more, we note that the normalization conditions are not sufficient to obtain the correct spectrum of eigenvalues. This deficiency can be overcome by imposing an additional regional condition, namely, the condition that the function be regular at the origin. If we demand that $1-(2Z/\varkappa) = -n$, $n = 0, 1, 2\ldots$, then Φ goes into the Laguerre polynomial, and consequently, the function R_2 is regular at $r = 0$. For $1-(2Z/\varkappa) = -n$ the functions R_1 and R_2 are lineraly dependent and for $1-(2Z/\varkappa) \neq -n$ they are linearly independent. In conclusion, we note that for $l = 0$ the theory of Weyl also demands some regional condition. In the present case the demand that the solution be regular corresponds to the choice of some fully determined boundary condition of Weyl for $r = 0$. In this way the demand of regularity guarantees the physical correctness of the spectrum of eigenvalues. It is also possible to substantiate the regional condition at $r = 0$ by making a use of the Hermiticity conditions[5].

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Nuclear Capture of Neutrons with an Energy of Several MEV

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THE object of the present note is to calculate the cross section for neutron capture by excitation of the nuclear volume vibrations.

The incident neutron, interacting with the vibrational motion, gives up its energy and goes into a bound state in the nucleus. We consider the excitation of the first radial vibration, the frequency of which ω_0 corresponds to an energy of the order 10 mev.[1] The corresponding level has an appreciable width γ as a result of dissipation. However, such a mechanism of capture makes sense if the width of the level is not too large compared with the vibrational energy.

The Schrödinger equation of the system, nucleus + neutron, is

$$[-(\hbar^2/2M)\nabla^2 + U_0(r) + U(\mathbf{r}, Q) + T + W]\Psi = E\Psi, \quad (1)$$

where $\mathbf{r}$ is the radius vector of the neutron, U_0 is a spherically symmetrical potential well of depth V_0 and radius R; $U(\mathbf{r}, Q)$ is the potential arising from the nuclear density vibrations described by coordinates Q; T and W are the kinetic and potential energies of vibration. According to Ref. 1, for a two-fluid compressible model of the nucleus

$$W = \frac{1}{2}\int (a\delta\rho_p^2 + 2b\delta\rho_p\delta\rho_n + a\delta\rho_n^2)\, d\tau, \quad (2)$$

where $\delta\rho_p$ and $\delta\rho_n$ are the deviations of neutron and proton densities from their static values and $\delta\rho_{n,p} = f_{n,p}(\mathbf{r})\, Q$;

$$f_p(r) = D_{1p}j_0(q_1 r) + D_{2p}j_0(q_2 r); \quad (3)$$

$$f_n(r) = D_{1n}j_0(q_1 r) + D_{2n}j_0(q_2 r).$$

Here the constants D and q depend only on the number of neutrons and protons in the nucleus; j_l is the spherical Bessel function of order l.

In zero order

$$\Psi = \psi(\mathbf{r})\varphi(Q);$$

$$-\frac{\hbar^2}{2M}\nabla^2\psi + U_0\psi = E^{(0)}\Psi; \quad (T+W)\varphi = \varepsilon\varphi.$$

Bearing in mind the short range of nuclear force, we consider that the interaction energy of the neutron with the vibrations is proportional to the density deviation $\delta\rho = \delta\rho_n + \delta\rho_p$ at that point where the neutron is located. We define the coefficient of proportionality such that the interaction with a single-fluid static distribution of charge could be described by a potential U_0.

