Magnitude of the Nuclear Spin-Orbit Interaction

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A WIDE group of interaction phenomena of nucleons with nuclei is at present sufficiently well described by a central potential with added spin-orbit term

$$A_1\rho(r) + A_2(\mathbf{l}, \vec{\sigma})\, d\rho/r\, dr, \qquad (1)$$

where $\rho(r)$ is a function representing the density distribution of nucleons in the nucleus. Polarization effects in elastic scattering of medium and high energy nucleons as well as the spin-orbit splitting of single particle levels belong to this group of phenomena. In the case of polarization and scattering the $A_{1,2}$ are complex, $A_{1,2} = V_{1,2}e^{i\alpha_{1,2}}$ and, besides, there is a basis for the assumption that Eq. (1) describes the scattering processes adequately in the case of not too large angles only[1]. In the case of single particle levels the $A_{1,2}$ are real ($\alpha_{1,2} = 0$), Eq. (1) being applicable, however, only in the case of nuclei with strongly closed kernel (nuclei of the type of He^5, O^{17}, etc.). In these cases the kernel is only little deformed by the external nucleon, the criterion of which is closeness of the values of the magnetic moments of such nuclei to their Schmidt values.

In this note we intend to present the relative values of the moduli V_2/V_1, the ratio of the central and of the spin-orbit potentials, based on all data available to the author.

1. The ratio V_2/V_1 can be obtained from the polarization data at high energies (100-300 mev). It is not necessary to determine the function $\rho(r)$ if we assume that the added spin-orbit term (so) in Eq. (1) has only a small influence upon the phases ($\delta_l^{so} \ll 1$) while the phase shift caused by the central part of the potential may be arbitrarily large. This approximation is correct for all nuclei and leads to an expression for the neutron polarization similar to the well-known Fermi formula[2], obtained by him, however, under the assumption of small phase shift

$$P(\theta) = 2\theta k^2 |V_2/V_1| \sin(\alpha_1 - \alpha_2) / [1 + k^4 |V_2/V_1|^2 \theta^2] \qquad (2)$$

(for details, see Ref. 3). It follows from (2) that $V_1/V_2 = k^2\theta_m$ and $\sin(\alpha_1 - \alpha_2) = P_m$, where θ_m is the angle of the maximum polarization P_m. Since the angle θ_m is not very small (12°-25°) we can obtain the value of V_2/V_1 from (2) with the help of well-known data[1] on the proton scattering polarization (see Table below). The mean value obtained for V_2/V_1 is 3.5×10^{-27} cm^2.

2. The estimation of the ratio V_2/V_1 from the low-energy scattering data on nuclear levels and shells requires the determination of the function $\rho(r)$. The evaluation is, however, correct in several cases:

1) Levels of He^5 and Li^5. The well-measured phase dependence of $P_{1/2}$ and $P_{3/2}$ levels in 1 to 15 mev nucleon scattering by He^4 can be exactly calculated assuming $\rho(r) \approx \exp(-r^2/a^2)$. Best agreement with experiment is obtained in Ref. 4 for the potential:

$$\left[1 - \beta(\mathbf{l}, \vec{\sigma})\frac{d}{r\, dr}\right] A \exp[-r^2/a^2],$$

where $a = 2.3 \times 10^{-13}$ cm, $A = 47.32$ mev, $\beta = 7.4\,(\hbar^2/Mc^2)$ (M is the nucleonic mass). We then obtain the value of $V_2/V_1 = 3.3 \times 10^{-27}$cm^2.

