Then, multiplying both sides of Eq. (7) by $(W+2E_p)/(W+2M)$, introducing the binding energy $\mathcal{E}=W-2M$ and passing to the coordinate representation, we obtain the equation

$$\left(\frac{\nabla^2}{M\rho}+\mathcal{E}\right)\Phi^{01}(\mathbf{r})=\int U_2(\mathbf{r}\mathbf{r}'W)\,\Phi^{01}(\mathbf{r}')\,d\mathbf{r}', \qquad (12)$$

where $\rho=1+\mathcal{E}/4M$ and

$$U_2(\mathbf{r}\mathbf{r}'W)=\int\frac{[(2E_{\mathbf{p}}+W)(2E_{\mathbf{p}'}+W)]^{1/2}}{4M\rho}\Delta_2(\mathbf{p}\mathbf{p}'W)\,e^{i(\mathbf{pr}-\mathbf{p}'\mathbf{r}')}d\mathbf{p}\,d\mathbf{p}'. \qquad (13)$$

The quantity M_ρ plays the role of reduced mass. Passing to the nonrelativistic limit $E_\rho\sim M$, $W\sim 2M$, we obtain the equation

$$(\nabla^2/M\rho+\mathcal{E})\,\Phi^{01}(\mathbf{r})=V_2(\mathbf{r})\,\Phi^{01}(\mathbf{r}), \qquad (14)$$

where

$$V_2(\mathbf{r})=-\frac{1}{3}(\tau_1\tau_2)\frac{g^2}{4\pi}\left(\frac{\mu}{2M}\right)^2 \qquad (15)$$

$$\times\left\{\sigma_1\sigma_2+S_{12}\left[1+\frac{3}{\mu r}+\frac{3}{(\mu r)^2}\right]\right\}\frac{e^{-\mu r}}{r}$$

$$-\frac{1}{3}(\sigma_1\sigma_2)(\tau_1\tau_2)(2M)^{-2}\,\delta(r)$$

$$+\frac{(\tau_1\tau_2+3)(\sigma_1\sigma_2-1)}{4\mu(2M-\mu)}\,\delta(r).$$

From this it is seen that the interaction potential between nucleon and antinucleon has a sign opposite to that of the interaction potential between two nucleons. The exchange term does not give a contribution in the nonrelativistic limit. Investigation in the relativistic region shows that the first term of equality (8) gives attraction at small distances, in contradistinction to the case of the two-nucleon system. The exchange term becomes large if $W\sim\mu$. This indicates that the possibility is not excluded of the formation of the bound nucleon-antinucleon system with large binding energy. This problem, which involves the relativistic region, requires a supplementary investigation. It would be interesting to investigate it also using the "new" Tamm method proposed by Dyson.

In conclusion, the author expresses his gratitude to Iu. M. Shirokov for advice and discussion of this problem.

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Translated by A. V. Bushkovitch
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Angular Distribution of Fission Fragments

V. M. Strutinskii

(Submitted to JETP editor October 13, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 606-608 (March, 1956)

THE anisotropic angular distribution of fission fragments from fission induced by fast neutrons[1,2], protons[3] and gamma rays[4,5] has been observed experimentally. A number of general features of the angular distribution of the fragments follow from the conservation of angular momentum. Thus the principal qualitative difference of the angular distribution of fragments from nucleon-induced fission from that of photofission, which is the maximum at $\vartheta=0$, as contrasted with the maximum at $\vartheta=\pi/2$ for photofission, results from the different spin orientation of the compound nucleus. When a fast nucleon is captured, the spin of the compound nucleus is oriented predominantly in a direction perpendicular to the beam. The component of the radiation moment along the direction of the beam is ± 1; thus for a dipole radiation the spin of the compound nucleus is oriented predominantly along the beam. This difference in the orientation of the compound nucleus spin results generally in the experimentally observed shape of the angular distribution.*

With the increase of nucleon energy the anisotropy of the angular distribution must increase because of the increased angular momentum transferred to the nucleus. When the target nucleus spin I_0 differs from zero the angular distribution will be more isotropic because of the greater isotropy of spin distribution of the compound nuclei. The angular distribution of photofission fragments is especially dependent on the initial spin when the angular momentum transferred to the nucleus is relatively small. Thus for $I_0\leq 1$ the angular distribution of fission fragments due to dipole radiation has maxima at $\vartheta = 0, \pi$; when the spin of the compound nucleus I is $I_0 \pm 1$ (compound nucleus spin oriented parallel to the photon moment), and for $\vartheta = \pi/2$ when $I = I_0$.

