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Generalized Method for Calculating Damping in Relativistic Quantum Field Theory

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An infinite system of coupled equations is constructed, of which each describes a process involving the emission and absorption of a certain number of particles. This system is shown to be equivalent to the Tomonaga-Schwinger equation. The solution, which is derived by a process of successive substitutions, leads to results for the S-matrix which generalize the results of the theory of radiation damping.

IT has often been pointed out that the solution of the problems of the quantum theory of fields by means of perturbation theory leads to difficulties in a number of cases. In order to eliminate these difficulties, and to study the limits of applicability of the results obtained by means of perturbation theory, one has to develop a more consistent method of solving the equations of field theory. The present paper represents an attempt to develop a method of solution which guarantees that the normalization remains correct, and from which one obtains as an approximation the results of damping theory, and, as a zero-order approximation, perturbation theory.

We start from the usual equation for the scattering matrix:

$$i\hbar c \delta U[\sigma] / \delta \sigma(x) = \mathcal{H}(x) U[\sigma], \qquad (1)$$

where $\mathcal{H}(x)$ as usual, is the operator of the interaction Hamiltonian, which, if we limit the discussion to the case of the electron-positron and the electromagnetic fields (we consider these fields for definiteness, although the result can be generalized directly) takes the form

$$\mathcal{H}(x) = -j_\mu A_\mu \qquad (2)$$

$$= -\frac{e}{2}[\bar{\psi}(x)\gamma_\mu\psi(x) - \psi(x)\bar{\psi}(x)\gamma_\mu] A_\mu(x).$$

The operators $\bar{\psi}$, ψ, A_μ satisfy the equations of free fields and therefore, if we write

$$\bar{\psi}(x) = \bar{u}(x) + v(x); \quad \psi(x) = u(x) + \bar{v}(x); \qquad (3)$$

$$A_\mu(x) = A_\mu^{(+)}(x) + A_\mu^{(-)}(x),$$

we may interpret $A_\mu^{(+)}$, $\bar{u}$ and v as free-particle creation operators (creating photons, electrons and positrons, respectively) and $A_\mu^{(-)}$, u and $\bar{v}$ as absorption operators for the same free particles.

This basic equation is quite general, i.e., it describes any arbitrary process involving the creation and annihilation of particles in arbitrary states due to their mutual interaction. Moreover, as has already been pointed out, [1] Eq. (1) does not describe each of these processes separately, but contains them all simultaneously. In the process of solving the equation by perturbation theory, this connection between different processes is lost sight of in practice, and each is considered in isolation. We shall try to put forward a method of solution which is free from this disadavantage, and which uses this connection between different processes as a starting-point.

The physical picture behind this change is the following: As a result of the interactions between the particles the probability of the initial state decreases in the course of time, while simultaneously the probabilities of the mutually com-

peting transitions into new states are building up; the transitions themselves look like the absorption of free particles with certain energies and momenta, and the creation of particles with others.

Our task is to construct a solution which corresponds to this physical picture. One of the simplest ways to this end is to put

$$U = \sum_{\xi=0}^{\infty} U^{(\xi)}, \qquad (4)$$

where (ξ) stands as a symbol for the set of suffixes (i,j; n,m; ρ,σ). The operator $U^{(\xi)}$ which we have introduced represents the transition matrix for a transition in which i photons, n electrons and p positrons present in the initial state are absorbed, and j photons, m electrons, and σ positrons emitted (the emitted particles belong to the final state). In this way we clearly take into account the fact that the general scattering matrix consists of a sum of the operators of all particular types of transition.

However, the mere "splitting" of the transition matrix U into a sum of partial transition matrices $U^{(\xi)}$ is not enough. One has to find a set of equations which connect all the $U^{(\xi)}$ and solve these with some or other degree of accuracy. We Have previously given such equations [1] for a simplified problem. The method used here of "splitting" the equations can be extended to the general case. The essence of the method is to replace the basic equation (10), which, with the use of (4), may be written in the form

$$i\hbar c \frac{\delta \sum_{\xi} U^{(\xi)}}{\delta\sigma(x)} = -\frac{e}{2}[(\bar{u}(x) \qquad (5)$$

$$+ v(x))\gamma_\mu (u(x) + \bar{v}(x)) - (u(x) + \bar{v}(x))(\bar{u}(x)$$

$$+ v(x))\gamma_\mu](A_\mu^{(+)}(x) + A_\mu^{(-)}(x))\sum_{\xi} U^{(\xi)}[\sigma],$$

by a system of equations which (a) when added together replace (5), (b) when taken separately, would each describe a process with the emission and absorption of a given number of particles, as distinct from (5) which contains the whole set of possible processes. The fact that an equation describes a process with a fixed number of emitted and absorbed particles means that each term in the equation must describe the emission and absorption of the same number of particles.

