we obtain

$$a' \sim a'_0 \left(1 - \frac{k^2}{m^2}\right)^{-(e^2/2\pi)\,(3-d_l^0)}, \qquad (12)$$

$$b' \sim b'_0 \left(1 - \frac{k^2}{m^2}\right)^{-(e^2/2\pi)\,(3-d_l^0)}.$$

Equations (9) and (12) which we have obtained as a qualitative illustration of the method of renormalization groups agrees with those results obtained earlier[3] by means of a summation of a series of Feynman "primary diagrams."

In conclusion, I wish to express my deepest thanks to Academician N. N. Bogoliubov under whose guidance this work was completed and, in addition, to D. V. Shirokov for discussion of this work.

[1] N. N. Bogoliubov and D. V. Shirkov, Dokl. Akad. Nauk SSSR 103, 203, 391 (1955).

[2] N. N. Bogoliubov and D. V. Shirkov, Usp. Fiz. Nauk 57, 87 (1955).

[3] L. D. Landau, A. A. Abrikosov and I.M. Khalatnikov, Dokl. Akad. Nauk SSSR 95, 497, 773 (1954); Dokl. Akad. Nauk SSSR 96, 261 (1954).

Translated by A. Skumanich
161

On the Mass of the Photon in Quantum Electrodynamics

D. A. KIRZHNITS

P. N. Lebedev Physical Institute,
Academy of Sciences, USSR
(Submitted to JETP editor January 18, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 796-797
(April, 1956)

IN the present quantum theory of fields, point (local) interaction is often considered as the limit of smeared (nonlocal) interaction[1,2]; this permits one to operate with finite expressions in the intermediate calculations. For this purpose a scalar smearing function F, a form-factor which contains a cut-off parameter Λ, is introduced into the interaction Lagrangian. This factor converges to unity when $\Lambda \to \infty$. In this point of view the interaction has the form

$$S \sim e_1 \int F(p, k, p-k, \Lambda)\, \overline{\psi}(p)\, \hat{A}(k)\, \psi(p-k)\, d^4p\, d^4k$$

$$+\text{charge interaction.} \qquad (1)$$

In order that the smearing function does not lead to a violation of physical reality (i.e., the Hermitian character of the Lagrangian) it must satisfy the condition[3,4]

$$F(p, k, p-k) = {}^*F^*(p-k, k, p). \qquad (2)$$

To each node in the Feynman diagram (or to each operator of the vertex portion) there will correspond a factor in F which depends on the momentum associated with the node.

It is known that the use of the simplest square-form smearing functions* leads to a violation of the gradient invariance; a violation which appears in the form of a nonzero photon mass. In this connection in Ref. 1, where a square-smearing function was used, the photon mass was eliminated by subtraction. In addition to this it was expressed by these authors (whose larger goal was the elimination of the divergences without the use of a subtractive procedure) that the assumption about the existence of such a smearing function leads automatically with its use to the falling-out of the photon mass. This note is devoted to a consideration of this question.

The mass of the photon corresponds to the value of the polarization operator at $k = 0$ (the symbolism of Ref. 1 is used here):

$$P_{\mu\nu}(0) \qquad (3)$$

$$= \frac{e_1^2}{\pi i}\, \mathrm{Sp} \int G(p)\, \Gamma_\mu(p, p, 0)\, G(p)\, \gamma_\nu \,|\, F(p, 0, p, \Lambda)\,|^2\, d^4p.$$

The appearance of the square of the modulus of F is associated with the Hermitian character of the Lagrangian [Eq. (2)]; it specifies the presence of two vertex parts in the diagram of the polarization operator for which the momenta differ only in direction**.

To study Eq. (3) we shall use the asymptotic expression[1,5]

$$\widetilde{G}(p) = \hat{p}^{-1}, \widetilde{\Gamma}_\mu(p, p, 0) = \gamma_\mu, \qquad (4)$$

which means we will consider the case where the longitudinal part of the photonic Green's function d_l is equal to zero. With the help of Eq. (4), we find

$$P_{\mu\nu}(0) = \breve{P}_{\mu\nu}(0) + [P_{\mu\nu}(0) - \widetilde{F}_{\mu\nu}(0)],$$

$$\widetilde{P}_{\mu\nu}(0) = \frac{e_1^2}{\pi i} \int \mathrm{Sp}\left(\frac{1}{\hat{p}}\gamma_\mu \frac{1}{\hat{p}}\gamma_\nu\right) |F|^2\, d^4p. \qquad (5)$$

