

the $M1$ fraction in the $2^+ \rightarrow 2^+$ transition is also smallest. For Pt nuclei, in addition, $|N - N_{\text{mag}}| \geq 8$, $|Z - Z_{\text{mag}}| \geq 4$, whereas for the other nuclei in the table $|N - N_{\text{mag}}| \geq 2$, $|Z - Z_{\text{mag}}|$

≥ 2 . This fact can be explained by the selection rules which were referred to above. If, indeed, it is assumed that for the states being considered $\Omega = 0$, then by (3) $\eta_i = \eta_f = +1$, so that magnetic radiation will be forbidden.

$2^+ \rightarrow 2^+$ transitions in even-even nuclei

Nucleus	$N - N_{\text{mag}}$ $Z - Z_{\text{mag}}$	% $M1$ in transition	transition energy in keV	Position of first excited level in keV
$^{76}\text{Se}_{34}^{42}$	+2 -6	20-66	650	560
$^{122}\text{Te}_{52}^{70}$	-12 +2	20	680	560
$^{114}\text{Cd}_{48}^{66}$	+16 -2	95,6	710	530
$^{194}\text{Pt}_{78}^{116}$	-8 -4	5-6	330	360
$^{196}\text{Pt}_{78}^{118}$	-10 -4	2	1480	330
$^{198}\text{Hg}_{80}^{118}$	-8 -2	30-50	680	410

In connection with these considerations it would be desirable to obtain the following experimental data: a) a comparison of the nonsphericity of the nuclei in the table through observation of the Coulomb excitation (at present data exist for Cd 114 6-7); b) more accurate information concerning the multipolarity of the $2^+ \rightarrow 2^+$ transition in Os 186 (at present we have only a reference to a private communication by the author of reference 5); c) a study of the level schemes and transition multiplicities of strongly nonspherical even-even nuclei of rare earths and heavy elements (in the majority of cases only the characteristics of the first excited levels are known at present; see Ref. 5).

⁷Temmer and Heydenburg, Phys. Rev. 98, 1308 (1955).

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Concerning the Correlation Function for Quantum Systems

IU. L. KLIMONTOVICH
Moscow State University

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*This is completely analogous to the Σ^+ and Σ^- terms of a diatomic molecule (see Ref. 2).

¹A. Bohr, Dansk Mat.-Fys. Medd. 26, 14 (1952);
A. Bohr and B. R. Mottelson, ibid, 27, 16 (1953).

²L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, GITTL, 1948.

³G. Scharff-Goldhaber and J. Weneser, Phys. Rev. 98, 212 (1955).

⁴G. Scharff-Goldhaber, Phys. Rev. 90, 587 (1953).

⁵J. J. Kraushaar and M. Goldhaber, Phys. Rev. 89, 1081 (1953).

⁶Mark, McClelland and Goodman, Phys. Rev. 98, 1245 (1955).

THE correlation function for a classical system of weakly interacting particles can be determined by an approximate solution of the equation for the binomial distribution function.¹ In this method the distribution function f_3 in the set of equations is approximately expressed in terms of the binomial distribution function f_2 .

The method of Bogoliubov is used in the present letter to determine the correlation function of a quantum system of interacting particles. Instead of a set of classical equations for the distribution function used by Bogoliubov¹ we use a set of 2 equations for the quantum distribution function. The approximation of the quantum distribution function f_3 by a binomial quantum distribution

function is made excluding the exchange effects. For the correlation function the following expression is obtained:

$$g(|q|) = g_{id} + (2\pi)^{-3} \int \frac{v(k) F(k)}{1 - v(k) F(k)} e^{ikq} dk, \quad (1)$$

where

$$g_{id} = 1 \pm \int f_0(p') f_0(p'') e^{i\mathbf{q} \cdot (\mathbf{p}' - \mathbf{p}'')} d\mathbf{p}' d\mathbf{p}'' \quad (2)$$

is the quantum correlation function of an ideal gas.³ This correlation function of ideal gas, dependent on exchange effects, was first considered by V. S. Fursov and A. D. Galanin. In Eq. (2)

$$f_0 = 1/(2\pi\hbar)^3 n_0 [A \exp(\mathbf{p}^2/2mkT \mp 1)] \quad (3)$$

is the momentum distribution function of particles distributed uniformly in space. The minus and plus signs apply to particles obeying Bose-Einstein and Fermi-Dirac statistics respectively.

