desired equations of motion in the interaction representation

$$(\gamma_\nu \partial/\partial x_\nu + \varkappa_0)\,\tilde{\psi}(x) = 0, \qquad \Box^2 \tilde{a}_\mu(x) = 0 \quad (14)$$

in accordance with Ref. 1.

---

*Notation of Refs. 1 and 2 is used throughout.

**The integral equations for $U[\sigma]$ and the vector of state $\Psi[\sigma]$. are, in contrast to the differential equations, applicable without reservation in the case of plane σ.

***Operators in interaction representation are labelled by ~.

1 E. M. Lipmanov, J. Exptl. Theoret. Phys. (U.S.S.R.) 27, 135 (1954).

2 J. Schwinger, Phys. Rev. 74, 1439 (1948).

Translated by H. Kasha
109

On the Theory of Asymmetric Fission of Heavy Nuclei

S. G. Ryzhanov
Kishinev State University

(Submitted to JETP editor December 2, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 599-601 (March, 1956)

IN the present communication we shall show that the dependenc of the yield of fragments on their charges and mass numbers, in the case of spontaneous fission as well as in the case of fission by thermal and fast neutrons, can be obtained by a single method, if one adopts the hypothesis suggested by Frenkel[1]. This hypothesis consists in regarding both spontaneous and induced fussion as a tunnel effect originating, respectively, from the ground or excited state of the fissioning nucleus. The potential barrier for fission is assumed to be due, on one hand, to the specific nuclear potential "well", and on the other, by the electric (Coulomb) repulsion of the fragments. From this point of view, α-decay is regarded as the most sharply asymmetric fission. The well-known quantum-mechanical formula for α-decay[2] can be easily extended to the asymmetric fission of a nucleus with charge Z_0 into fragments of charge Z_1 and Z_2.

Let E be the sum of the kinetic energies of the fragments; $v = \sqrt{2E/M}$ their relative velocity (M is the reduced mass of the fragments); ρ the radius of the specific nuclear potential well; $\sigma = Z_1 Z_2 e^2/\rho E$. It is easy to obtain the following formula for the probability of fission w:

$$w = w_0 \times \exp\left\{-\frac{4Z_1Z_2e^2}{\hbar v}\left(\arccos\sigma^{-1/2} - \sigma^{-1}(\sigma-1)^{1/2}\right)\right\}, \quad (1)$$

where the coefficient w_0 in front of the exponential is of the order of magnitude of the fundamental frequency of nuclear vibrations ($w_0 \approx 3\times10^{21}$ sec^{-1}). The sum of the kinetic energies of the fragments if found from the relation

$$E = E_1 + E_2 - E_0 - Q, \quad (2)$$

where E_0, E_1 and E_2 are the binding energies of the fissioning nucleus and of the two fragments, and Q the energy of their internal excitation[3]. The binding energies are calculated from the semi-empirical formula[4]. Formula (1) has been applied to the fission of uranium nuclei. The correct order of magnitude for the period of spontaneous fission, 10^{21} sec, is obtained with $E = 150$ mev and the radius $\rho = (A_1^{1/3} + A_2^{1/3})\ 1.5\times10^{-13}$ cm. (Here a preliminary approximate integration of Eq. (1) has been carried out over all fragments A_1 and A_2 satisfying the condition $A_1 + A_2 = A_0$). The hypothesis underlying the calculation of τ_{sp} assumes that the fission takes place through the intermediate stage of spheres in contact, which in no way contradicts the thermo-hydrodynamical mechanism of fission. An analogous calculation for Th^{232}_{90} gives the period of spontaneous fission as $\tau_{sp} > 10^{18}$ years. The asymmetry of the fission can be easily obtained from (1). Let us denote by $\overline{w}$ the probability of symmetric fission and let us expand $\ln(w/\overline{w})$ into a series according to powers of the variables

$$\xi = \left(\frac{Z_0 - 2Z_1}{Z_0}\right); \quad \eta = \frac{\overline{E} - E}{\overline{E}}$$

($\overline{E}$ is E for symmetric fission). We then obtain, with sufficient accuracy, the formula

$$\ln\frac{w}{\overline{w}} = \frac{Z_0^2e^2}{\hbar\overline{v}}\left\{\xi \arccos(\overline{\sigma})^{-1/2} - \left(\arccos(\overline{\sigma})^{-1/2} + \frac{\sqrt{\overline{\sigma}-1}}{\overline{\sigma}}\right)\frac{\eta}{2}\right\}. \quad (3)$$

The results of calculations with this formula are given in Fig. 1; conversion of fragment charges into their mass numbers was carried out with the aid of the semi-empirical tables of Ref. 5; the same reference was used for comparison with experimental data. In these calculations account was taken of the dependence of E on the mass numbers A_1 or A_2, given in Fig. 2, which was constructed using Eq. (2). In view of the symmetry of the plot of the fission yield with respect to the light and the heavy fragment groups, in Fig. 1 only the mass numbers of the fragments of the heavy group are marked on the axis of abscissas; Fig. 2 is similarly constructed. Calculations according to (3) show that the influence of the asymmetry of the charges of the fragments on the asymmetry of probability of fission is considerably more important than asymmetry in the masses, in the sense of the original explanation given in Ref. 6.