If, as is customary (see Ref. 2), $F(p, 0, p, \Lambda) = f(p^2/\Lambda^2)$, then

$$\tilde{P}_{\mu\nu}(0) = \frac{e_1^2}{\pi i} \Lambda^2 \delta_{\mu\nu} \int_0^\infty |f(x)|^2 dx. \tag{6}$$

$$P_{\mu\nu}(0) = -\frac{e_1^2}{\pi i} \int_0^\Lambda \frac{\partial}{\partial p_\mu} \mathrm{Sp}\,(G\gamma_\nu)\, d^4 p, \tag{8}$$

The eliminated part of Eq. (5) was of the order of $\ln(\Lambda^2)$. An analogous evaluation occurs in the more general case of $F(p, \Lambda) = f(p^2/\Lambda^2, p^2)$; at the same time, of course, one assumes that $F \to 1$ as $\Lambda \to \infty$.

In this fashion the problem of dealing with the elimination of the photon mass for any sufficiently large Λ by means of the selection of a suitable form for the smearing function rests, in all cases, upon the necessity of reducing Eq. (6) to zero. This leads to the condition

$$\int_0^\infty |f(x)|^2 dx = 0, \tag{7}$$

which cannot be satisfied because of the essential positive definite form for the integrand.

It may be remarked that the elimination of the photon mass by the Pauli-Villars method[6] is possible because the method does not depend on the smearing of the Lagrangian; therefore, the form-factor.

$$\int_{\lambda_0}^\infty \frac{\lambda^4 G(\lambda)}{(p^2 + \lambda^2)^2} d\lambda$$

with $\lambda_0 \gg m$, which corresponds to $|F(p, \Lambda)|^2$ in the present method, is an oscillating quantity. Eq. (7) assumes the well-known form $\int_{\lambda_0}^\infty \lambda^2 G(\lambda) d\lambda = 0$, then the condition can be satisfied.

For the case of a nonzero longitudinal Green's function d_l, $P_{\mu\nu}(0)$ becomes an explicit function of d_l inasmuch as the phase factors in G and Γ[1] which depend on d_l cannot be completely reduced in expression (3). Consequently if, indeed, one could eliminate $P_{\mu\nu}(0)$ with the aid of smearing, then the corresponding function F should depend explicitly on the arbitrary and abstract (i.e., without direct physical meaning) quantity d_l.

In this exceptional case the use of a square-smearing function allows one to calculate the magnitude of $P_{\mu\nu}(0)$ exactly. Transforming Eq. (3) with the help of Warden's equality

$$\Gamma_\mu(p, p, 0) = \partial G^{-1}(p)\ \partial p_\mu,$$

we obtain the expression***

to which the Gauss theorem can be applied. At the same time, the surface integral around the point $p^2 = m^2$ disappears in view of the proximity of a series of poles of G at this point[7]. As a result, it appears that $P_{\mu\nu}(0)$, like its value in the second order perturbation theory, is cut off at the momentum Λ. This is associated with the fact that over the surface of radius Λ nonexcited functions must be inserted in Eq. (3). In the exceptional case investigated it appears that $P_{\mu\nu}(0)$ does not depend on d_l because the region $p^2 < \Lambda^2$, where the phase factors are different from unity, does not contribute to Eq. (8).

Thus, in the elimination of the photon mass by means of a smeared (nonlocal) interaction through the use of a scalar form factor F one meets with a serious difficulty; in view of this fact one cannot possibly avoid the use of one or another form of the subtraction process.

* A square-shaped smearing function ($F(p, \Lambda) = 0$ for $p^2 > \Lambda^2$ and $F(p, \Lambda) = 1$ for $p^2 < \Lambda^2$) corresponds to the customary cut-off for integrals in momentum space.

** The question of possibly using a non-Hermitian smearing requires special study.

*** In the derivation of Eq. (8), the smearing function is essentially constant for $p^2 < \Lambda^2$ (and equal to unity).

[1] L. D. Landau, A. A. Abrikosov and I. M. Khalatnikov, Dokl. Akad. Nauk SSSR 95, 497, 773, 1177 (1954).

[2] A. A. Abrikosov and I. M. Khalatnikov, Dokl. Akad. Nauk SSSR **103**, 993 (1955); I. Ia. Pomeranchuk, Dokl. Akad. Nauk SSSR **103**, 1003 (1955).

[3] P. Kriestensen and C. Moller, Det. Kgl. Danske. **27**, 7 (1952).

[4] B. V. Medvedev, Dokl. Akad. Nauk SSSR **100**, 3, 433 (1955).

[5] E. S. Fradkin, J. Exptl. Theoret. Phys. (U.S.S.R.) **28**, 750 (1955); Soviet Phys. JETP **1**, 604 (1955).