After summation over all final states of the captured neutron, the cross section is, to first approximation in perturbation theory;

$$\sigma = \sum_{nl}\sigma_{nl} = (2\pi/\hbar)\sum_{nl}|U_{nl}|^2, \quad (4)$$

where l is the angular momentum of the captured neutron, n is the quantum number of the bound state with momentum l;

$$U_{nl} = -\frac{4\pi r_0^3 V_0}{3M}\int \psi_{nl}\varphi(Q)\,\delta\rho\psi_k\,h_0(Q)\,d\tau\,dQ, \quad (5)$$

where $R = r_0 A^{1/3}$, $\psi_{n,l}$ and ψ_k are the wave functions of the captured and incident neutron, $h_0(Q)$ is the wave function of the oscillator ground state. Because of the rapid fall off of the latter, in the integration indicated in Eq. (5) small Q are important and therefore the wave function of an excited level of width γ is [2]

$$\varphi(Q) = \{\gamma/\pi[(\varepsilon-\varepsilon_0)^2+\gamma^2]\}^{1/2} h_1(Q), \quad (6)$$

where $h_l(Q)$ is the wave function of the first excited state of the oscillator.

Carrying out the integration in Eq. (5) and using Eq. (2), we obtain

$$\sigma_{nl} = \frac{4\pi^4(2l+1)r_0^4 V_0^2}{9A^{2/3}a\hbar^2 MkK^2} \times \frac{\cos\eta_l j_l(kR) + (-1)^l \sin\eta_l j_{-l-1}(kR)}{j_l(KR)} \times \frac{\hbar\omega_0\gamma}{(E_k^{(0)} - E_{nl}^{(0)} - \hbar\omega_0)^2+\gamma^2} F_{nl}^2(k), \quad (7)$$

where $\hbar k = \sqrt{2ME_k^{(0)}}$; $\hbar K = \sqrt{2M(E_k^{(0)}+V_0)}$; η_l is the lth phase shift from the potential U_0^3;

$$F_{nl}(k) = F_{nl}^{(1)}(k) + F_{nl}^{(2)}(k), \quad F_{nl}^{(i)}(k) \quad (8)$$

$$= \alpha_i\left[P_l\left(\frac{K^2+\varkappa_{nl}^2-q_i^2}{2K\varkappa_{nl}}\right) - \frac{\cos(q_i+\varkappa_{nl}-K)R}{\pi(q_i+\varkappa_{nl}-K)R}\right],$$

$$\hbar\varkappa_{nl} = \sqrt{2M(E_{nl}^{(0)}+V_0)},$$

where P_l is the Legendre polynomial of order l, α_i is a dimensionless constant of order unity, which can be expressed in terms of D and q and, consequently, depends only on the number of neutrons and protons in the nucleus. In carrying out the integration of Eq. (5) we use the relation [4]

$$\int_0^\infty j_0(qr)\,j_l(Kr)\,j_l(\varkappa r)\,r^2\,dr = \frac{\pi}{4K\varkappa q} P_l\left(\frac{K^2+\varkappa^2-q^2}{2K\varkappa}\right), \quad (9)$$

valid for $K-\varkappa < q < K+\varkappa$; $q_i R$ is the order of several units; if [5] $V_0 = 40$ mev, the inequality is satisfied. The formula (7) is valid only for $l \ll \varkappa R$, which is also satisfied, in so far as in Eq. (4) only those terms are important for which $E_k^{(0)} - E_{nl}^{(0)} - \hbar\omega_0 \sim \gamma$ and so far as for large l there are no bound states.

Taking [1] $a = 3.4\times10^5$ cm^5 gm^{-1} sec^{-2} and considering the order of several mev, we obtain a cross section of order 10^{-26} cm^2.

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Characteristics of the Levels of Nonspherical Even-Even Nuclei

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IN the adiabatic approximation the state of motion of external nucleons in a nonspherical nucleus which possesses an axis and center of symmetry is defined by quantum numbers that are completely analogous to the quantum characteristics of electronic terms for a diatomic molecule containing identical nuclei. [1] In this case the sum Ω of the projections w_i of the nucleonic spins on the axis of symmetry of the nucleus and the parity of the state $P = \pm 1$ are integrals of the motion.

In the present note we consider the characteristics of levels with $\Omega = 0$. Analogous to the Σ terms of a diatomic molecule, in the case of $\Omega = 0$ there is an additional quantum number $\eta = \pm 1$ which characterizes the behavior of the wave function under reflection with respect to a plane passing through the nuclear axis of symmetry,