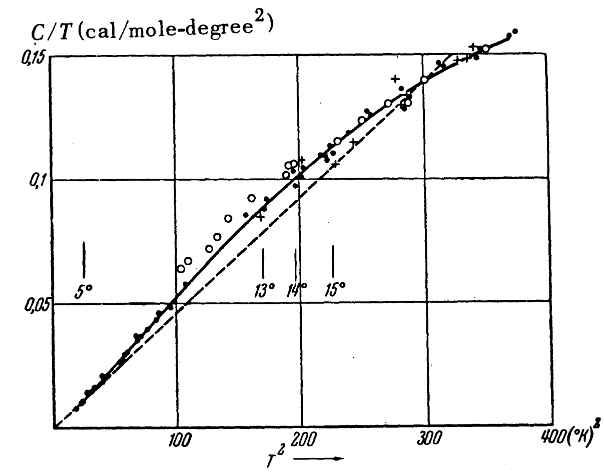

●- Data of author, O- Data of Clusius, +- Data of Giauque and Johnston.

The results of three series of measurements of the heat capacity of solid oxygen between 20° and 4° K are given in the Table. These same results are plotted in the graph (which gives C/T as a function of T^2) and are compared with the results of the measurements of Clusius[3] and Giauque and Johnston[4] which extend, respectively, to 10° and 13° K. The dotted line in the drawing represents an extrapolation of the cubic temperature dependence of the heat capacity of oxygen, found in measurements between 4° and 1.6° K [1]. It is evident from these results that the heat capacity of solid oxygen increases smoothly for the temperature range 4°-20° K, while, beginning at 5°K, the departure from the cubic law of change of heat capacity with temperature increases. The smooth character of the change in the heat capacity between 4° and 10° K bears witness to the absence of any antiferromagnetic transformation in solid oxygen in the temperature range investigated.

The measurements were carried out at the Institute for Physical Problems of the Academy of Sciences, USSR.

* A thermometer made from a radio resistor was kindly lent by B. N. Samoilov.

[1] M. O. Kostriukova and P. G. Strelkov, Dokl. Akad. Nauk SSSR **90**, 525 (1953).

[2] M. O. Kostriukova, Dokl. Akad. Nauk SSSR **96**, 959 (1954).

[3] K. Clusius, Z. Phys. Chem. **3**, 41 (1929).

[4] W. F. Giauque and H. L. Johnston, J. Am. Chem. Soc. **51**, 2300 (1929).

Translated by R. T. Beyer
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The Theory of Cyclotron Resonance In Metals

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THE present work predicts and studies theoretically a new form of resonance in metals, which differs fundamentally from diamagnetic resonance.[1] In metals close to resonance, the skin-depth δ is much less than the radius r of the orbit in the magnetic field.* Thus in a constant magnetic field $\mathbf{H}(H_x=H,\ H_y=H_z=0)$, parallel to the surface of the metal $z=0$, an electron which moves in a helical orbit through a number of revolutions ($l/2\pi r >> 1$ where l is the mean free path) will return several times into the layer of thickness $\delta << r$ where the electric field is large. Thus the motion is similar to that of an electron in a cyclotron with a single dec, so that for a value of ω which is a multiple of the 'cyclotron' frequency $\Omega_0 = eH/mc$ ($\omega = q\Omega_0$, $q = 1, 2, \ldots$) we shall have resonance. This resonance in metals we shall call cyclotron resonance (as distinct from diamagnetic resonance,** which occurs only in semiconductors, for $\omega = \Omega_0$).

If the magnetic field is not parallel to the surface of the metal, the electrons will pass through the layer once only, and resonance will be absent since the impedance does not depend on the magnetic field.

The condition for cyclotron resonance $\delta << r << l$ corresponds to the anomalous skin effect, so that the system is governed by Maxwell's equations together with the kinetic equation for $f_1(z, E, p, \tau)$, the perturbation to the Fermi distribution function (E – energy, p – momentum, $\tau = (eH/m_0c)t$, t – periodic time of electron in orbit[2], m_0 – effective mass of electron). The role of boundary condition on f_1 is played by the requirement that f_1 shall be periodic with respect to τ with period $\theta = m_0^{-1}\, dS/dE$, together with the condition of diffuse reflection at the surface.[3] The problem is solved under the most general conditions of the electron theory of metals – for arbitrary energy dependence $E = E(p)$ and arbitrary collision term $(df_1/dT)_{\text{coll.}}$. It turns out that in the anomalous skin effect region, because of the particular form of f_1, the collision integral can be put in the form $f_1/t_0(p)$ (in the zero approximation where $\delta/l \ll 1$.