In "splitting" Eq. (5) one must note that $U^{(\xi)}$ contains i photon absorption operators $A^{(-)}$, and j photon creation operators $A^{(+)}$. Similarly, it contains n operators u, m operators $\bar{u}$, ρ operators v and σ operators $\bar{v}$. The system of equations obtained after this "splitting" may be written schematically in the form

$$i\hbar c \delta U^{(\xi)} / \delta\sigma = \sum H_{(\xi')}^{(\xi)} U^{(\xi')}, \qquad (6)$$

where $H_{(\xi')}^{(\xi)}$ is an interaction operator which represents one of the sixteen terms in $\mathcal{H}(x)$, and where the labels in ξ' may differ from the corresponding labels in ξ on the left-hand side at most be one each (some of the numbers n, n' m, m'; ρ, ρ'; σ, σ' may also be the same). One may show without difficulty that the system of equations (6) is consistent, but we shall not give the proof here.

We shall not write out the right-hand side of Eq (6) explicitly, because of its unwieldy form (it contains 128 terms) but we shall only discuss the method of construction of these terms, and consider a number of typical examples.

The values which the labels ξ' may take in a term on the right-hand side of (6) depend on which of the operators $A^{(+)}$, $A^{(-1)}$, $\bar{u}$, u, $\bar{v}$, v occur in that term. For terms containing $A^{(-)}$, $i'=i-1$; $j'=j$, or $i'=i$, $j'=j+1$. In the terms with $A^{(+)}$, $i'=i$, $j'=j-1$, or $i'=i+1$; $j'=j$. In quite a similar way one can express the labels n', m' and $\rho'\sigma'$ in terms of n, m, ρ and σ according to which operators of the electron-positron field occur in the term under discussion. The only difference is that each term contains two electron-positron field operators, which may be grouped in pairs: u and $\bar{u}$, v and $\bar{v}$, u and v, and $\bar{u}$ and $\bar{v}$, whereas the electromagnetic field operator occurs only once. This difference reflects the inportant fact that photons are absorbed and emitted singly, whereas the creation and annihilation of the particles of the electron-positron field can occur only in pairs (electron and positron). This causes no essential complication in the determination of the labels n', m', ρ' and σ', but in writing them down one has to remember the effect of the various spinor operators.

Consider a few terms as examples:

1) $\bar{u}\gamma_\mu A_\mu^{(-)} u U^{(i-1,\, j;\, n-1,\, m-1;\, \rho,\, \sigma)}$.

This term describes the absorption of a photon, and the absorption of an electron in one state, and its emission in another; the total number of spinor particles therefore remains the same, and the process looks like the transition of an electron from one state to another with the absorption of a photon.

2) $\bar{u}\gamma_\mu A_\mu^{(+)} u U^{(i,\, j-1;\, n,\, m;\, \rho,\, \sigma)}$.

This term corresponds to the emission of a photon in the transition of an electron from one state to another. However, in distinction from the case considered above, the last transition is connected withthe absorption (and emission) of an electron which was previously emitted (absorbed), and not withthe absorption or emission of "new" electrons.

3) $\bar{u}\gamma_\mu A_\mu^{(-)}\bar{v}U^{(i,\,j+1;\,n,\,m-1;\,\rho,\,\sigma-1)}$.

This term corresponds to the absorption of a previously emitted photon and the simultaneous creation of an electron-positron pair.

4) $v\gamma_\mu A_\mu^{(+)}uU^{(i+1,\,j;\,n,\,m+1;\,\rho,\,\sigma+1)}$.

This term corresponds to the emission of a photon which was previously absorbed, and the absorption of a previously emitted electron-positron pair.

5) $v\gamma_\mu A_\mu^{(+)}\bar{v}U^{(i,\,j-1;\,n,\,m;\,\rho-1,\,\sigma-1)}$.