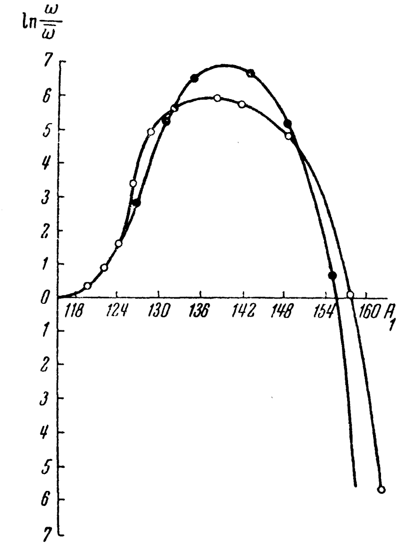

FIG. 1. ● — calculated using formula (1), o — experimental values from Ref. 5.

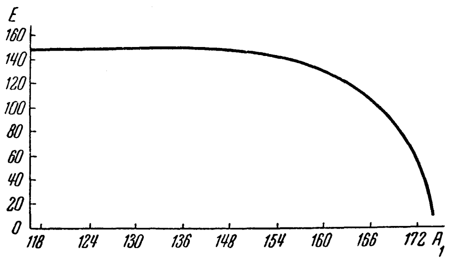

FIG. 2.

Formula (1) makes it possible to interpret the dependence of τ_{sp} on the stability parameter $x = Z^2A^{-1}/(Z^2A^{-1})_{st}$. This parameter is proportional to the ratio of the electric (Coulomb) energy of the nucleus to its surface energy (the term $\gamma A^{2/3}$ in the binding energy). The subscript "st" indicates "stability limit" with respect to fission.

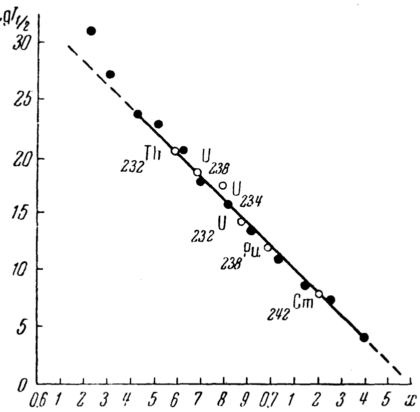

FIG. 3. ● — calculated using formula (4), o — experimental values from Ref. 7.; dotted line is Seaborg's.

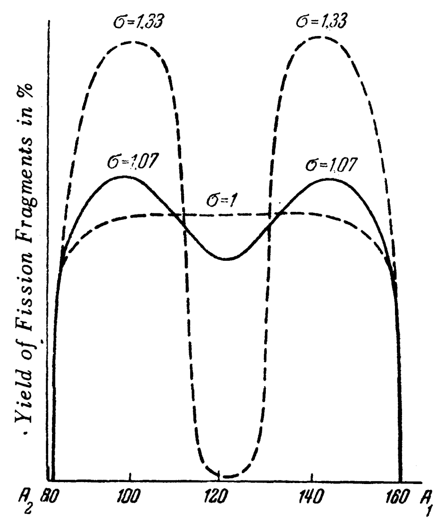

FIG. 4.

The dependence of the period of spontaneous fission of the stability parameter for the sequence of nuclei $Th^{232}_{90}, U^{238}_{92} - U^{232}_{92}, Pu^{238}_{94}, Cm^{242}_{96}$ was experimentally determined in Ref. 7. As a result of an approximate integration of (1) the following formula has been obtained for the half-period of spontaneous fission $T_{1/2}$ (years)

$$\lg T_{1/2} \approx -28 + 1.027\,(M_p \gamma r_0^2/\hbar^2)^{1/2} A^{7/6} x \Delta (3x-1)^{-1/2}. \tag{4}$$

