performed over all values of $|\varkappa| \leq I$. Fission with $S < I$ gives an anisotropic contribution to the angular distribution with the maximum of the angular distribution, just as for spinless fragments, perpendicular to the compound nucleus spin, although it decreases as S increases. For high excitation energies the fragment angular distribution from photofission of an even-even nucleus induced by dipole radiation may be written as $1/2 \sin^2 \text{theta} + \delta$ where δ is the relative probability for the formation of fragments with spin $S \geq 1$. We note that equal probabilities for different $\varkappa$, which follows from (5), also means equal probability of different orbital moments l which are allowed by a given S.

The experimentally observed dependence of the anisotropy of angular distribution on the mass ratio of the fragments is clearly due to the different excitations of the fissioning nuclei in symmetric and asymmetric fission.

Symmetry properties impose certain restrictions on fission and angular distribution. Thus it follows from (3) that with $\varkappa = 0$ fission into two equal fragments is forbidden if the spin of the compound nucleus is odd (particularly in the fission of an even-even nucleus by dipole radiation). The parity of the spatial wave function of these nuclei agrees with the parity of S[7] but the coefficients C^{I0}_{l0S0} vanish if the sum $I + S + l$ is odd. It is clear, however that this and other similar cases of forbidding of individual instances of fission are not of great importance.

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* This is obvious for spinless fragments; in this case the direction of fragment flight perpendicular to the fragment orbital momentum is also perpendicular to the compound nucleus spin.

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$$D^I_{M0} = \sqrt{\frac{4\pi}{2I+1}}\, Y_{IM},$$

$$\sum_{\varkappa = \pm|\varkappa| > 0} |D^I_{M\varkappa}|^2$$

$$= 2(-1)^{|\varkappa| - M} \sum_{\substack{\lambda=0 \\ (\text{even})}}^{2I} C^{\lambda 0}_{IMI-M} C^{\lambda 0}_{I|\varkappa|I-|\varkappa|} P_\lambda(\cos\vartheta).$$

*** Rotation of the system can be neglected if the level spacing of the fissioning nucleus is large compared with the characteristic rotational energy $E_{\text{rot}} \sim l(l+1)/2\mu R^2$ where μ is of the order of the reduced mass of the fragments.

**** The distribution of $\varkappa$ existing at the instant of fission is preserved at infinity since the centrifugal energy of the fragments can always be neglected compared with their kinetic energy.

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Density of Linear Boiling of a Superheated Liquid Along the Track of an Ionizing Particle

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THE prospects for the utilization of bubble chambers[1-3] in experimental nuclear physics depend on the possibility of determining the ionizing power of particles from the bubble density distribution along a track. A number of theoretical papers[4-6] devoted to the design and study of the operation of bubble chambers throw no light on this very important feature of the new particle recorder, although the literature contains indications of the comparitively strong dependence of initial boiling density on the superheating of the liquid, which reflects the difficulty of calibrating the instrument.

The elementary theory of induced boiling which is developed in the present article is based on the electrodynamical model of stimulated disturbance of superheated liquids. Here it is assumed that the disturbance of the liquid results from the formation and mutual repulsion of closely situated, similarly charged, multimolecular groups (forming the walls of newly created microcavities) which are collected by ions appearing at the passage of an ionizing particle. (The theory developed below also contains the elements necessary for the construction of a theory of other possible forms of the electrodynamical model.)

In the calculation we shall assume that for the induced production of a bubble in the superheated liquid under normal operating conditions, it is sufficient that at a distance not exceeding the "localization interval" λ two positive ions should be formed whose electrons would receive during ionization the energy $\epsilon > \epsilon_d$. In fact, the condition for the localization of the ions and the condition for sufficient withdrawal of the electron component may be regarded as the definitions of the parameters λ and ϵ_d. It is evident that these parameters are independent of the ionizing power of the particles but are determined by the properties of the medium and the degree of superheating; the parameter ϵ_d takes into consideration the average condition under which the approach of oppositely charged centers is unimportant, at least during the time of formation of the bubble nucleus.