The second term in Eq. (1) is determined by the interaction of the particles. The functions $F(\mathbf{k})$ and $v(\mathbf{k})$ entering into it are evaluated in the following way

$$F(\mathbf{k}) = \hbar^{-1} \int \frac{f_0(\mathbf{p} + \hbar\mathbf{k}/2) - f_0(\mathbf{p} - \hbar\mathbf{k}/2)}{pk/m} d\mathbf{p}, \quad (4)$$

$$v(\mathbf{k}) = \int \Phi(|\mathbf{q}|) e^{-i\mathbf{k}\mathbf{q}} d\mathbf{q}, \quad (5)$$

where $\Phi(|\mathbf{q}|)$ is the potential energy of interaction of a pair of particles.

For $\hbar = 0$ Eq. (1) goes over into the expression for a correlation function of a classical system.

$$g(|\mathbf{q}|) = 1 - (2\pi)^{-3} \int \frac{v(\mathbf{k})}{n_0 v(\mathbf{k}) + kT} e^{i\mathbf{k}\mathbf{q}} d\mathbf{k}, \quad (6)$$

obtained by Zubarev.⁴ For completely ionized gas and $\hbar = 0$ one obtains the Debye correlation function¹ from Eq. (1).

We shall consider several particular examples. For a completely degenerate Bose gas, Eq. (1) assumes the form

$$g(|\mathbf{q}|) = g_{id} - (2\pi)^{-3} \int \frac{v(\mathbf{k})}{v(\mathbf{k}) n_0 + \hbar^2 k^2/2m} e^{i\mathbf{k}\mathbf{q}} d\mathbf{k}. \quad (7)$$

For small momenta we may set $v(\mathbf{k}) = v(0)$. Then we obtain from Eq. (4)

$$g(|\mathbf{q}|) = g_{id} - (1/4 \pi n_0 r_c^2 |\mathbf{q}|) \exp(-|\mathbf{q}|/r_c), \quad (8)$$

where $r_c = \hbar/2mc$ is the correlation radius, $c = \sqrt{v(0) n_0/m}$ is the velocity of sound in the gas. For helium $r_c = 10^{-8}$ cm. Equation (7) for the correlation function agrees with the expression that can be obtained from the work of Bogoliubov and Zubarev⁵ and Zubarev⁶. From Eq. (7) we can also obtain the correlation function for charged Bose gas.

For a completely degenerate system obeying Fermi-Dirac statistics

$$F(\mathbf{k}) = -\frac{3m}{2p_0^2} + \frac{3m}{4p_0^3 \hbar k} \left[\left(\frac{\hbar k}{2} \right)^2 - p_0^2 \right] \ln \left(\frac{p_0 + \hbar k/2}{p_0 - \hbar k/2} \right) \quad (9)$$

for $\hbar k \ll p_0$ $F(\mathbf{k}) = -3m/p_0^2$. If the considered system is plasma, then we obtain for $\hbar k \ll p_0$ the following expression for the correlation function of electrons

$$g(|\mathbf{q}|) = g_{id} - (1/4 \pi n_0 r_d^2 |\mathbf{q}|) \exp(-|\mathbf{q}|/r_d), \quad (10)$$

where

$$r_d = (p_0^2/12\pi e^2 n_0)^{1/2} \quad (11)$$

is the Debye radius for degenerate Fermi gas. This determination of the correlation function of Fermi gas is for the case in which the average interaction energy is less than the limiting energy of the Fermi distribution.

In the above approximation for completely ionized gas it is possible to disregard the set of equations for quantum distribution function and one may limit oneself to a solution for the quantum distribution function² of the kinetic energy equation with self-consistent field. Indeed to compute the thermodynamic functions of completely ionized gas one must know the magnitude of the energy in excess of the energy of ideal gas. The magnitude of this additional energy is determined by the distribution of the potential U around an arbitrary ion.