This term corresponds tothetransition of a positron from one state to another with the emission of a photon.

These examples will suffice to showthat all kinds of possible processes find their expression in the equations (6).

The unwieldy form ofthe equations (6) is no unsurmountable obstacle for their analysis. In a number of cases a kind of "operator dimension rule" proves convenient: all terms in the equation must contain $A^{(+)}$, $A^{(-)}$, u, $\bar{u}$, v, $\bar{v}$ to similar degrees; it should, however, be remembered that certain bilinear combinations of creation and annihilation operators of certain fields have "zero" dimension (if the operators relate to the same particles) in other words they are c-numbers.

It is easy to see that the sum of all the equations (6) gives the original Eq (5) , as it should do. Indeed, the sum of the left-hand sides of (6) gives

$$i\hbar c\sum_\xi \frac{\delta U^{(\xi)}}{\delta\sigma} = i\hbar c\,\frac{\delta}{\delta\sigma}\left[\sum_\xi U^{(\xi)}\right] = i\hbar c\,\frac{\delta U}{\delta\sigma}\,. \tag{7}$$

The sum of the right-hand sides is

$$\sum_\xi\sum_{\xi'} H_{(\xi')}^{(\xi)}U^{(\xi')} = \sum_{\xi'}\mathcal{H}U^{(\xi')} = \mathcal{H}U. \tag{8}$$

It is easy to see that the sum of (6) contains all the terms which occur on the right-hand side of (5) and that each term occurs only once.

One can also show that the original equation (10) and the requirements (a) and (b) determine the system (6) uniquely. Indeed, if we add to the various equations (6) further terms whose sum vanishes, then requirement (a) would remain satisfied, but (b) would be violated. And, similarly, any addition to (6) which would satisfy (b), would violate (a). Hence the transition from (1) to (6) subject to the conditions (a) and (b) is unambiguous.

We now turn to the problem of solving (6), i.e.,, of finding $U^{(\xi)}$. Consider first time-dependent problems (stationary states will be discussed later) i.e. we require a solution which satisfies the initial condition

$$U^{(0)}[\sigma_0] = 1,\quad U^{(\xi)}[\sigma_0] = 0, \tag{9}$$

if (ξ) (i.e. at least one of the labels i, j; n, m; ρ, σ) differs from zero.

We look for the solution by the method of substitution, which consists in inserting, on the right-hand side of (6) for the operator $U^{(\xi')}$ its value

$$U^{(\xi')} = -\frac{i}{\hbar c}\left(\frac{\delta}{\delta\sigma}\right)^{-1}\sum_{\xi''}H_{(\xi'')}^{(\xi')}U^{\xi''}. \tag{10}$$

The operator $(\delta/\delta\sigma)^{-1}$ which is the inverse of a functional derivative, may in all the cases which we shall meet here, be interpreted as an integral over the four-dimensional region bounded by the hypersurfaces σ_0 and σ.

As a result of substituting (10) on the right-hand side of (6) there occurs a $U^{(\xi'')}$ with new labels (ξ''). This we can in turn express in terms of $U^{(\xi)}$, etc. As a result of such successive substitutions one obtains an equation of the type

$$\frac{\delta U^{(\xi)}}{\delta\sigma} = \sum_{q,\,\xi',\,\xi'',\,\ldots}\left(\frac{-i}{\hbar c}\right)^{i+j+2q} \left[H_{(\xi')}^{(\xi)}\left(\frac{\delta}{\delta\sigma}\right)^{-1}\cdots\left(\frac{\delta}{\delta\sigma}\right)^{-1}H_{(\xi_{i+j+2q})}^{(\xi_{i+j+2q-1})}\right]U^{(0)} + \sum_{l,\,\xi',\,\xi'',\,\ldots}\left(\frac{-i}{\hbar c}\right)^{2l} \left[H_{(\xi')}^{(\xi)}\left(\frac{\delta}{\delta\sigma}\right)^{1}\cdots\left(\frac{\delta}{\delta\sigma}\right)^{-1}H_{(\xi_{2l})}^{(\xi_{2l-1})}\right]U^{(\xi)} + Q. \tag{11}$$

Here we have divided the terms on the right-hand side into three groups. The first consists of the terms which contain $U^{(0)}$. Each of these contains i, n, ρ annihilation and j, m, σ creation operators for photons, electrons and positrons, respectively. In addition, these terms may contain q operators $A^{(+)}$ and q operators $A^{(-)}$, which form a combination $(A^{(+)}A^{(-)})$ of "operator dimension zero". *

*And a corresponding number of combinations of "operator dimension zero" of electron-photon field operators (uu), (vv) since to every electromagnetic field operator belong two operators of the electron-positron field.