Omitting all calculations, we give the final equation for the surface impedance $Z_j = R_j + iX_j$ ($j = x', y'$) close to resonance, where

$$\omega \sim \frac{|e|H}{m_0 c}, \qquad (1)$$

$$\frac{m_0 c}{|e|\,t_0} \ll H \ll v\sqrt{2\pi m_0 n}; \quad \frac{|\omega - q\Omega_0|}{\omega} \ll 1.$$

Under these conditions

$$Z_j \equiv -\frac{4\pi i\omega}{c^2}\frac{E_j(0)}{E_j'(0)} \qquad (2)$$

$$= 2I\left(\frac{\sqrt{3}\,\pi\omega^2}{c^4 B_j}\right)^{1/3} e^{i\pi/3}; \quad I \approx 1,$$

where x', y' are the principal axes, and B_j is the principal (diagonal) value of the tensor B_{ik}***,

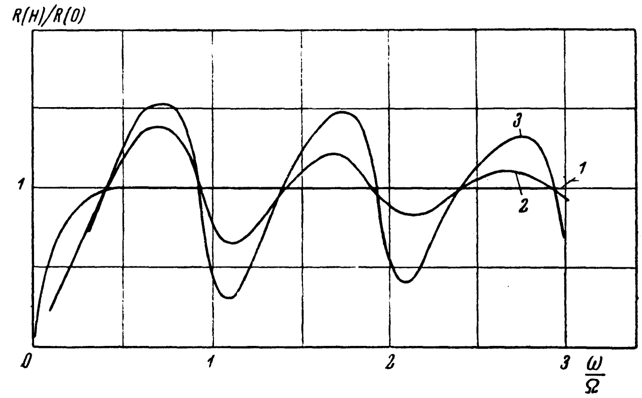
$$B_{ik} = \frac{16e^2 m_0}{3h^3}\int \frac{v_i(\tau_1)\,v_k(\tau_1)}{|v_z'(\tau_1)|} \qquad (3)$$

$$\times\left[1 - \exp\left\{-\frac{2\pi}{\Omega}\overline{\left(\frac{1}{t_0}\right)} - 2\pi i\frac{\omega}{\Omega}\right\}\right]^{-1}_{\varepsilon=\varepsilon_0} dp_x;$$

$$\Omega = \frac{2\pi|e|H}{c\,|\partial S/\partial\varepsilon|}; \quad v_z(\varepsilon_0, p_x, \tau_1) = 0;$$

$$p_y(\varepsilon_0, p_x, \tau_1) > 0; \quad \overline{\left(\frac{1}{t_0}\right)} = \frac{1}{\theta}\int_0^\theta \frac{d\tau}{t_0};$$

S – cross-sectional area of Fermi surface $E(p) = E_0$ by the plane p_x = const., (assuming the Fermi surface is closed; if the orbit is open, resonance is absent)****



1 — $\omega t_0 = 1$; 2 — $\omega t_0 = 10$; 3 — $\omega t_0 = 50$

If $\Omega \gg 1/t_0$, and Ω is not close to a resonance ω/q ($q = 1, 2, \ldots$) the denominator of the integrand in Eq. (3) is near unity, and Z does not depend on the collision integral. For $\Omega \approx /q$, the denominator becomes small, and for

$$\Omega_{res} \equiv \frac{2\pi|e|H_{res}}{c\,|\partial S/\partial\varepsilon|_{ext}} = \frac{\omega}{q}(1+\Delta); \qquad (4)$$

$$|\Delta| \ll 1; \quad q = 1, 2, \ldots \ll \frac{1}{2\pi}\left(\frac{r}{\delta}\right)^{2/3}$$

we have resonance. The relative heights of resonance, R_{res}/R_0 and X_{res}/X_0, are determined by $K = 1/\omega t_{00}$ and differ considerably in the following cases:

1. If the surface $E(p) = E_0$ is an ellipsoid, dS/dE is independent of p_x;

$$R_{res}/R_0 \sim \varkappa^{2/3}, \quad X_{res}/X_0 \sim \varkappa^{1/3}. \qquad (5)$$

2. If $E(p) = E_0$ is not an ellipsoid, and the extremum of dS/dE is a minimum;

$$R_{res}/R_0 \sim \varkappa^{4/9}, \quad X_{res}/X_0 \sim \varkappa^{1/6}. \qquad (6)$$

If the extremum of dS/dE is a maximum:

$$R_{res}/R_0 \sim \varkappa^{1/6}, \quad X_{res}/X_0 \sim \varkappa^{1/6}. \qquad (7)$$

Here Z_0 is the surface impedance in zero magnetic field; $1/t_\infty$ is the value of $1/t_0$ for $p_x = p_0$; p_0 is the value of p_x at which dS/dE has its extreme value.

The relative shift of resonant frequency Δ [cf. Eq. (4)] differs for R and X. The shift of order K which occurs for X in all cases and for R in case (7) is connected with the growth of the number of revolutions of the electron between collisions with increase of H. A shift of order $K^{1/2}$ (for R in cases (5), (6)), although it leads to a phase change of π in the electric field after $(2q|\Delta|)^{-1}$ revolutions of the electron, also turns out to be profitable, since it is equivalent to a small change in phase of the electric field in the layer δ ($X \gg R$).