To find the equation for the potential U of a quantum system we shall use the kinetic equation with self-consistent field for quantum distribution function, which can be written for electron-ion plasma in the form²

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \frac{\partial f}{\partial \mathbf{q}} \quad (12)$$

$$= i \frac{e}{\hbar} \int \left[U \left(\mathbf{q} - \frac{\hbar \vec{\tau}}{2} \right) - U \left(\mathbf{q} + \frac{\hbar \vec{\tau}}{2} \right) \right]$$

$$\times f(\vec{\eta}, \mathbf{p}) \exp [i \vec{\tau} \cdot (\vec{\eta} - \mathbf{p})] d\vec{\tau} d\vec{\eta};$$

$$\Delta U = -4\pi e \left\{ \int f(\mathbf{q}, \mathbf{p}) d\mathbf{p} - n_i \right\}.$$

From Eq. (12) we obtain the relation for U assuming that the deviation of the distribution function from f_0 is small, i.e.

$$f(\mathbf{q}, \mathbf{p}) = f_0(\mathbf{p}) + f''(\mathbf{q}, \mathbf{p}); f''(\mathbf{q}, \mathbf{p}) \ll f_0(\mathbf{p}). \quad (13)$$

Substituting Eq. (13) into Eq. (12) and retaining only the first order terms, we obtain for $\partial t / \partial t = 0$ the following equation for the potential

$$\Delta U = -4\pi e^2 \int F(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{q} - \mathbf{q}')} U(\mathbf{q}') d\mathbf{k} d\mathbf{q}'. \quad (14)$$

In Eq. (14) the function $F(\mathbf{k})$ is given by Eq. (4). For $\hbar = 0$ Eq. (14) turns into the expression derived by the Debye theory. For $\hbar k \ll p_0$ we obtain for the potential of a quantum system the equation

$$\Delta U = r_d^{-2} U, \quad (15)$$

which agrees with the relation obtained in the Debye theory, except that it is for a different correlation radius. For a completely degenerate Fermi gas, the correlation radius is determined by Eq. (11). This result agrees with the results of Landau and Lifshitz.³

The expressions obtained for the correlation function are therefore correct in the case of weak interactions both for classical and quantum systems of particles with central interactions at arbitrary temperatures.

¹N. N. Bogliubov, *Problems of the Dynamic Theory in Statistical Physics*, State Publishing Co., 1946.

²Iu. L. Klimontovich and V. P. Silin, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **23**, 151 (1952).

³L. D. Landau and E. M. Lifshitz, *Statistical Physics* GITTL, Moscow, 1951.

⁴D. N. Zubarev, Dissertation, Moscow State University, 1953.

⁵N. N. Bogoliubov and D. N. Zubarev, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **28**, 129 (1955); *Soviet Phys. JETP* **1**, 83 (1955).

⁶D. N. Zubarev, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **29**, 881 (1955); *Soviet Phys. JETP* **2**, 745 (1956).

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Interaction of π^- -Mesons with Protons at 4.5 BEV

A. I. NIKISHOV

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THE experimental data on the n - p interaction at 1.7 beV and the π^- - p interaction at 1.37 beV do not contradict the statistical theory which takes into account the isobaric states of nuclei^{1,2}. We have carried the calculation for the π^- - p interaction, the energy of the incident meson being equal to 4.5 beV in the laboratory system.

In the calculation of the final state density, we have taken into account the conservation of momentum and the indistinguishability of mesons, as well as the exact nucleonic mass (or the isobaric state). We have, however, neglected the mass of the meson. The last approximation is justified by the fact that, as we shall see, in the most important processes there are no more than four particles in the final (or the intermediate isobaric) state. Even in the worst case (process $3N'$) the kinetic energy per particle amounts to 0.365 beV (total energy in the center-of-mass system 3.1 beV). If every one of the four mesons, created in the process of annihilation of a nucleon and an antinucleon, possessed such an energy, the correction factor to the meson mass would amount to 0.7 (according to I. L. Rozental' and V. M. Maksimenko). We take exactly into account only one (the heaviest) mass and the correction, therefore, will be even smaller. Only the processes $4N'$ and $5N$ may substantially depend on it, but their role is small at such high energies.

We shall use the following notation^{1,2}: N --nucleon, N' --isobaric state; nN --state with n pions and one nucleon, nN' --state with n pions and one isobar. The statistical weights of the processes under consideration are given in Table I.

As usual, we take $R = 1.4 \times 10^{-13}$ (R is the parameter determining the nonshortened interaction volume $V_0 = 4/3 \pi R^3$).

Table II gives the division of the charge states for all processes.

Some of the implications of our calculations can be already compared with preliminary experimental results on the π^- - p interaction at 4.5 beV³. In Ref. 3 the relative probabilities are given of elastic nondiffractive collisions and of inelastic collisions producing two-, four- and six-prong stars.