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# Properties of Energy Surfaces of Heavy Nuclei

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A detailed investigation is carried out of the properties of the energy surface of heavy nuclei for each of the four types of nuclear parity. Along with a more precise formulation of known empirical laws and parameters of the energy surface, it is shown that for given mass numbers $A$, the masses of the nuclei reach a minimum value for several different values of $Z$ depending on the parity of $Z$. The curvature of the isobar parabolas does not depend on the parity of $Z$ and is apparently several times larger for nuclei with even $A$. The character of the shell $N = 126$ and $Z = 82$ is made clear. The results are compared with the usual formulas for the binding energy.

## 1. INTRODUCTION

ACCORDING to statistical theory, the masses of the nuclear isobars $M(A, Z)$ reach a minimum value $M^*$ for certain values $Z = Z^*$ for which $\partial M(A, Z)/\partial Z = 0$[1]. This conclusion is found in qualitative agreement with the empirical rule of Mattauch-Shchukarev, according to which there is only one $\beta$-stable nucleus among isobaric nuclei of odd mass. The statistical theory also points to the parabolic character of the cross section of the energy surface, at least in the neighborhood of $Z = Z^*$. As the systematics of nuclear mass show (which have been given in a number of works[2-8]), the parabolic character of the isobaric energy cross sections in general agrees with experiment. There is also established empirically the dependence of the energy of the nucleus on the type of parity. Under otherwise equal conditions (one and the same $A$ and $Z$) the masses of even-even nuclei ($Z_e$, $N_e$) are less than the masses of nuclei with odd mass numbers, which in turn have smaller masses than the odd-odd nuclei ($Z_o$,$N_o$). This difference of nuclear masses, which is connected with their parity, is taken into account by the well-known correction to the parity[1]. There is also evidence on the existence of small differences in the mass of nuclei with odd mass numbers, depending on the parity of $Z$[9-11]. Finally, a sharp change has been discovered in the energy of nuclei in the neighborhood of places of filling of the neutron and proton shells[3-8].

Thus, the principal properties of the energy surface of nuclei are sufficiently well understood at the present time. However, many details still need to be made more exact. In particular, it would be desirable to determine whether the real isobaric curves are exactly quadratic parabolas or, whether one can consider the parabolas to be some sort of approximation. Moreover, there is interest in making more precise the parameters of the energy surfaces, especially taking into account all possible corrections to the parity, the role of the shells, and other possible individual peculiarities of the nuclei. Here there is known a significant number of total energies of isobaric transitions, in part measured experimentally, in part calculated from the energy of $\alpha$ - $\beta$ cycles[7,8]. At present, in the interval $210 < A < 240$, significant appearances of any shells or subshells[3,8,13,19,23] which would alter the general regularities have not been discovered.

In the present work the energies of $\beta$-transitions of heavy nuclei ($A > 210$) have been used for purposes of empirical analysis. Here no previous assumptions are made on the character of the mutual position of the energy surfaces in dependence on the parity of $A$ or $Z$. It is only noted that there exist no more than 4 different types of nuclei, connected with the parity of $A$ and $Z$, the masses of which, under otherwise equal conditions, can be different.

## 2. SYSTEMATICS OF THE ENERGY OF $\beta$-TRANSITIONS

The necessary energies of $\beta$-transitions $Q_\beta$ were taken from tables[14] which contain, along with the experimental values, values computed from the energetics of $\alpha$- $\beta$ cycles. We also have used the data on the systematics of the energy of $\alpha$-decay[9,12,13] which permit us to calculate a certain number of energies $Q_\beta$ in addition. For confirmation, the latter values were computed in independent fashion from $n\beta$- and $n\,\alpha$-cycles. For this purpose, the necessary experimental values of the binding energy of the neutrons were taken from Refs. 15-17.