For the purpose of calculating the probability of establishing conditions sufficient for the creation of a bubble we divide the track of a δ-electron with energy ϵ into sections equal to λ. The average number of all positive ions produced by the δ-electron in the i the section is:

$$\bar{\nu}_i \approx (1/\varepsilon^*_{\text{ion}})\,(d\varepsilon/dx)_{x=x_i}\lambda;$$

(ϵ^*_{ion} is the portion of the entire energy expended required per ion). The average number of ions whose electrons received in ionization the energy $\epsilon > \epsilon_d$ is

$$\bar{\nu}_{i\,\text{d}} \approx (1/\varepsilon^*_{\text{d}})\,(d\varepsilon/dx)_{x=x_i}\lambda.$$

(Here ϵ^*_d is the portion of the total expended energy required per ion with sufficiently distant electron. We separate these ions out, and formally raise the effective ionization energy to ϵ_d. It is clear that ϵ^*_d exceeds ϵ_d but both are of the same order of magnitude, in analogy to the relation between ϵ_{ion} and the ionization energy.)

According to Poisson's equation, the probability of producing two ions in the distance λ_i with electrons sufficiently distant is

$$w(2\text{ in }\lambda_i) \approx \frac{\bar{\nu}^2_{i\,\text{d}}}{2!}\exp(-\bar{\nu}_{i\,\text{d}}) \approx \frac{\bar{\nu}^2_{i\,\text{d}}}{2},$$

since $\bar{\nu}_{i\,d} \ll 1$ (the coefficient 2 can be dropped since allowance for the fluctuations of ions at both ends of the interval gives a coefficient close to unity. Such a procedure amounts to the addition of the probability of production of two ions (with distant electrons) in the i th interval of the net of path subdivisions shifted by $\lambda/2$.

The probability of creating sufficient conditions for the formation of a cavity somewhere along the path of the δ-electron under consideration with initial energy ϵ is

$$\sum_i w(2\text{in}\lambda_i) \approx \varepsilon^{*-2}_{\text{d}} \sum (d\varepsilon/dx)^2_i \lambda^2_i$$

$$\approx \lambda\varepsilon^{*-2}_{\text{d}} \int \left(\frac{d\varepsilon}{dx}\right)^2 dx \approx \lambda\varepsilon^{*-2}_{\text{d}} \int_{\varepsilon^*_{\text{d}}}^{\varepsilon} \frac{d\varepsilon}{dx}\,d\varepsilon.$$

Using the well-known equation for energy loss $d\epsilon/dx \approx a/\epsilon$, which holds true in a wide range of δ-electron energies, we obtain

$$w(\varepsilon) \approx \lambda a \varepsilon^{*-2}_{\text{d}} \ln(\varepsilon/\varepsilon^*_{\text{d}}) + w(1\text{in}\lambda_1).$$

The number of δ-electrons with energy from ϵ to $\epsilon + d\epsilon$ produced by the ionizing particle per unit track length is

$$dN(\varepsilon) = KZ^2\beta^{-2}d\varepsilon/\varepsilon^2$$

(Ze and βc are the charge and velocity of the ionizing particle). Consequently,the mean number of bubbles per unit track length is

$$\bar{n} \approx \int_{\varepsilon^*_{\text{d}}}^{\varepsilon_{\max}} w(\varepsilon)\,dN(\varepsilon) \approx \frac{\lambda K Z^2 a}{\beta^2 \varepsilon^{*3}_{\text{d}}},$$

since the maximum possible transfer of energy to the electron is $\epsilon_{\max} \gg \epsilon_d{}^*$

The principal qualitative results of the theory are (1) the separation of the variables which characterize the ionizing particles and the variables which characterize the state of the medium, and (2) the linear dependence of the boiling density on the ionizing power of the particles: $\bar{n} \approx Z^2\beta^2\Phi$, where Φ is the state function of the medium.

We shall now estimate the order of magnitude of $\bar{n}$ for a relativistic, singly charged particle passing through diethyl ether at a temperature of 140 ° (the ether density under the given operating conditions is $\rho \approx 0.5$ g/cm^3). We have

$$K \approx \pi N_0 r_0^2 \rho mc^2 \approx 0{,}075\ mc^2$$

(N_0 is Avogrado's number; r_0 and m are the classical radius and mass of the electron). We also assume that $a \approx (mc^2/2)^2$. (For an estimate of the order of magnitude of a we have converted to density from the empirical equation for the energy losses of electrons in aluminum $d\beta_e/dx = 2.2\beta_e^{-3}$ cm^{-1} [7]).