The second group of terms in (11) contains $U^{(\xi)}$, where (ξ) is that label which occurs on the left-hand side of the equation. The structure of the terms of this second group differs from that of the first group in that the numbers of photon emission and absorption operators contained in them are equal (and equal to l).

Finally, a third group of terms, which we have denoted by Q, contain those operators $U^{(\xi')}$ whose labels (ξ') are neither zero, nor equal to that which stands on the left-hand side of (11). By carrying the process of substitution far enough, we may make the order of the terms Q as high as we like, so that, at any rate in electrodynamics where the coupling constant is small, Q may be neglected.

In solving concrete problems, we shall restrict the discussion to terms of a certain order, and neglect Q. To neglect Q is similar to the device of cutting off the chain of linked equations in the Tamm Dancoff method.[2] Omitting Q, we are left with the following equation to be solved:

$$\delta U^{(\xi)}[\sigma]/\delta\sigma(x) = K_0^{(\xi)} U^{(0)}[\sigma] + K_l^{(\xi)} U^{(\xi)}[\sigma], \tag{12}$$

where

$$K_0^{(\xi)} = \sum_{q=0}^{\infty} \left(\frac{-i}{\hbar c}\right)^{i+j+2q} B^{(i+q,\, j+q)} \left\{ H_{(\xi')}^{(\xi)} \left(\frac{\delta}{\delta\sigma}\right)^{-1} \cdots \left(\frac{\delta}{\delta\sigma}\right)^{-1} H_{(\xi_{i+j+2q})}^{(\xi_{i+j+2q-1})} \right\}; \tag{13}$$

$$K_l^{(\xi)} = \sum_{l=1}^{\infty} \left(\frac{-i}{\hbar c}\right)^{2l} B^{(l,\, l)} \left\{ H_{(\xi')}^{(\xi)} \left(\frac{\delta}{\delta\sigma}\right)^{-1} \cdots \left(\frac{\delta}{\delta\sigma}\right)^{-1} H_{(\xi_{2l})}^{(\xi_{2l-1})} \right\}.$$

In line with what was said above, the operators $B^{(i+q,\, j+q)}\{\ldots\}$ denote the sum of all possible permutations of $A^{(+)}$ and $A^{(-)}$ in operators of the type

$$H_{(\xi')}^{(\xi)} \left(\frac{\delta}{\delta\sigma}\right)^{-1} \cdots \left(\frac{\delta}{\delta\sigma}\right)^{-1} H_{(\xi_i)}^{(\xi_{i-1})},$$

containing $i + q$ operators $A^{(-)}$ and $j + q$ operators $A^{(+)}$, in such a way that q pairs of such operators refer to the same photon states (and form bilinear combinations which are c-numbers).

Equation (12) by itself is not sufficient. For a complete formulation of the problem, we must also write down an equation for $U^{(0)}[\sigma]$. This equation could be derived by the method of successive substitutions from the system (6); however, we shall not derive it in that manner, but obtain it directly from (12) by putting there $(\xi) = (0)$:

$$\frac{\delta U^{(0)}[\sigma]}{\delta\sigma(x)} = \sum_{l=1}^{\infty} \left(\frac{-i}{\hbar c}\right)^{2l} B^{(l,\, l)} \left\{ H_{(\xi')}^{(\xi)} \left(\frac{\delta}{\delta\sigma}\right)^{-1} \cdots \left(\frac{\delta}{\delta\sigma}\right)^{-1} H_{(\xi_{2l})}^{(\xi_{2l-1})} \right\} U^{(0)}[\sigma] = K^{(0)} U^{(0)}[\sigma]. \tag{14}$$

The final form of the equations to be investigated is

$$U^{(\xi)}[\sigma] = \int_{\sigma_0}^{\sigma} K_0^{(\xi)} U^{(0)}[\sigma]\, d\omega + \int_{\sigma_0}^{\sigma} K_l^{(\xi)} U^{(\xi)}[\sigma]\, d\omega; \tag{15a}$$

$$U^{(0)}[\sigma] = 1 + \int_{\sigma_0}^{\sigma} K^0 U^{(0)}[\sigma]\, d\omega. \tag{15b}$$

In forming the Eqs. (15) we have made use of the initial condition (9).