[6] W. Pauli and F. Villars, Rev. Mod. Phys. **21**, 434 (1949).

[7] A. A. Abrikosov, Dissertation Int. Fiz. Problem, AN SSSR (1955); N. N. Bogoliubov and D. V. Shirkov, Dokl. Akad. Nauk SSSR **104**, 1391 (1955).

Translated by A. Skumanich
162

Effect of the Rate of Flow of a He II Film on Its Thickness

V. M. KONTOROVICH

*Physico-Technical Institute,
Academy of Sciences, Ukrainian SSR*

(Submitted to JETP editor January 27, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 805 (April, 1956)

IT is well known that below the λ-temperature point the wall of a container of liquid helium becomes covered with a thin ($\sim 3 \times 10^{-16}$ cm), rapidly creeping film which moves (under isothermal conditions) in the direction of a lower level[1-4] (i.e., toward a lower gravitational potential). It is of interest to ascertain how the motion influences the thickness of the film. The fact that some influence is expected follows from the dependence of the thermodynamic potential of He II on the rate of relative motion of the superfluid and the normal component[5,6].

We shall consider the leaking of a film over a vertical wall. The rate of overflow of the film, i.e., the volume of fluid Q, transported by the film in unit time and across a unit perimeter, shall be considered as a known quantity. The hydrodynamic equations of motion in a constant potential field are obtained in the customary manner from the conservational laws and the requirement of the existence of a velocity potential for the superfluid motion[5,6]. The equations of motion for the superfluid component prove to be

$$\mathbf{v}_s + \nabla \{ {}^{1}/_{2} v_s^2 + \tilde{\mu} + U \} = 0. \tag{1}$$

Here $\tilde{\mu}$ is the chemical potential of helium per unit mass in the absence of a field, U is the potential energy per unit mass of fluid. For the problem under consideration, U equals $\beta y^{-3} + gz$, where β is a constant specifying the Van der Wall's interaction with the wall. The y-axis is directed along the normal to the wall and lies in the plane that contains the horizontal surface of the helium in the container.

We shall consider the motion quasi-stationary. In view of the extremely small film thickness we can neglect the motion of the normal component in comparison to the superfluid: $v_n = 0$. If we insert $\tilde{\mu} = \mu(P, T) - (\rho_n/2\rho) v_s^2$ into Eq. (1) and integrate over the free surface of the film, we find that

$$(\rho_s / 2\rho) v_s^2 + \mu (P, T) + gz - \beta \delta^{-3} = \mu (P_0, T_0), \tag{2}$$

where δ is the thickness of the film at height z and P_0, T_0 is the pressure and temperature, respectively, at the horizontal surface. We shall consider the flow isothermal. Then

$$\mu (P, T) - \mu (P_0, T_0) = (P - P_0) / \rho. \tag{3}$$

The boundary condition at the free surface requires

$$P_\sigma + P = P_0, \tag{4}$$

where P_σ is the pressure associated with the curved surface. The change in the pressure of helium vapor with height is neglected. If we consider a z sufficiently large in comparison with the capillary constant of helium so that the film can be considered plane-parallel, then we obtain from Eq. (2) through (4)

$$(\rho_s / 2\rho) v_s^2 + gz - \beta \delta^{-3} = 0. \tag{5}$$

In this part of the film the flow rate is

$$Q \equiv \frac{\rho_s}{\rho} \int_0^\delta v_{sz} \, dy \approx v_s \delta \frac{\rho_s}{\rho} .$$

Expressing v_s in terms of Q and allowing for the fact that

$$(\rho/6\rho_s) (Q^2/\beta) (\beta / gz)^{1/3} \equiv q \ll 1,$$

we find from Eq. (5) that

$$\delta = (\beta / gz)^{1/3} (1 - q). \tag{6}$$

For observed values of Q we find that $q \sim 10^{-1}$ to 10^{-2} [1,2]. For $Q = 0$ we obtain the usual equation for thin stationary helium films[7]. We note that the form of the first term in Eq. (5) is specified by the dependence of the chemical potential on the relative rate of flow.

I use this opportunity to express my sincere gratitude to Prof. I. M. Lifshitz for detailed discussions on the problems considered in this note.

[1] V. Keezom, *Helium, with "Supplement"* by I. M. Lifshitz and E. L. Andronikashvili, IIL, 1949.

[2] B. N. Esel'son and B. G. Lazarev, J. Exptl. Theoret. Phys. (U.S.S.R.) 23, 552 (1952).

[3] K. R. Atkins, Proc. Roy. Soc. (London) **A203**, 119, 240 (1950).

[4] G. S. Picus, Phys. Rev. **94**, 1459 (1954).