	5+5 —	30±20 —	27±10 70±5	38±5 —	~20 ~90	~40 ~0	— 22±10	— 58±3
$\Sigma^\pm$	7±5	20±20	10±10					

hypothesis of a decay-scheme of the hyperon: $\Sigma^+ \rightarrow \pi^+ + n$ fits best the observed values of the momentum, of the angle and of the ionization ratio pertaining to the primary and secondary particles. The observed half-life of the particle is also in good agreement with this hypothesis. For the energy of decay of the hyperon we get

$Q = (125^{+175}_{-20})$ mev.

Figure 2 shows the photograph of case 120.54: generation of V^0-particles in a shower. Two types of neutral V-particles are known: the Λ^0 and θ^0-particles. The analysisof the decay-schemes of these particles [$\Lambda^0 \rightarrow p + \pi^-$ and $\theta^0 \rightarrow \pi^+ + \pi^-$] has shown that, in the observed case, a Λ^0-particle decayed into a fast proton and a slow π^--meson. In case 112.66 one also observes the decay of a V^0-particle formed on the beryllium plate. The positively charged secondary particle cannot be a proton because of the observed values of the momentum and of the ionization. One must then assume that the decay follows the scheme $\theta^0 \rightarrow \pi^+ + \pi^- + 214$ mev. In this case, the momentum of particle l must be equal to 6.3×10^8 ev, which is in good agreement with the experimental value. In all the observed cases the direction of the charged particle (which generated the V-particle on a Be nucleus) is known; hence, one can measure the angle φ between the plane of generation of the V-particle and the plane of its decay (see Table II).

Table III shows the data on angles φ for all cases known in the literature of pair generation of hyperons and K-particles resulting from irradiation of hydrogen by π -mesons.

For all 9 observed cases of formation of hyperons in a π^-_p interaction, the angle $\varphi$is such that $\varphi \leq 40°$; this indicates that hyperons have large spins. At the same time, for hyperons formed on a Be nucleus, we have $\varphi \geq 40°$ (Table II). This is probably due to the Be nucleus (such as scattering of hyperons or their generation by secondary particles of the shower).

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Linearization of the Hartree Equations

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IN addition to the existing methods of description of collective interactions[1-4] we may consider another one based on the linearization of the Hartree equation near the solutions with constant density.

In the equations

$$i\hbar \frac{\partial\psi_i}{\partial t} + \frac{\hbar^2}{4m}\Delta\psi_i - \left\{\int G(|\mathbf{r}-\mathbf{r}'|) \times \sum_j |\psi_j(\mathbf{r}')|^2 d\mathbf{r}'\right\}\psi_i(\mathbf{r}) = 0 \quad (1)$$

let us make the substitution

$$\psi_i(\mathbf{r}, t) = \sqrt{P_i(\mathbf{r}, t)} \exp\{-iS_i(\mathbf{r}, t)/\hbar\}.$$

This leads to the system of equations

$$\partial P_i/\partial t + m^{-1}\,\mathrm{div}\,(P_i \nabla S_i) = 0, \quad (2)$$

$$\frac{\partial}{\partial t} S_i + \frac{1}{2m}(\nabla S_i)^2 + \int G(|\mathbf{r}-\mathbf{r}'|) \times \sum_j P_j(\mathbf{r}')\,dr' - \frac{\hbar^2}{4m}\left\{\frac{\Delta P_i}{P_i} - \frac{1}{2}\left(\frac{\nabla P_i}{P_i}\right)^2\right\} = 0.$$

The form of these equations is identical to the form of the equations of irrotational motion of an ideal compressible fluid. The states of the system which are close to a constant space density of particles can be described by equations obtained by the linearization of equations (2) near the solutions, with $P_j^0 = \mathrm{const} = P_0$, $S_j^0 = E_j^0 t + \bar{S}_j^0(\mathbf{r})$; $\Delta S_j^0 = m v_j^0$ [$\mathbf{v}_j^0$ is the velocity of the jth particle in the state of a uniform space density of particles, $E_j^0 = m(v_j^0)^2/2$].

Let us look for the solutions $P_j S_j$ of the linearized equations in the form of a superposition of plane waves [$\sim \exp(i\mathbf{kr} - i\omega t)$]. The conditions of the solvability of homogeneous algebraic equa-