The character of the resonance can be seen in the sketch, where is shown $R(H)/R_0$ versus $\omega/\Omega = mc\,\omega/eH$ for various ratios of ω to $1/t_0$, in the simplest case of an ellipsoidal Fermi surface with l/t_0 independent of p_x.

The conclusions reached above are also valid when several zones are present. An experimental study would in principle allow one : (a) to find out, from the existence or not of the resonance, whether the surface $E(p)=E_0$ is closed, (b) to determine the degree of filling of the zones, i. e. how far the Fermi surface differed from ellipsoidal shape; (c) to establish the speed of electrons at the Fermi surface (4), by determining from H_{res} the value of $(dS/dE)_{ext}$. In the presence of several surfaces, we can determine the speed on each in turn; in equation (4) only $(dS/dE)_{ext}$ enters. Note that here we discuss only the main surfaces, not the anomalously small zones.[5]

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* $\delta/r \sim Hm^{1/2}h^{-1}n^{-5/6} \sim 10^{-6}\,H \ll 1$. In semiconductors, where diamagnetic resonance is observed, $\delta/r \gg cm(|e|t_0)^{-1}(nkT)^{-1/2} \gtrsim 1$ (t_0 — time of free path, n — density of electrons, $\omega t_0 >> 1$, ω — angular frequency of electromagnetic field, T — temperature).

**Unfortunately, diamagnetic resonance has often been called cyclotron resonance in the literature. The present nomenclature seems more appropriate.

***It turns out that near resonance, for a non-quadratic law of dispersion the complex tensor B_{ik} can be reduced to principal axes. For a quadratic law of dispersion $E(p) = \frac{1}{2}\mu_{ik}p_ip_k$ and $1/t_0$ independent of p_x, this is possible for all ω and H, and equation (2) is valid for $\delta << r < l$ and becomes an interpolation formula for all $H \ll v\sqrt{2\pi m_0 n} \sim 10^6$ G.

****The derivation of these equations, and detailed discussion of some further points, will be the subject of a separate article.

Note added in proof: Quite recently a paper has appeared[6] on a resonance in bismuth: this is to be distinguished from the resonance discussed here, since the latter (1) occurs at multiple frequencies, (2) occurs independently of the sign of the magnetic field, (3) occurs only for magnetic fields exactly parallel to the surface of the specimen (the angle ϕ must satisfy $\phi > (\delta/r)^{2/5}$). In particular, condition (3) is not fulfilled in the work referred to.

[1] Ia. G. Dorfman, Dokl. Akad. Nauk SSSR 81, 765 (1951); R. B. Dingle, Proc. Roy. Soc. (London) A212, 38 (1952); Dresselhaus, Kip and Kittel, Phys. Rev. 98, 368 (1955).

[2] Lifshitz, Azbel and Kaganov, J. Exptl. Theoret. Phys (U.S.S.R.) 30, 220, 1956; Soviet Phys. JETP 3, 143(1956).

[3] K. Fuchs. Proc. Camb. Phil. Soc. 34, 100 (1938).

[4] I. M. Lifshitz and A. B. Pogorelov, Dokl. Akad. Nauk SSSR 96, 1143 (1954).

[5] I. M. Lifshitz and A. M. Kosevitch, Dokl. Akad. Nauk SSSR 96, 963 (1954).

[6] Galt, Yager, Merritt, Cetlin and Dail, Phys. Rev. 100, 748 (1955); R. N. Dexter and B. Lax, Phys. Rev. 100, 1216 (1955).

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Quantum Theory of Electrical Conduction in a Magnetic Field

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IN a previous paper we have developed a theory of galvanomagnetic phenomena in strong magnetic fields, treating the electrons classically as Fermi-particles with a general dispersion law $E = E(p)$.[1] There, however, we did not treat certain specific phenomena connected with the quantisation of the electronic energy levels (for example, the oscillations in resistance as the magnetic field changes). Such effects are observed experimentally[2], but previous theoretical investigations[3] are not entirely satisfactory. In the present paper we shall construct a consistent quantum-mechanical theory of metallic conduction in a magnetic field.

1. In quasi-classical approximation, the spacing of levels in a magnetic field in the z direction is given by[4]

$$\Delta\varepsilon_n = \varepsilon_{n+1} - \varepsilon_n = \mu^* H; \tag{1}$$

$$\mu^* = \frac{e\hbar}{m^* c}; \quad 2\pi m^* = \frac{\partial S}{\partial \varepsilon},$$

where $S = S(E, p_z)$ is the area cut by the surface $E(p) = E$ in the plane p_z = constant. Thus the essentially quantum-mechanical effects appear when $\mu^* H \sim kT$.

The Hamiltonian $\mathcal{H}$ of an electron in a magnetic field $H_z = H$ and an electric field E may be written