$$\eta(x) = \begin{cases} 1, & x_0 > 0 \\ 0, & x_0 < 0 \end{cases};$$

and the summation in (3) is over the number of mesons n and the number of nucleons and antinucleons λ.

The matrix element of the S-matrix for a transition of a meson from a state of momentum q to a state of momentum q' is related to the Green's function in the following way:

$$\langle p', q' \mid S \mid p, q\rangle \tag{4}$$
$$= -\int f^*_{q'}(x) f_q(y) K_x K_y \langle p' \mid x, y \mid p\rangle \, dx\, dy,$$

where $f_q(x) = (2q_0)^{-1/2} \exp[iqx]$ is a solution of the Klein-Gordon equation;

$$qx = \mathbf{qx} - q_0 x_0, \quad q_0 = \sqrt{\mathbf{q}^2 + \mu^2}.$$

Using this relation, and also the relations

$$\sum_{\mathbf{q}} f_q(\xi) f^*_q(\eta) = i\Delta^{(+)}(\xi - \eta);$$

$$\eta(x) = \frac{1}{2\pi i} \int_{-\infty}^{+\infty} \frac{e^{i\alpha x}}{\alpha - i\varepsilon} \, d\alpha,$$

we immediately obtain from (3)

$$\langle p', q' \mid S \mid p, q\rangle \tag{5}$$
$$= \frac{1}{2\pi i} \sum_{\mathbf{q}_1 \ldots \mathbf{q}_n, n, \lambda} \Big\{ \int \frac{d\alpha}{\alpha - i\varepsilon} \langle p', q' + \alpha \mid S \mid \lambda, q_1 \ldots q_n\rangle$$
$$\langle \lambda, q_1 \ldots q_n \mid S \mid p, q + \alpha\rangle$$
$$+ \int \frac{d\alpha}{\alpha - i\varepsilon} \langle p', -q + \alpha \mid S \mid \lambda, q_1 \ldots q_n\rangle$$
$$\langle \lambda, q_1 \ldots q_n \mid S \mid p_1, -q' + \alpha\rangle \Big\}$$
$$+ \int dx\, dy\, \delta(x_0 - y_0) f^*_{q'}(x) f_q(y) \langle p' \mid [K_x \varphi(x), \dot\varphi(y)]_- \mid p\rangle.$$

Here $\langle \lambda q_1 \ldots q_n \mid S \mid p, q + \alpha\rangle$ is the matrix element of the S-matrix for a transition of a meson with momentum q, energy $q + \alpha$ and a nucleon with energy-momentum p to a state with n mesons and λ nucleons and antinucleons. The last term on the right-hand side of (5) is zero if the interaction is linear in the meson field. If the sum in (5) is restricted to the states ($n = 0$, $\lambda = 0$) and ($n = 1$, $\lambda = 1$) and we go over to matrix elements on the energy surface then we get Low's equation.

We emphasize that the relation (5) with the restriction given above becomes an equality only if there is provided the inhomogeneous term ($n = 0$, $\lambda = 1$), which depends on the concrete form of the interaction. The inhomogeneous term can be found exactly only in the limit $q, q' \to 0$, therefore the applicability of the equation is essentially limited to the nonrelativistic domains of nucleon energy and meson energy. For arbitrary q, q' the inhomogeneous term can be expressed in terms of the exact renormalized one-particle Green's function and vertex part. Therefore, attempts to treat the equations found by Low relativistically run into kinds of problems; first, the exact forms of the renormalized one-particle Green's function and vertex part are unknown, and secondly, as shown in the works cited,[3] the renormalized Green's function possesses nonphysical poles, reflecting the fact that the renormalized coupling constant becomes zero.[4]

Note added in proof. Analogous questions are considered in recently appearing work.[5]

[1]F. E. Low, Phys. Rev. 97, 1392 (1955).

[2]Lehman, Symanzik and Zimmerman, Nuovo Cimento 1, 205 (1955).

[3]Landau, Abrikosov and Khalatnikov, Dokl. Akad. Nauk SSSR 95, 497, 773, 1177 (1954); 96, 261 (1954). Abrikosov, Galanin, and Khalatnikov, Dokl. Akad. Nauk SSSR 97, 793 (1954).

[4]E. S. Fradkin, Soviet Phys. JETP 1, 604 (1955). L. D. Landau and I. Ia. Pomeranchuk, Dokl. Akad. Nauk SSSR 102, 489 (1955). L. D. Landau and I. Ia. Pomeranchuk, Dokl. Akad. Nauk SSSR 102, 489 (1955).

[5]Duimio, Gulmanelli and Scotti, Nuov. Cim. 2, 1132 (1955).