Here M_p is the mass of the nucleon, r is the rigid radius of the nucleon, γ is the coefficient in front of $A^{2/3}$ in the binding energy of the fissioning nucleus, A is its mass number and

$$\Delta = \arccos \bar{\sigma}^{-1/2} - (\bar{\sigma}-1)^{1/2}\bar{\sigma}^{-1}$$

for symmetric fission. Figure 3 illustrates the agreement between calculation and experimental data. The period of fission of U^{236}_{92} nuclei by capture of thermal neutrons, according to Ref. 8 $\sim 10^{-12}$ sec., which is, in fact, obtained from Eq. (1), if one assumes that the radius of the potential well for the excited level is 1.2 times greater than for the ground level (the energy difference of these levels $\sim$ 6 mev). This assumption is entirely acceptable, since the bottom of the well has parabolic shape. This assumption is equivalent to taking into account of the influence of the deformation of the nuclear surface on the probability of tunnel fission; this has been done in the case of α-decay in Ref. 9, where it was also shown that deformation of the nucleus very considerably increases the probability of the tunnel transition. Extension of these calculations to asymmetric fission shows that it is sufficient to take the relative deformation of the nucleus due to polarization by the captured neutron as equal to 0.5 in order to explain the difference in the periods of spontaneous and induced fissions. Deformations of this order of magnitude are entirely reasonable.[10] It is interesting to note that the influence of deformation on $\ln(\omega/\bar{\omega})$ is so slight that on a logarithmic scale asymmetric fission (spontaneous as well as induced by thermal neutron capture) is represented by almost the same graph. Application of (1) to fission by fast particles, with energy of the order of 15 – 25 mev, is illustrated in Fig. 4 and shows good agreement with experiment.[11] In the calculations it was assumed that the kinetic energy of the (fast) particle causing fission is completely transferred into the energy of the fission fragments. Thus the hypothesis, put forward in Ref. 1, does in fact give a unified explanation of the basic regularities of asymmetric fission. An explanation of the fission threshold by slow neutrons in the nuclei Th^{232}_{90}, Pa^{231}_{91}, U^{238}_{92} can also be given from the point of view here set forth and does not require the assumption of a supra-tunnel mechanism of fission.[12]

[1] Ia. I. Frenkel, *Principles of the Theory of Atomic Nuclei*, Academy of Sciences Press, 1950, p. 159.

[2] H. Bethe, *Nuclear Physics*, Part II, GITTL, 1948, p. 184 (Russian translation).

[3] K. Way and E. Wigner, Phys. Rev. 73, 1318 (1948).

[4] E. Fermi, *Nuclear Physics*, IIL, 1950, p. 14. (Russian translation).

[5] Zh. Sigel, Usp. Fiz. Nauk 33, 1, 77 (1947).

[6] Ia. I. Frenkel, Izv. Akad. Nauk SSSR Fiz. Ser. 10, 415 (1946).

[7] G. T. Seaborg, Phys. Rev., 85, 157 (1952).

[8] R. R. Wilson, Phys. Rev. 72, 98 (1947).

[9] L. A. Sliv and L. K. Peker, Dokl. Akad. Nauk SSSR 99, 727 (1954).

[10] A. Bohr and B. R. Mottelson, Problems of Contemporary Physics, 9, p. 34, 1955. (Russian translation).

[11] A. Newton, Phys. Rev. 75, 17 (1949); R. W. Spence, BNL – C – 9, 1949.

[12] N. Bohr and J. A. Wheeler, Phys. Rev. 56, 426 (1939).

Translated by A. V. Bushkovitch
118

Pion-proton Interactions at 1.4 BEV

A. I. NIKISHOV

(Submitted to JETP editor July 12, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 601-603 (March, 1956)

Eisberg et al.[1] have made a detailed study of the interaction of negative pions with protons at 1.37 bev. The analysis by Belen'kii and Nikishov[2] of $n-p$ collisions at 1.7 bev shows that a direct comparison of experimental results for multiple particle production with Fermi's[3] statistical theory in its original form leads to considerable divergence of theory and experiment. When, however, the statistical theory also allows for the formation of isobar states the discrepancy disappears, leaving a quite satisfactory agreement with experiment.

It is also of interest to compare with the statistical theory including isobar states the experimental data on the interaction of negative pions with protons. We shall use the notation N for nucleon, N' for isobar state and π for pion. The important cases here are those which give the following states as a result of pion-nucleon collisions: $N\pi$ – elastic scattering; $N'\pi$, $N\pi\pi$ – the production of one secondary meson; $N'\pi\pi$, $N\pi\pi\pi$ – the production of two secondary mesons. The statistical weights of these states are calculated as in Refs. 2 and 4. For brevity we shall write $N'\pi\pi \equiv 2N'$ etc.

After correcting for identity, spin and isotopic spin conservation, the relative statistical weights of processes $1N$, $2N$, $1N'$, $2N'$, $3N$ and $3N'$ (in %) are, respectively, 21, 30, 29, 13, 6 and 1. The last process can thus be neglected. Furthermore,