For convenience in the subsequent analysis, we systematized the energies of the $\beta$-transitions in a fashion somewhat different than was done in

Refs. 3, 7 and 8. In the graph of the dependence of the energy on $Z$, the energies $E_{\beta-}$ for $\beta^-$-transitions and the energies $E_{\beta+}$ for $\beta^+$-transitions ($K$ capture) of isobars ($A$ = const) of a given parity (for example, odd-odd) are plotted in Fig. 1. For convenience, we used for the energies of the $\beta^-$- and $\beta^+$-transitions, not the quantities $Q_{\beta-}$ and $Q_{\beta+}$, but

$$E_{\beta-} = Q_{\beta-} + 0.51 \text{ mev and}$$

$$E_{\beta+} = Q_{\beta+} - 0.51 \text{ mev.}$$

$E_{\beta-}$ (and also $E_{\beta+}$) represent the differences of masses of nuclear isobars between which $\beta^-$ ($\beta^+$) transitions occur (in contrast to $Q_{\beta-}$ and $Q_{\beta+}$ which correspond to mass differences of the neutral atoms). In those cases, when the isobar $X_Z^A$ was shown to be stable, for example, in the behavior of $\beta^-$-decay, the energy $E_{\beta+}$ with negative sign is plotted on the graph for a given $Z$. This energy is that with which the nucleus $X_{Z+1}^A$ is converted into the given nucleus $X_Z^A$. The same was achieved when the given isobar $X_Z^A$ was shown to be stable relative to conversion into the isobar $X_{Z-1}^A$. The points of $\beta^-$-transitions and those of $\beta^+$-transitions (for $A$ = const) are separately joined together on the graph. As is seen from Fig. 1, the lines of the $\beta^-$- and $\beta^+$-transitions for each of the values of $A$ represent very nearly straight lines, which intersect crosswise (at a point which we shall denote by $Z^0$). The slopes of each of the branches are independent of the mass numbers $A$ within the limits of experimental error.

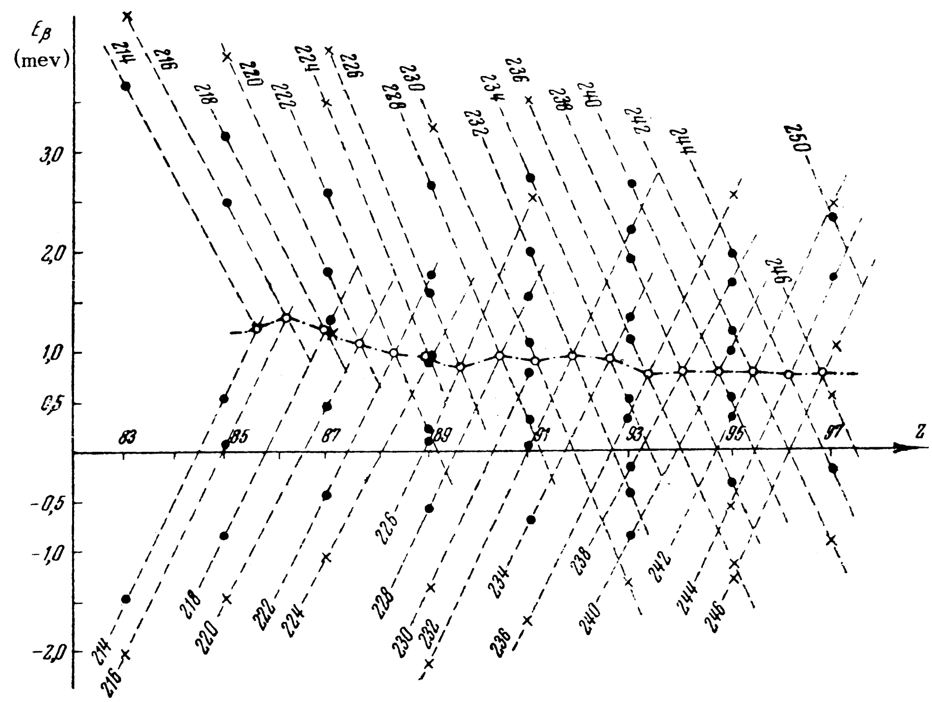


FIG. 1. Energies of $\beta$-transitions of odd-odd nuclei. ●–experimental values also obtained from $\alpha$-$\beta$ cycles, ×–values found with the help of extrapolated values of $E_\alpha$, o–points of intersection of $\beta^-$ and $\beta^+$ branches.