The angular distribution of the fragments is generally more isotropic the larger the spin of the fragments; however, the angular distribution of the fragments depends on the fission mechanism, especially when the spin of the fragments is comparable with the spin of the compound nucleus. The general theory of the angular distribution of particles with spin has been discussed frequently (see, for example, Ref. 6). For analysis of the experimental data on the angular distribution of fission fragments it is convenient to have simpler expressions giving the distribution of the components of fragment spins along the line joining the centers of mass of the fragments.

The general form of the wave function of two fragments in states τ_1 and τ_2 with orbital momentum l, and resulting from decay of a compound nucleus with spin (I, M), is

$$\psi_{IM} = \sum_{lS\tau} f_{lS\tau}(r) \sum_{m\mu} C^{IM}_{lmS\mu} \chi_{S\mu\tau} Y_{lm}(\mathbf{n}), \qquad (1)$$

where $\overline{n}$ is the unit separation vector of the fragments; $C^{c\gamma}_{a\alpha b\beta}$ are Clebsch-Gordon coefficients; $\chi_{S\mu\tau}$ is the internal wave function of the fragments for spin S; $|j_1 - j_2| \leqslant S \leqslant j_1 + j_2$, $\tau \equiv (\tau_1, \tau_2)$. In (1) $f_{lS\tau}$ is the radial wave function at infinity

$$f_{lS\tau} \sim \alpha_{lS\tau} \frac{1}{r} \exp\{ik_{S\tau} r\}.$$

In (1) we transform $\chi_{S\mu\tau}$ to a system of coordinates whose axis z' is parallel to $\overline{n}$

$$\chi_{S\mu\tau} = \sum_{\varkappa} D^S_{\mu\varkappa}(\mathbf{n}) \chi_{S\varkappa\tau},$$

where $D^S_{\mu\varkappa}$ is the transformation matrix [7], and $\varkappa$ is the component of S along z'. Then ψ_{IM} can be written as

$$\psi_{IM} = \sum g_{S\tau\varkappa} D^I_{\mu\varkappa}(\mathbf{n}) \chi_{S\varkappa\tau}, \qquad (2)$$

$$g_{S\varkappa\tau} = \sum_l \sqrt{\frac{2l+1}{4\pi}} f_{lS\tau} C^{I\varkappa}_{l0S\varkappa}$$

At infinity

$$g_{S\varkappa\tau} \sim \beta_{S\varkappa\tau} \frac{1}{r} \exp\{ik_{S\tau} r\}.$$

The relative probability $b_{S\varkappa\tau}$ of fragment production in the state $(S\varkappa\tau)$ is:

$$b_{S\varkappa\tau} = |\beta_{S\varkappa\tau}|^2 = \left|\sum_l \alpha_{lS\tau} \sqrt{\frac{2l+1}{4\pi}} C^{I\varkappa}_{l0S\varkappa}\right|^2. \quad (3)$$

In accordance with (2) the angular distribution of fission fragments from the compound nucleus with s; in (I,M) is*

$$f_{IM}(\vartheta) = \sum_{\substack{S \\ |\varkappa| \leqslant \min(I,S)}} b_{S|\varkappa|} \sum_{\varkappa = \pm|\varkappa|} |D^I_{M\varkappa}(\mathbf{n})|^2, \qquad (4)$$

where $b_{S|\varkappa|}$ denotes the probability for production of fragments in state $(S, \varkappa)$:

$$b_{S|\varkappa|} = \overline{b_{S\varkappa\tau}} = \frac{1}{4\pi} \sum_l (2l+1) \overline{|\alpha_{lS\tau}|^2} (C^{I\varkappa}_{l0S\varkappa})^2. \qquad (5)$$

The bar denotes summation over all observed states. In (5) interference terms (with $l \neq l'$) are neglected since these drop out through averaging over a large number of states of the system. The angular distribution of fragments for this reaction is

$$F_{I_0}(\vartheta) = \sum_{M_0 IM} \rho_{IM;\, I_0 M_0} f_{IM}(\vartheta), \qquad (6)$$

where $\rho_{IM;\, I_0M_0}$ is the probability for the formation of a compound nucleus in a state with spin (I, M), when the initial nucleus was in a state with spin (I_0, M_0). In Eq. (6), $M = M_\sigma + 1/2$ for nucleon-induced fission and $M = M_0 \pm 1$ for photofission.