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Isomeric States of Deformed Nuclei

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As is well known, the term "isomer" denotes an excited state of a nucleus characterized by a long lifetime. The small probability of decay of such states may be due to a large difference between their spins and that of the ground state ($\Delta I \geq 3$).[1] Besides this, the decay probability of the excited state depends substantially on the nature of the levels between which γ-transitions are taking place, and in some cases (the nuclei of Lu^{177}, Ta^{181}, Re^{187}, Np^{237}, Pu^{239} and others) the decay probability turns out to be small (and the lifetime large), although the spins of the isomeric states in the nuclei mentioned differ from the spins of lower levels by one or two units.[2]

In the present note, by "isomeric states" we shall mean only those states, whose large lifetime is due to a large difference of their spin and the spin of the ground state; we shall further restrict our considerations to nuclei of odd mass number A.

The explanation of the properties of such isomeric states has been one of the most substantial successes of the one-particle nuclear shell model.[2-6] Within the framework of this model it turned out to be possible to describe correctly, in the overwhelming majority of cases, the multipolarity of isomeric transitions of type M4 and E3, and also to explain the concentration of isomers in the so-called "islands of isomerism" $39 \leq Z, N \leq 49$: $63 \leq Z, N \leq 81$; $93 \leq Z, N \leq 125$[3,4]. Out of the 86 known cases of isomerism in nuclei with odd A only two ($_{42}Mo^{93}_{51}$ and $_{31}Ga^{65}_{34}$) lie outside of the boundaries of the islands, while in four or five cases multipole order of isomeric transitions of type M3, E4 is observed which is not describable by the single-particle model.

However, a more detailed analysis of isomeric states with the single-particle model meets with a number of difficulties. According to the model isomers with an odd number of protons and an odd number of neutrons should occur equally often, and the boundaries of the islands for them should coincide. In reality it turns out that, in general, they do not coincide. Thus, in odd neutron nuclei, isomeric states are observed with $63 \leq N \leq 81$, while in odd proton nuclei, only with $77 \leq Z \leq 81$. In the next island, $93 \leq N \leq 125$, isomeric transitions described by the single-particle model (type M4) are observed only with $N > 113$ (10 cases), while in nuclei with $93 \leq N \leq 115$, only 5 isomeric transitions of an anomalous type (E3, M3, E4) are observed. Finally, the single-particle shell model does not account for isomeric transitions of type E3 which occur with $Z, N = 41, 43, 45, 47$.[6]

Recently, attempts have been made to overcome some of these difficulties[5,7,8]. However, they cannot be regarded as satisfactory, since they do not explain all of the facts. It will be shown below that taking into account of the deformation of the nucleus makes it possible to remove all of the mentioned difficulties without any additional assumptions. From an analysis of various experimental data (quadrupole moments, isotope shifts, etc.) it follows that the deformation of nuclei in which at least one of the shells (proton or neutron) is close to being filled, is small. For such nuclei, in calculating the single-particle levels in the first approximation, one can use a spherically-symmetrical potential of the same type as in the single-particle model.[5] For this reason, the states of weakly deformed nuclei may be characterized by an exact quantum number j (j is the angular momentum of the odd particle), and hence they may be described in the terminology of the single-particle model as $p_{1/2}$; $g_{9/2}$; $d_{5/2}$; $d_{3/2}$; $h_{11/2}$ etc. For such nuclei all the basic results of the single-particle model hold, and in the overwhelming majority of cases one finds in these nuclei the usual isomeric transitions of types M4 or E3. However, as soon as we pass to nuclei in which both shells are far from being filled, the situation changes essentially, since here it becomes necessary to take into account the nonsphericity of the nuclei. In such nuclei the potential will not be spherically symmetrical. In the case of axial symmetry in deformed nuclei, not the total angular momentum j, but its projection Ω on the axis of symmetry, is an exact quantum number.

In order for the phenomenon of isomerism to occur, it is necessary that either the ground or the excited state of the nucleus possess a large spin $I = \Omega = 9/2, 11/2$ or $13/2$. In slightly deformed nuclei, large values of the spin $I = j$ will occur in all nuclei with an odd number of nucleons in the level j. It is easy to show that in filling the levels $j = 9/2; 11/2; 13/2$ the number of such nuclei is 5,6,7 $[(2j+1)/2]$. At the same time in nuclei with a marked deformation, the maximum value of the spin $I = \Omega = j$ occurs only once, since in a level with a given Ω (in particular, the largest one) only two nucleons may be present. Thus it may be expected that levels with large spins in deformed nuclei will occur considerably more rarely than predicted by the single-particle model. Since the deformation removes the degeneracy of the energy levels in the quantum number Ω, their number is considerably increased. As a result, the probability of occurrence of levels with strongly varying spins ($\Delta I \geq 3$) next to each other, is substantially reduced.