Our next task consists in considering the Eq. (15).

It goes without saying that the method of successive substitutions, with the omission of the Q terms, which we used in obtaining (15) is not the only way of solving the fundamental set (6). Even considering only approximate solutions one could have given other ways of approach, which might be more convenient, for example, in the case of strong coupling. However, we shall confine our attention to the discussion of the Eqs. (15) in view of their relative simplicity. Indeed, as has already been stated, our initial set (6) is most unwieldy. Although we have obtained our Eqs. (15) from (6), they can in practice also be written down directly without reference to (6). For this purpose we must only observe the rule of "operator dimensions", according to which all the terms in each equation must describe the emission and absorption of the same number of particles (not only of photons, but of each other kind). One must then also allow for the fact that bilinear expressions of absorption and

emission operators relating to the same particle have "operator dimension" zero.

From this "operator dimension" rule it is at once evident, in particular, that $U^{(\xi)}$ may differ from the corresponding quantity obtained from perturbation theory, $U^{(\xi)}_{\text{pert}} = U^{(\xi)}_0$ only by a c-number factor (i.e., a factor of "operator dimension" zero). Using this fact, and noting also that

$$U^{(\xi)}_0 = \int_{\sigma_0}^{\sigma} K^{(\xi)}_0 \, d\omega, \tag{16}$$

we shall look for a solution of (15) in the form

$$U^{(\xi)}[\sigma] = \int_{\sigma_0}^{\sigma} K^{(\xi)}_0 \, U^{(0)} \varepsilon^{(\xi)} \, d\omega, \tag{17}$$

where $\epsilon^{(\xi)}$ is a certain unknown c-number quantity which remains to be determined, and $U^{(0)}[\sigma]$ is found from Eq. (15b). It is easy to see that the insertion in (12) of the value of $U^{(0)}[\sigma]$ obtained from (15b) by iteration is equivalent to an increase of q in the expression for $K^{(\xi)}_0$.

We may therefore write down the following equation for the determination of the quantity $\epsilon^{(\xi)}$:

$$K^{(\xi)}_0 \varepsilon^{(\xi)} = K^{(\xi)}_0 + K^{(\xi)}_l \int_{\sigma_0}^{\sigma} K^{(\xi)}_0 \varepsilon^{(\xi)} \, d\omega. \tag{18}$$

We shall be interested only in the asymptotic solution for $\sigma \to \infty$, $\sigma_0 \to -\infty$, and this makes it possible to write the equation for $\epsilon^{(\xi)}$ in the form

$$\varepsilon^{(\xi)} = 1 + \overline{K}^{(\xi)}_0 K^{(\xi)}_l \int_{-\infty}^{\infty} K^{(\xi)}_0 \varepsilon^{(\xi)} \, d\omega \Big/ \overline{K}^{(\xi)}_0 K^{(\xi)}_0, \tag{19}$$

where $\overline{K}_0(\lambda)$ is the conjugate of $K^{(\xi)}_0$.

We have written (19) in a form which makes it obvious that $\xi^{(\xi)}$ is a c-number. Indeed all coefficients in (19) are operators in diagonal form, and this proves our statement that $\xi^{(\xi)}$ is a c-number.

We see from the relation (17) that the result of perturbation theory corresponds to the zero-order approximation $\xi^{(\xi)} = 1$ in the solution of (19).

If we represent $\xi^{(\xi)}$ in the form

$$\varepsilon^{(\xi)} = \frac{1}{(1 - R^{(\xi)})}, \quad R^{(\xi)} = \frac{K^{(\xi)}_0 K^{(\xi)}_l \int_{-\infty}^{\infty} K^{(\xi)}_0 \, d\omega}{\overline{K}^{(\xi)}_0 K^{(\xi)}_0}, \tag{20}$$

we find the following expression for the S matrix:

$$S^{(\xi)} = \int_{-\infty}^{\infty} K^{(\xi)}_0 \frac{d\omega}{1 - R^{(\xi)}}. \tag{21}$$

If we restrict the discussion to one-electron problems, and neglect in the expression for $K^{(\xi)}$ all terms with $q > 0$, then the value (21) for $S^{(\xi)}$ reduces to that obtained earlier from the generalized damping theory. Thus our solution reduces, in appropriate limits, to damping theory and to perturbation theory.