Substituting the calculated values of the constants, we obtain

$$\bar{n} \approx 0{,}02\lambda\,(mc^2/\varepsilon^*_{\text{d}})^3 \approx 30 \text{ bubbles/cm}.$$

for admissible parameter values $\lambda \sim 3 \times 10^{-7}$ cm and $\epsilon_d \sim 0.3$ kev. (Experiment gives $n \sim$ from 10 to 100 depending on the degree of superheating.) A certain arbitrariness in the choice of specific values of the parameters λ and ϵ_d* which we have assumed for illustrative purposes in our estimate of the order of magnitude of $\bar{n}$, was due to the absence of data regarding the dependence of the parameters on the degree of superheating. This dependence is unimportant, however, for a comparison of the ionizing powers of two particles passing through the liquid when the extent of superheating is either unknown or not accurately determined. The comparative estimate is particularly useful when the bubble chamber is used with an accelerator, when, simultaneously with the tracks of particles participating in the investigated nuclear reactions, there are also recorded the tracks of particles of known energy emitted by the accelerator.

It is evident that all that has been said above may apply not only to a superheated liquid but also to a supersaturated solution of a gas in a liquid.[8] A gassy liquid behaves very much like a superheated one[9] when pressure is released, the decreased resistance of the liquid to disturbance (because of excess dissolved gas) lowers the required operating pressures and justifies an experimental attempt to realize a gas bubble chamber.

It can be expected that the above considerations concerning the role of the formation of molecular groups and the introduction of the concepts of the localization interval, withdrawal energy and sufficient conditions for the creation of microcavities will facilitate the study of the peculiar and complicated phenomena which take place in bubble chambers.

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The Elastic Scattering of Protons by Tritium

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THE experimental data on the angular distribution of $p - H^3$ scattering[1] fit very well on a curve of the form $a + b\cos\theta + c\cos^2\theta$ (see the Figure). In the angles measured (up to 54.7°) there is little Coulomb scattering. It is possible to carry through a phase analysis of the experimental data taking into consideration moments of relative motion $l = 0$ and 1. The vector scheme of the decomposition in this instance is:

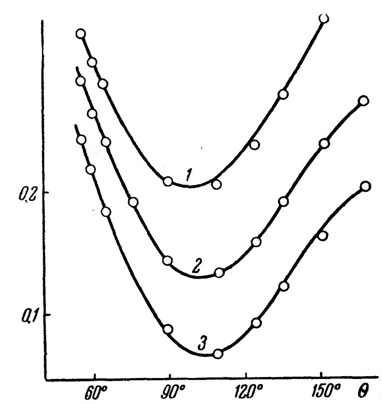
$$s_{1/2} + 1/2 = 0^+ + 1^+,\ p_{1/2} + 1/2$$
$$= 0^- + 1^-,\ p_{3/2} + 1/2 = 1^- + 2^-.$$

In agreement Ref. 2 we assume that the principal role is played by the phases

$$\delta_0 = \delta(s_{1/2},\ 0^+),\qquad \delta_1 = \delta(p_{3/2},\ 1^-)$$
$$\delta_2 = \delta(p_{3/2},\ 2^-).$$

The analysis shows that the experimental data cannot be accounted for if only three phases are used. It is necessary to take into account the phase which can materially increase the terms in $\cos\theta$ and $\cos^2\theta$, i.e., the phase $\delta_3 = \delta(p_{1/2}, 1)$.

For three values of proton energy we have obtained the phases which describe satisfactorily the angular distribution of the scattering (see the figure):



Curves of the shape $a + b\cos + c\cos^2\theta$ obtained from the phases mentioned in the text (θ is the angle in the center of mass system). o — experimental points. The curves are displaced relative to each other by 0.05; *1* — 2.54; *2* — 3.03; *3* — 3.50 mev