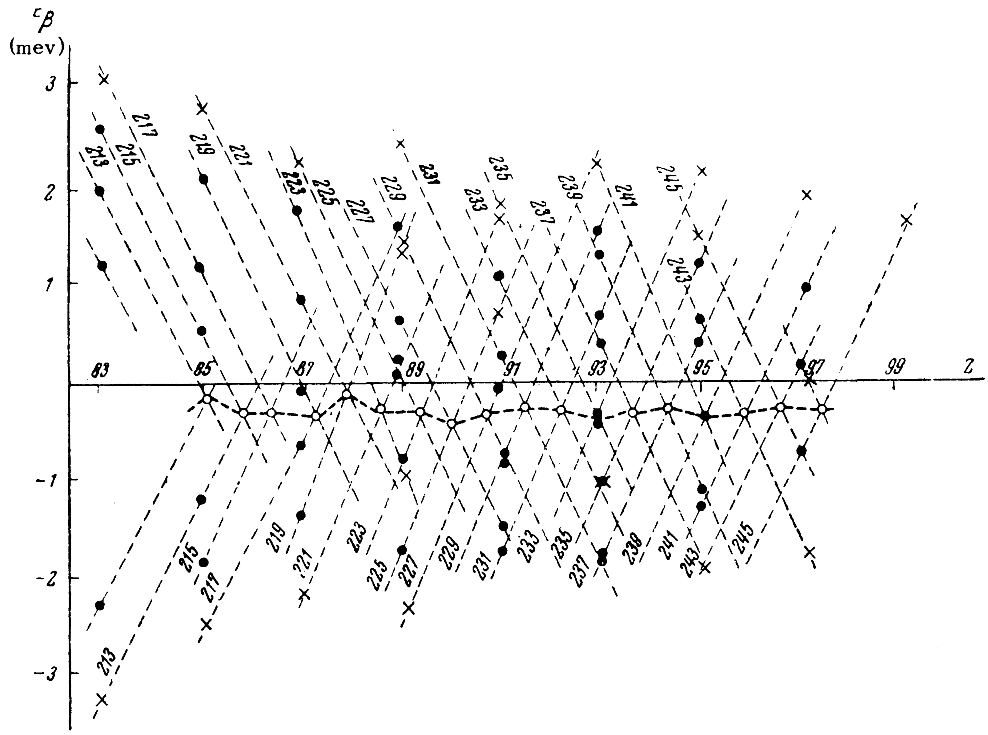


FIG. 1a. Energies of $\beta$-transitions of nuclei with $A$ odd. •--experimental values also found from $\alpha$ - $\beta$ cycles, ×--values in the finding of which the data of $\alpha$-decay were used, o--intersection points of $\beta^-$ and $\beta^+$ branches.

In order to establish this, we combine in one graph the curves which pertain to different $A$, by means of a plane parallel shift such that the points $Z^0$ coincide. As is seen from Fig. 2, where such a shifting has been carried out, all the points lie on two crosswise intersecting straight lines.

If, in place of the odd-odd nuclei considered here, one chooses a nucleus of different parity (see Fig. 1$a$), then an analogous picture is obtained. We note one special circumstance, which was discovered upon consideration of Fig. 2 (and others analogous to it for other types of parity), in which the slopes of the right and left branches of the "cross" are different according to the following Table I.

TABLE I

| Type of Parity | | Tangent of the angle of slope |
|---|---|---|
| Left Branch | Right Branch | |
| $Z_o, N_e, (A_o)$ | $Z_e, N_o, (A_o)$ | 1.18 |
| $Z_o, N_o, (A_e)$ | $Z_e, N_e, (A_e)$ | 1.14 |
| $Z_e, N_o, (A_o)$ | $Z_o, N_e, (A_o)$ | 1.08 |
| $Z_e, N_e, (A_e)$ | $Z_o, N_o, (A_e)$ | 1.06 |

## 3. ANALYSIS OF THE EMPIRICAL DATA

Making use of the results of the preceding section, we can draw concrete conclusions about the property of the energy surface in the region of heavy nuclei.