2) Single particle doublet levels of O^{17} and Pb^{209}. The estimation of the absolute value of V_2 in Eq. (1) was done by Blin-Stoyle[5] for the levels of Pb^{209}, for the case of an infinitely deep well. We have obtained a more exact result, solving the Schrödinger equation for a finite potential well with flat bottom and diffuse walls, making use of Airy functions. The expression for V_2/V_1 in the first approximation of the perturbation theory is:

$$\frac{V_2}{V_1} = \frac{\Delta E_{co}}{V_1}\frac{R_0^2}{2l+1}\frac{(\hbar^2/4m R_0^2 E_{co})[(E_l + 1/2)^2 - (l+1/2)^2] + (E_l + 1/2)\delta + 1/2 E_l^2\delta^2}{(1 + E_l\delta + 1/3 E_l^2\delta^2)(1 + 1/2\delta)}, \qquad (3)$$

where ΔE_{co} is the value of the doublet splitting, E_{co} is the center-of-mass energy of the doublet (defined as the binding energy of the lower doublet component $+\dfrac{l+1}{2l+1}\Delta E_{co}$), R_0 is the total radius of the nucleus (taking the diffusion into account): $R_0 = (1.33\,A^{1/3} + 1.3)\times 10^{-13}$ cm, δ is the relative value of the diffusion*, given by:

$$-E_l = \frac{R_0}{\hbar}\sqrt{2mE_{co}}\,\frac{K'_{l+1/2}(R_0\hbar^{-1}\sqrt{2mE_{co}})}{K_{l+1/2}(R_0\hbar^{-1}\sqrt{2mE_{co}})}+\frac{1}{2},$$

Data on V_2/V_1

Source	Nucleus	Energy in mev	$(V_2/V_1)\times 10^{27}\mathrm{cm}^2$
Polarization of high-energy protons	He	315	3.3
	C	290	3.4
	C	130	3.5
	Be	315	4
	Be	130	3.5
	Al	290	3.5—4.5
	Al	130	4.5
	Fe	130	3.5
	Cd	130	4
	Jn	130	3
	Bi	130	3.5
	U	130	3.5
Scattering	He^4-p	1—10	3.9
	He^4-n	1—18	3.3
Single-particle levels	O^{17}	$1D_{5/2-3/2}$	3.9
	Pb^{209}	$3D_{5/2-3/2}$	3.0
	Pb^{209}	$2G_{9/2-7/2}$	3.3
Shells	—	—	4.

where $K_{l+1/2}$ is the modified Bessel function. The values of V_1 for $\delta \ll 1$ are determined from the logarithmic derivative condition:

$$F_l^{-1}\{R_0(1-\delta)\,\hbar^{-1}\sqrt{2m(V_1-E_{co})}\} - E_l^{-1}\{R_0\hbar^{-1}\sqrt{2mE_{co}}\} = \delta.$$

E_l and F_l are tabulated in Ref. 7. We obtain from the above $V_1(Pb^{209}) = 54$ mev for $\delta = 0.29$.

There exist two single particle doublets in the case of Pb^{209} [8]:

$$3D_{5/2-7/2}\ (\Delta E_{co} = 0.98 \text{ mev},\ E_{co} = 1.91 \text{ mev}),$$
$$2G_{9/2-7/2}\ (\Delta E_{co} = 2.03 \text{ mev},\ E_{co} = 2.9 \text{ mev}).$$

In the case of O^{17}, one doublet[9]:

$$1D_{5/2-3/2}\ (E_{co} = 1.81 \text{ mev},\ \Delta E_{co} = 5.08 \text{ mev}).$$

In general, the values of V_2/V_1 and δ can be determined for the case of Pb^{209} in a unique way with the help of Eq. (3). However, the simultaneous solution of Eq. (3) for the both doublets gives a much too low value of $\delta(\sim 0.08)$. This fact is evidently connected with a substantial deformation of the kernel of the Pb^{209} nucleus. We have, therefore, for the estimation of the ratio V_2/V_1, chosen the values of δ and V_1 according to the potential of Ref. 9: The values of V_2/V_1 in the Table are calculated for $\delta = 0.57$ (O^{17}), $\delta=0$ (Pb^{209}) and $V_1 = 50$ mev.