Neglecting the rotation of the system as a whole** it is reasonable to assume that the axial symmetry of the system is not disturbed at any stage of the fission process. In this case $\varkappa$ is the integral of motion and $\varkappa$ can denote the state of the deformed nucleus.*** The value of $\varkappa$ in the fissioning nucleus determines the orientation of the nuclear axis of symmetry with respect to the spin direction of the compound nucleus (the compound nucleus spin I component along the nuclear axis is $\varkappa$) and, consequently, the distribution of $\varkappa$ determines the angular distribution of the fragments. A comparison with experiment enables us to determine the probability of fission for a given value of $\varkappa$. Thus, a comparison with the experimentally observed angular distribution of fragments from the photofission of Th^{232} (4, $E_\gamma \approx 8$ mev) gives the value 0.29 for b ($\varkappa = \pm 1$) with b ($\varkappa = 0$) taken as unity. As the excitation energy increases the level separation of the fissioning nucleus for different $\varkappa$ decreases, the distribution of $\varkappa$ becomes more uniform and the angular distribution becomes more isotropic. For neutron-induced fission this decrease of anisotropy associated with higher excitation energy may be compensated by an increase of angular momentum transferred to the nucleus. For sufficiently high excitation energies the different values of $\varkappa$ (with fixed S) are of practically equal probability: all spin orientations of the fragments become equally probable. Then fission with $S \geq I$ contributes isotropically to the angular distribution since with $S \geq I$ in (4) the summation is performed over all values of $|\varkappa| \leq I$. Fission with $S < I$ gives an anisotropic contribution to the angular distribution with the maximum of the angular distribution, just as for spinless fragments, perpendicular to the compound nucleus spin, although it decreases as S increases. For high excitation energies the fragment angular distribution from photofission of an even-even nucleus induced by dipole radiation may be written as $1/2 \sin^2\theta + \delta$ where δ is the relative probability for the formation of fragments with spin $S \geq 1$. We note that equal probabilities for different $\varkappa$, which follows from (5), also means equal probability of different orbital moments l which are allowed by a given S.

The experimentally observed dependence of the anisotropy of angular distribution on the mass ratio of the fragments is clearly due to the different excitations of the fissioning nuclei in symmetric and asymmetric fission.

Symmetry properties impose certain restrictions on fission and angular distribution. Thus it follows from (3) that with $\varkappa = 0$ fission into two equal fragments is forbidden if the spin of the compound nucleus is odd (particularly in the fission of an even-even nucleus by dipole radiation). The parity of the spatial wave function of these nuclei agrees with the parity of S[7] but the coefficients C^{I0}_{l0S0} vanish if the sum $I + S + l$ is odd. It is clear, however that this and other similar cases of forbidding of individual instances of fission are not of great importance.

I am profoundly grateful to A. B. Migdal, B. T. Geilikman and V. M. Galitskii for their interest and for valuable discussions.

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* This is obvious for spinless fragments; in this case the direction of fragment flight perpendicular to the fragment orbital momentum is also perpendicular to the compound nucleus spin.

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$$D^I_{M0} = \sqrt{\frac{4\pi}{2I+1}}\, Y_{IM},$$

$$\sum_{\varkappa = \pm|\varkappa| > 0} |D^I_{M\varkappa}|^2 = 2(-1)^{|\varkappa| - M} \sum_{\substack{\lambda=0 \\ (\text{even})}}^{2I} C^{\lambda 0}_{IMI-M} C^{\lambda 0}_{I|\varkappa|I-|\varkappa|} P_\lambda(\cos\vartheta).$$

*** Rotation of the system can be neglected if the level spacing of the fissioning nucleus is large compared with the characteristic rotational energy $E_{\text{rot}} \sim l(l+1)/2\mu R^2$ where μ is of the order of the reduced mass of the fragments.

**** The distribution of $\varkappa$ existing at the instant of fission is preserved at infinity since the centrifugal energy of the fragments can always be neglected compared with their kinetic energy.

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Translated by I. Emin
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Density of Linear Boiling of a Superheated Liquid Along the Track of an Ionizing Particle

G. A. ASKAR'IAN

(Submitted to JETP editor November 26, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 610-611 (March, 1956)

THE prospects for the utilization of bubble chambers[1-3] in experimental nuclear physics depend on the possibility of determining the ionizing power of particles from the bubble density distribution along a track. A number of theoretical papers[4-6] devoted to the design and study of the operation of bubble chambers throw no light on this very important feature of the new particle recorder, although the literature contains indications of the comparitively strong dependence of initial boiling density on the superheating of the liquid, which reflects the difficulty of calibrating the instrument.

The elementary theory of induced boiling which is developed in the present article is based on the electrodynamical model of stimulated disturbance of superheated liquids. Here it is assumed that the disturbance of the liquid results from the formation and mutual repulsion of closely situated, similarly charged, multimolecular groups (forming the walls of newly created microcavities) which are collected by ions appearing at the passage of an ionizing particle. (The theory developed below also contains the elements necessary for the construction of a theory of other possible forms of the electrodynamical model.)