Let us consider a typical case of the intersection of the branches of $\beta^-$ and $\beta^+$-decay for any fixed value of $A$ (Fig. 3). The equation of the line $AB$ will be

$$E_{\beta^-}(A_e, Z_o) = D_{oo} - \operatorname{tg}\alpha_{oo}(Z - Z^0_{oo}) \quad (1)$$

(the subscripts $nn$ are inserted to show that in the given case we are considering $\beta^-$-decay of nuclei of the type $Z_o, N_o$). Similarly, for the branch corresponding to $\beta^+$-transitions, $Z_o \rightarrow (Z-1)_e$ (the line $CD$ in Fig. 3):

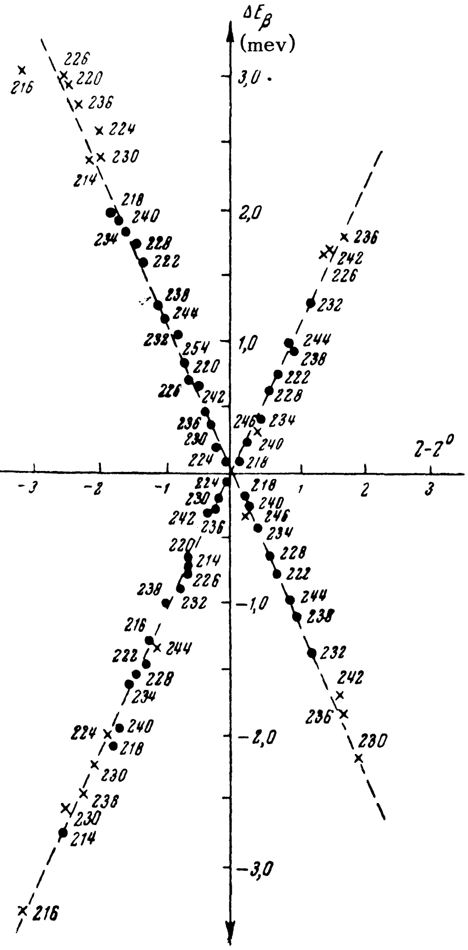


FIG. 2. Reduced energies of $\beta$-transitions.

$$E_{\beta^+}(A_e, Z_o) = D_{oo} + \mathrm{tg}\,\beta_{oo}(Z - Z^0_{oo}). \quad (1a)$$

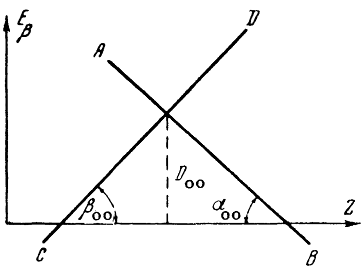


FIG. 3

Moreover, the following relations are valid:

$$E_{\beta^-}(A_e, Z_o) = -E_{\beta^+}(A_e, Z_o + 1);$$

$$E_{\beta^+}(A_e, Z_o) = -E_{\beta^-}(A_e, Z_o - 1).$$

On the other hand, $E_{\beta^-}(A_e, Z_o)$ must be equal to the difference of the masses of the initial $M(A_e, Z_o)$ and the final $M(A_e, Z_o + 1)$ of the nuclei:

$$E_{\beta^-}(A_e, Z_o) = M(A_e, Z_o) - M(A_e, Z_o + 1)$$

$$= D_{oo} - \mathrm{tg}\,\alpha(Z - Z^0_{oo}). \quad (2)$$

Further, in similar fashion,

$$M(A_e, Z_o + 1) - M(A_e, Z_o + 2) \quad (2a)$$

$$= -D_{oo} - \mathrm{tg}\,\beta_{oo}(Z + 2 - Z^0_{oo}).$$

Combining (2) and (2a), we find

$$M(A_e, Z_o) - M(A_e, Z_o + 2)$$

$$= -(\mathrm{tg}\,\alpha_{oo} + \mathrm{tg}\,\beta_{oo})(Z - Z^0_{oo}) - 2\,\mathrm{tg}\,\beta_{oo}.$$

Since $Z^0_{oo} = f(A)$, then we can write in place of the foregoing:

$$M(A_e, X_o) - M(A_e, X_o + 2) \quad (3)$$

$$= -\lambda_{oo} X_o - \mu_{oo},$$

where

$$X_o = Z - Z^0_{oo};$$

$$\lambda_{oo} = \mathrm{tg}\,\alpha_{oo} + \mathrm{tg}\,\beta_{oo};$$

$$\mu_{oo} = 2\,\mathrm{tg}\,\beta_{oo}.$$

The functional Eq. (3) has the following solution:

$$M(A_e, Z_o) = a_{oo}(A) + {}^1/_4\,\lambda_{oo} X_o^2 + {}^1/_2\,\nu_{oo} X, \quad (4)$$

where $\nu_{oo} = \tan\beta_{oo} - \tan\alpha_{oo}$. Thus,

$$M(A_e, Z_o)$$

$$= a_{oo}(A) + {}^1/_4\lambda_{oo}(Z - Z^0_{oo})^2 + {}^1/_2\nu_{oo}(Z - Z^0_{oo}).$$