The two above-mentioned factors have as a consequence that in strongly deformed nuclei ($63 \leq Z \leq 75$; $93 \leq N \leq 115$) favorable conditions for the existence of isomeric states are not realized. If one uses the single-particle level scheme obtained by Nillsen which takes deformation into account[10], it is possible to indicate in which nulcei of this region isomeric states should occur, and also to determine the possible multipolarities of the transitions into these levels. An analysis based on this scheme shows that such nuclei must be: $_{66}Dy_{95}^{165}$ with isomeric transition of type E3[2]; $_{72}Hf_{107}^{179}$ (E3, M3)[2]; $_{74}W_{109}^{183}$ (E3, M3)[2]; $_{74}W_{111}^{185}$ (E3, M3)[2]; $_{76}Os_{115}^{191}$ (M3 + E4)[11]; as well as $_{68}Er_{99}^{167}$; $_{72}Hf_{105}^{177}$; $_{70}Yb_{101}^{171}$. In the first five nuclei isomeric states have already been observed, and their expected multipolarity (which agrees with experimental data) is given in parentheses. In an analogous manner it is possible to explain the absence of an isomeric state in the heaviest isotope $_{43}Tc_{58}^{101}$ [12], since this nucleus is more strongly deformed than the lighter isotopes Tc^{93-99} in which isomeric states are observed. Taking account of deformation makes it also possible to explain the appearance in the island $41 \leq Z, N \leq 47$ of isomeric transitions of type E3[9].

Thus, taking into account of deformation makes it possible to explain the absence of isomeric states in nuclei with $63 \leq Z \leq 75$. the small number of isomeric transitions and their anomalous multipolarity with $93 \leq N \leq 115$, as well as the absence of an isomeric state in Tc^{101} and the appearance of transitions of type E3 in the region $39 < Z, N < 49$.

In conclusion, it is a pleasant duty to express my deep gratitude to L. A. Sliv for detailed discussion of this paper.

1 C. E. Weizsäcker, Naturwiss 24, 813 (1936).

2 M. Goldhaber and R. D. Hill, Rev. Mod. Phys. 24, 179 (1952).

3 L. W. Nordheim, Phys. Rev. 75, 1894 (1949).

4 E. Feenberg and K. C. Hammack, Phys. Rev. 75, 1877 (1949).

5 M. G. Mayer, Phys. Rev. 78, 16 (1950).

6 M. Goldhaber and A. W. Sunyar, Phys. Rev. 83, 906 (1951).

7 R. Katz, Phys. Rev. 91, 1487 (1953).

8 G. Scharff-Goldhaber, Phys. Rev. 90, 587 (1953).

9 A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab. Mat. – fys. Medd., 27, 16 (1953).

10 S. Nillsen, Kgl. Danske Videnskab, Selskab., Mat. – fys. Medd. 29, 14 (1955).

11 W. Michelich, Bull. Amer. Phys. Soc. 30, 46 (1955).

12 D. R. Wiles, Phys. Rev. 93, 181 (1954).

Translated by A. V. Bushkovitch
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Energy Spectrum of High Energy Ionizing Particles Passed Through a Thick Layer of Matter

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THE energy spectrum of a fast ionizing particle after passage through a thin layer of matter for which the average energy loss $\overline{\Delta E} \ll E$ is determined by the relation

$$s \equiv \overline{\Delta E}/\varepsilon_{\max} L_i,$$

where L_i is the well-known ionization logarithm and $\epsilon_{\max}$ (E) is the maximum energy loss per ionizing collision where m is the electron rest mass,

$$L_i = 2\left(\ln \frac{2m\beta^2}{I_{\text{cp}}(1-\beta^2)} - \beta^2\right), \tag{1}$$

$$\varepsilon_{\max} = \frac{2m(E^2 - \mu^2)}{\mu^2}, \quad \beta = v/c,$$

where m is the rest mass of an electron, μ the rest mass of the ionizing particle and I_{ave} is the ionization potential. If $W(\epsilon, E)$ is the probability per unit path length of a collision with an energy loss ϵ, then the distribution function for energy loss Δ after passage through a layer δx has the form[1,2]:

$$f(\delta x, \Delta) = (2\pi)^{-1} \tag{2}$$

$$\times \int_{-\infty}^{\infty} dz \exp\left\{iz\Delta - \delta x \int_0^{\infty} W(\varepsilon, E)(1 - e^{-iz\varepsilon})\, d\varepsilon\right\}.$$

For $S \ll 1$ Eq. (2) gives the curve due to Landau,[1] and for $S \gg 1$ it yields a Gaussian distribution.[2]

For the case of a thick layer ($S \gg 1$) we shall obtain a more accurate distribution function than the Gaussian. If we allow first $s \gg 1$, but with $\overline{\Delta E} \ll E$, then the distribution function can be expanded in Hermite polynomials:

$$\varphi(\delta x, y) = (2\pi)^{-1/2} e^{-y^2/2}\left(1 + \sum_{n \geqslant 3} a_n H_n(y)\right); \tag{3}$$

$$H_n(y) = (-1)^n e^{y^2/2} \frac{d^n}{dy^n} e^{-y^2/2}, \quad a_n(\delta x) \tag{4}$$

$$= (n!)^{-1} \int_{-\infty}^{\infty} \varphi(\delta x, y) H_n(y)\, dy,$$