3. The sequence of level filling in shells, calculated using potential of a form in-between the oscillatory and the square-well potentials, gives the value of $V_2/V_1 = 4\times 10^{-27}$ cm^2.[10]

The relative value of the spin-orbit potential is therefore constant for wide variations of nucleon energies and of the size of the nucleus being equal approximately to 3.5×10^{-27} cm^2. This fact can be interpreted as an indication that the effective nuclear potential is the mean potential of the nucleons forming the nucleus. It is essental that, averaging over closed shells, forces depending on the products of spins of the external and the nuclear nucleons (e.g., tensor forces) do not appear in the first approximation. The forces leading to the desired interaction are of the form

$V(\mathbf{r})\{1(\vec{\sigma}_i+\vec{\sigma}_j)\}$[5]. The existence of a strong spin-orbit interaction is an indication of the presence of such forces, and the interaction constant expresses their magnitude in relation to central forces in the nucleon-nucleon interaction.

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* R_0 and δ are chosen according to the potential of Woods and Saxon[6] $V(r)=V_0(1+i\xi)\{1+\exp\times[(r-r_0)/a]\}^{-1}$, $r_0=1.33\,A^{1/3}\times10^{-13}$ cm, $a=0.49\times10^{-13}$ cm. In our case this potential can be approximated by a trapezium-shaped well with $R_0=(1.33\,A^{1/3}+1.3)\times10^{-13}$ cm, $\delta=4.3\,a/R_0$, $V_0=50$ mev, $\xi=0$.

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Modification of Double Proton Scattering Experiments

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SCATTERING of nonrelativistic protons by nuclei of zero spin is described by a two-row scattering matrix.

$$f(\theta,\varphi)=A(\theta)+B(\theta)(\mathbf{n}\vec{\sigma}), \tag{1}$$

where $\mathbf{n}$ denotes the normal to the plane of scattering, and functions A and B are expressed by phases according to known formulas. The function

$$P(\theta)=(AB^*+A^*B)/(|A|^2+|B|^2) \tag{2}$$

determines the polarization as a result of scattering of the originally nonpolarized beam as well as the azimuthal asymmetry observed when polarized protons are scattered.

For the experimental determination of P it is necessary to subject the protons to double scattering. By measuring the ratio of intensities for $\mathbf{n}_1=\mathbf{n}_2$ and $\mathbf{n}_1=-\mathbf{n}_2$ equal to

$$R=(1+P_1P_2)/(1-P_1P_2), \tag{3}$$

we find the product P_1P_2, and if the factor P_1 is known we obtain P_2 (or vice versa). The success of measurement of this kind therefore depends on the existence of a "standard" scattering process for which the dependence of P on the angle of scattering and energy E is previously known. Note that even if both scatterings are by the same kind of nuclei and $\theta_1=\theta_2$, $P_1\neq P_2$ since the energy E_2 before the second scattering is smaller than the initial energy E_1 by the amount of energy imparted to the nucleus, which is considerable in the case of scattering by light nuclei.

The purpose of this note is to describe an experiment which in principle permits the direct determination of P_2. Let the beam of hydrogen molecular ions H_2^+ incident on the scattering target be such that its ratio q of the number of parahydrogen ions to the number of orthohydrogen ions is considerably different from the equilibrium value $q_0=1/3$. Upon entering the target the molecular ions "break up" and the two protons are scattered independently of each other. If two counters register coincidences due to protons from the same molecular ion scattered in directions θ, φ_1 and θ, φ_2, the number of coincidences D is proportional as follows:

$$D\sim\frac{q}{1+q}(F\chi_0,F\chi_0)+\frac{1}{3(1+q)}\sum_{m=-1}^{1}(F\chi_1^m,F\chi_1^m), \tag{4}$$

where χ_0 and χ^m are wave functions of the proton spins for the para- and orthomolecules and F is the "direct product" of matrices f_1 and f_2

$$F=f_1(\theta,\varphi_1)\times f_2(\theta,\varphi_2); \tag{5}$$