Consequently,

$$M(A_e, Z_e)$$

$$= a_{oo}(A) + {}^1/_4\lambda_{oo}(Z - Z^0_{oo})^2 - D_{oo} - {}^1/_4\lambda_{oo}.$$

From the condition $(\partial M_{oo}(A, Z)/\partial Z)_A = 0$ we find

$$Z^*_{oo} = Z^0_{oo} - \nu_{oo}/\lambda_{oo}, \quad (Z^*_{ee} = Z^0_{oo}).$$

Therefore, $M(A_e, Z_o)$ can be written in the form

$$M(A_e, Z_o)$$

$$= a_{oo}(A) + k_{oo}(Z - Z^*_{oo})^2 + (\nu_{oo})^2/16k_{oo},$$

where $k_{oo} = 1/4\,\lambda_{oo}$ represents the curvature of the isobar parabolas. One can determine the correction to the parity $2\delta^{ee}_{oo}$ between nuclei of the type $Z_e$, $N_e$ and nuclei of the type $Z_o$, $N_o$ as the difference of the energy between the minimum corresponding parabolas. In such a case,

$$\delta^{ee}_{oo} = {}^1/_2\{M(A_e, Z^*_{oo}) - M(A_e, Z^*_{ee})\}$$

$$= {}^1/_2\{D_{oo} + k_{oo} + \nu^2_{oo}/16k_{oo}\}. \quad (5)$$

Since $\nu_{oo} \ll k_{oo}$, then

$$\delta^{ee}_{oo} \approx {}^1/_2(D_{oo} + k_{oo}). \quad (5a)$$

Thus we can write

$$M(A_e, Z_o) \approx b_{oo}(A) + k_{oo}(Z - Z^*_{oo})^2 - \delta^{ee}_{oo}, \quad (6)$$
$$M(A_e, Z_e) \approx b_{oo}(A) + k_{oo}(Z - Z^0_{oo})^2 + \delta^{ee}_{oo},$$

where

$$b_{oo} = a_{oo} + (\nu^2_{oo}/16k_{oo}) - \delta^{ee}_{oo}.$$

Completely analogous to the foregoing, for the case of odd mass numbers $A_o$ we get the following dependencies of the masses of the nuclei $M(A, Z)$ on $Z$ and $A$:

$$M(A_o, Z_o) = b_{oe}(A) + k_{oe}(Z - Z^*_{oe})^2 + \delta^{eo}_{oe},$$
$$M(A_o, Z_e) = b_{oe}(A) + k_{oe}(Z - Z^0_{oe})^2 - \delta^{eo}_{oe} \quad (7)$$

(oe denotes an odd number of protons, even number of neutrons; eo, an even number of protons, odd number of neutrons). We note that in Eq. (7) the correction $\delta^{eo}_{oe}$ appears, which takes into account the dependence of the mass of odd nuclei ($A$-odd) on the parity of $Z$. The mean value of such a correction in the interval of mass numbers under consideration is about 0.15 mev, in agreement with the estimates given in Refs. 9-11 and 19. This fact shows that the coupling energy of the protons is somewhat larger than the coupling energy of the neutrons. An analogous correction $\delta^{ee}_{oo}$ between even-even and odd-odd $Z$ and $N$ is shown to be equal in mean to 0.75 mev. The dependence of $\delta^{eo}_{oe}$ and $\delta^{ee}_{oo}$ on $A$, obtained from Eq. (5a) upon consideration of values of $D$ and $k$ found empirically (see Fig. 1), is plotted in Fig. 4. There is also shown the dependence of the radius of curvature of the isobar parabolas $k$ on the mass number $A$ for the case of even and odd values of $A$.