We do not need to investigate specially whether the normalization is correct, since this is already proved within the framework of damping theory. Thus the solution which we have constructed satisfies the requirements set out in the beginning of this paper. *

We see from (20) that the perturbation theory gives a satisfactory approximation when $R^{(\xi)} \to 0$, which is usually the case at low energies. But if $R^{(\xi)}$ is appreciable, then the result may differ appreciably from that of perturbation theory. It has been shown in a number of examples that $\xi^{(\xi)}$ may play the part of a cut-off factor, which will reduce the general magnitude of the cross sections at high energies. However, the appearance of the general solution does not in general exclude other possibilities. The detailed discussion has to be carried out separately for each case.

We still have to refer to the elimination of divergences. Since the whole theory has been formulated in a manifestly covariant way, we may use the same methods for eliminating the divergences as in the usual theory. The results obtained after removing the infinities, may, however, differ from the results of perturbation theory. We shall not examine the questions of self-energy and charge, since they do not lie within the scope of this paper.

The method explained above is applicable not only to the case of the electron-positron and the electromagnetic fields, but also to any arbitrary combination of fields. As we increase the number of interactions which are considered, we must add to (ξ) more labels; the total number of pairs of labels in (ξ) equals the number of types of elementary particles.

However, our method is general not only in its applicability to any types of field. This method may be used also to study various kinds of generalized theory (those containing derivatives of higher order, nonlinear interactions, some kind of nonlocal generalizations, etc.). We note incidentally that in theories with nonlinear interactions ($H \sim \varphi^n$) the damping must play a much more significant part than

* Actually our solution corresponds to a partial summation of a series of successive approximations.

with the usual linear interactions.

The method may also be generalized to stationary problems for which one cannot determine initial conditions. In that case the problem reduces to a set of homogeneous integral equations which may be obtained from (12) by omitting the terms in U^0 In solving stationary problems one may use the method of residues. Problems involving several particles lead to equations of the Bethe-Salpeter type.

In the region of low energies the method proposed above leads, as a rule, to the same results as perturbation theory (in corresponding approximations). At high energies, however, the corrections may be appreciable, in particular as regards the relative importance of various processes.

It is a valuable duty to thank I. M. Lomsadze for valuable discussions.

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Theory of Wave Motion of an Electron Plasma

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A general investigation of the nonlinear wave motions of an electron plasma has been carried out for arbitrary electron velocities. Temperature effects are not taken into account, and the state of the plasma is characterized not by a distribution function, but by the particle density. A correspondence is established between the wave motion of the plasma and the motion of a nonrelativistic particle in a certain potential field. The variation of the frequency of longitudinal vibrations on the velocity amplitude has been determined. Nonlinear transverse vibrations of the plasma, and vibrations close to these, are also considered, and their frequencies determined. A number of relations are established for the complicated longitudinal transverse plasma oscillations.

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1. FUNDAMENTAL EQUATIONS

IN the study of the oscillatory behavior of an electron plasma, i.e., of an electron gas neutralized by ions, or a neutralized electron beam, it is usually assumed that the electron velocities and the density fluctuations are small, so that one may use a linearized system of equations. This scheme makes it possible to determine the frequencies of oscillation and to discuss, by means of gas-kinetic considerations, the part played by temperature effects[1,2], which turn out in general to be unimportant.

Nonlinear plasma oscillations were considered in a previous paper[3], in which temperature effects were neglected, and the electron velocity was assumed to be finite, but essentially nonrelativistic. Under these assumptions it is found that the oscillation frequency is independent of the velocity amplitude, and obeys the classical formula of Langmuir.

The purpose of the present paper is to investigate the oscillatory motion of the plasma quite generally, for arbitrary velocities. But, as in Ref. 3, we shall neglect temperature effects, i.e, we shall assume the plasma temperature to be zero. This approximation is very natural when we are investigating nonlinear oscillations even in a "high-temperature" plasma, and even more so in the study of plasma oscillations in electron beams, where the temperature is practically zero. Under these conditions it is not necessary to introduce a distribution function to specify the state of the plasma, but one