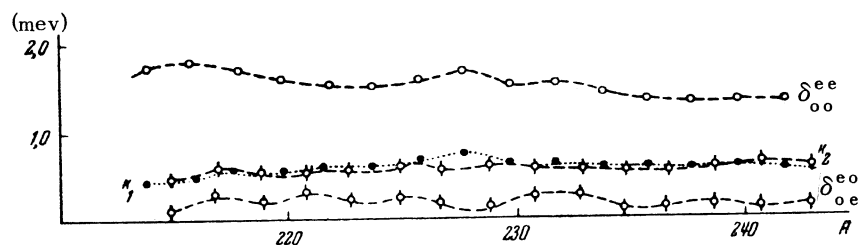


FIG. 4. Corrections to the parity $\delta$ and coefficient $k$ as functions of $A$. $k_1$ are the curvatures of the isobar parabolas for $A$ even, $k_2$ for $A$ odd.

As Figs. 4 and 1 show, $k$ does not change with increase in $A$, even though it does experience small fluctuations. The mean values of $k$ are equal to 0.56 and 0.54 for the case of even and odd mass numbers, respectively, and do not depend on the parity of $A$.

In accord with the above, the values of $Z^*$ determined from the condition $\partial M/\partial Z = 0$ are equal for all four types of parity, and thus, instead of one curve $Z^* = f(A)$, we have in fact a curve for nuclei of each of the four types (4 curves in all). However, these curves are very close together. One can represent approximately the curve for $Z^*$[11,19,22,23] (not taking into account the small deviations from this curve resulting from the individual properties of the nuclei: parity, etc.) in the range of mass numbers under consideration ($214 < A < 244$) by the following empirical formula

$$Z^*_{av} = 0{,}356\,A + 9{,}0. \quad (8)$$

## 4. THE CHARACTER OF THE NUCLEAR SHELLS $N = 126$ AND $Z = 82$

For the calculation of the jumps or "anomalies" in the energies of $\beta$-decay, associated with nuclear shells, one must determine the difference between the experimental energies of $\beta$-transitions, $E_\beta$ and the values of $E_\beta$ which do not take into consideration the effect of the shells, and which can be calculated by Eqs. (1) and (2). Equal energy differences, $E_\beta -$ are different from zero in the following cases (all values in mev):

$Tl^{210}(+1.5)$, $Tl^{209}(+1.6)$, $Tl^{208}(+2.1)$, $Tl^{206}(-0.5)$,
$Tl^{205}(-1.2)$, $Pb^{214}(-0.4)$, $Pb^{213}(-0.7)$, $Pb^{212}(-0.5)$,
$Pb^{211}(-0.6)$, $Pb^{210}(-0.6)$, $Pb^{209}(-0.3)$, $Bi^{208}(+2.2)$,
$Bi^{215}(-0.3)$, $Bi^{214}(-0.4)$, $Bi^{213}(-0.2)$, $Bi^{212}(-0.4)$,
$Bi^{211}(-0.5)$, $Bi^{210}(-0.5)$, $Po^{209}(+2.3?)$.

In order to make it clear whether these peculiarities are connected with the relative increase in the binding energy of the initial nucleus, or with the decrease in the binding energy of the final nucleus, between which nuclei $\beta$-transitions occur, it is quite proper to use the energy of addition of neutrons and protons, as was done in Refs. 4-6. However, such energies are rather difficult to compare amongst themselves, and it is most difficult to make a reliable separation of the effect of the shells. Using the results of the previous investigation, we can, however, bring all the isobaric nuclei to a homogeneous condition. For this purpose, we calculate the addition energy of the neutron $E^*_{n\gamma}$ which the nucleus with $(A, Z^*_A)$ ought to have, i.e., the nucleus located on the stability curve of $Z^*$.

It follows from the energy cycle that

$$E^*_{n\gamma} = E_{n\gamma} + E_1 - E_2,$$

where $E_1$ is the mass difference for nuclei of like parity $(A, Z)$ and $(A, Z^*_A)$; $E_2$ is the mass difference for nuclei $(A + 1, Z)$ and $(A + 1, Z^*_A)$. $E_1$ and $E_2$ can be calculated easily if we make use of the empirical smoothed values of $Z^*$ and $k$.

The reduced energies $E^*_{n\gamma}$ found for isobars of one type of parity ought to coincide exactly if only the masses of the nuclei are given by Eqs. (6) and (7). Deviations ought to appear only in places of anomalously high or low binding energies upon filling of the nuclear shells. If $b_{oo}(A)$ and $b_{oe}(A)$ [see Eqs. (6) and (7)] are smooth functions of $A$ (and do not, in particular, depend on the parity of $A$) and $b_{oo}(A) = b_{oe}(A)$, then all the reduced energies $E^*_{n\gamma}$ ought to be distributed along two smooth curves. The distance between these curves must, in accordance with Eqs. (6) and (7), be equal to

$$\Delta E^*_{n\gamma} \approx 2\,(\delta^{ee}_{oo} - \delta^{eo}_{oe}).$$

As is seen from Fig. 5, where the dependence of $E^*_{n\gamma}$ on $A$ is shown, the laws referred to are actually demonstrated. Moreover, the anomalously high addition energies of neutrons, if their number is less than 126 (in comparison with the addition energy in the region $N > 126$) are also shown.

For a complete analysis from $p$-$n$ – $\beta$-cycles, the binding energies of protons $E^*_{p\gamma}$ were also calculated. The values of the deviation of $E^*_{n\gamma}$ and $E^*_{p\gamma}$ from the mean values (from the curves in Fig. 5) are plotted in Fig. 6.

Consideration of this scheme of "anomalies" shows that (a) the binding energies of neutrons in the region $N < 126$ are raised on the average by 1.5 mev (in comparison with the mean values) and by $\sim 2.1$ mev relative to nuclei with $N = 127, 128$; (b) the binding energies of protons in the region $Z < 82$ are higher, relative to the mean, by $\sim 1.4$ mev, and the nuclei with $Z = 83, 84$ have energies $E^*_{p\gamma}$ that are lower by $\sim 0.5$ mev[13,4]. The character of the neutron shell $N = 126$ agrees well with the assumptions made on it from its spin-orbit behavior. Actually, the conclusions drawn above, from the point of view of the one particle model, can be interpreted in the following way. For $N < 126$, the neutron single particle levels are placed comparatively close to one another, with approximately equal spacing. The level which corresponds to $N = 127$ and the neutron one-particle levels following it are greatly increased relative to the preceding levels. The separation of the levels $N = 126$ and 127 is larger by $\sim 2.1$ mev than that between neighboring levels. The character of the proton levels of $Z = 82$ is quite similar.

## 5. COMPARISON WITH THE BETHE-WEIZSÄCKER FORMULA

For a comparison of the results of our analysis with the Bethe-Weizsäcker formula, we rewrite the latter in the form [19]

$$E_{\text{bind}} = \varepsilon A - \gamma A^{2/3} - \alpha A\left\{\left(\frac{I}{A}\right)^2 + \frac{A^{2/3}}{\beta}\left(1 - \frac{I}{A}\right)^2\right\} + \delta, \tag{9}$$

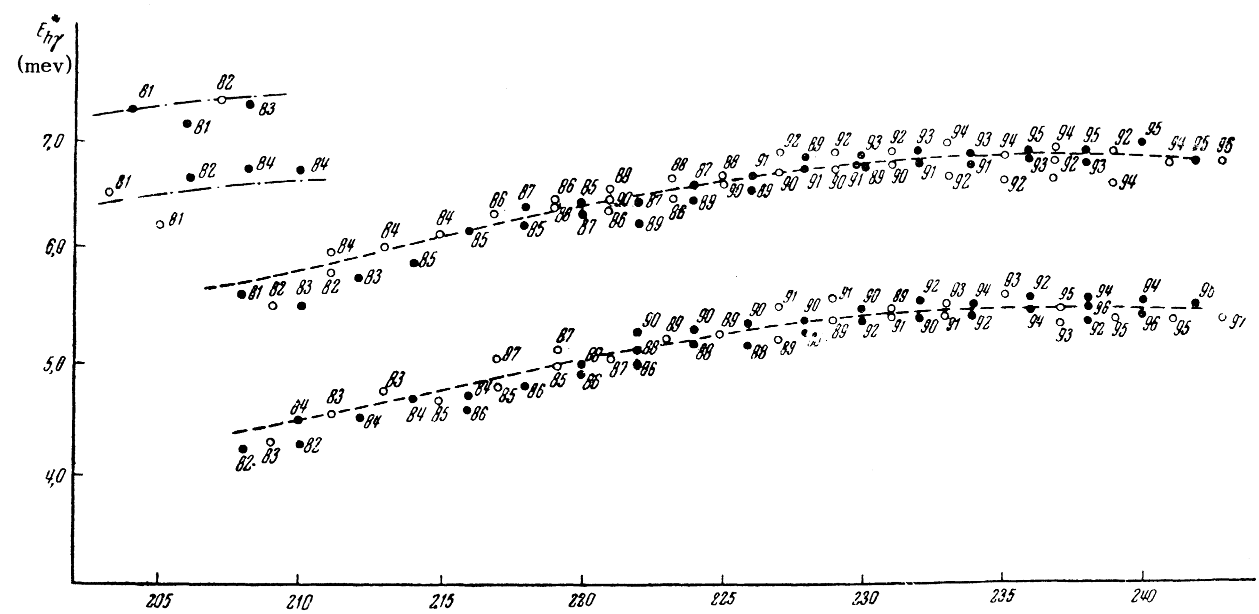


FIG. 5. Reduced energies of addition of neutrons. •—even $A$, o—odd $A$. The numbers by each of the points denote the number of protons.

where the first term characterizes the volume energy, the second the surface, the third the symmetric, the fourth the Coulomb; $\epsilon$, $\gamma$, $\beta$, $\alpha$ are numerical coefficients. The term $\delta$ characterizes the correction to the parity, which, in accord with the foregoing analysis, can be written for the given mass number interval in the following form:

$$\delta = \begin{cases} +\delta^{eo}_{oe} \approx +0.1 & \text{for } A_o,\ Z_e, \\ -\delta^{eo}_{oe} \approx -0.1 & \text{for } A_o,\ Z_o, \\ +\delta^{ee}_{oo} \approx +(2.77 - 0.0088A) & \text{for } A_e,\ Z_e, \\ -\delta^{ee}_{oo} \approx -(2.27 - 0.0088A) & \text{for } A_e,\ Z_e. \end{cases} \quad (10)$$

Equation (9) can be rewritten in the form[19]

$$E_{bind} = \varepsilon A - \gamma A^{2/3} - \alpha I_A - k_I (I - I_A)^2 + \delta, \quad (11)$$

where $I_A$ is determined by the condition $(\partial E_{bind}/\partial I)_A = 0$, whence $I_A = A/(1+\beta A^{-2/3})$, in which $I_A$ is related to $Z^*$ by the following:

$$I_A = I^* + 0.78/k = A - 2Z^* + 0.78/k, \quad (12)$$

$$k_I = k/4 = -\tfrac{1}{2}(\partial^2 E_{CB}/\partial I^2)_A = \alpha/A\,(1 + A^{2/3}/\beta). \quad (13)$$

Comparing the empirical values of $Z^*$ and $k$ with Eqs. (12) and (13), we find the following dependence of the coefficients $\alpha$ and $\beta$ on $A$:

$$\alpha \approx 0.1\ A + 2.32, \quad (14)$$

$$\alpha/\beta \approx 0.03 + 0.00067\ A,$$

i.e., the coefficients $\alpha$ for the term of symmetric energy, and $\alpha/\beta$ for the Coulomb energy term, do not remain constant, but increase with increase in the mass number $A$. One can undertake to connect this fact with the fact that in the range considered the effective radii increase somewhat more slowly than according to the law $A^{1/3}$. Actually, a decrease in the radius leads to an increase in the coefficient for the Coulomb energy term, and also for the symmetric energy term, inasmuch as the kinetic energy which makes the essential contribution to the term increases upon compression of the nucleus. The mean values of the coefficients for Coulomb and symmetric terms are close to the mean values of the same quantities found by other methods[1,2,20,21].

## CONCLUSION

The study of the properties of the energy surface in the region of heavy nuclei $210 < A < 244$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Tl | +2.2 | +1.8 | +1.55 | −0.3 | −0.1 | | | | Z=81 |
| Pb | +2.4 / +1.2 | +2.3 / +1.55 | +1.8 / +1.8 | −0.4 / +1.7 | −0.3 / +1.3 | −0.1 / +1.45 | 0 | +0.2 | Z=82 |
| Bi | | −0.7 | +1.6 / −0.7 | −0.5 / −0.8 | −0.2 / −0.7 | +0.1 / −0.6 | +0.1 / −0.5 | −0.2 / −0.9 | Z=83 |
| Po | | −0.7 | +1.5 / −0.8 | −0.3 / −0.6 | 0 / −0.4 | +0.2 / −0.2 | −0.1 / −0.4 | −0.1 / −0.3 | Z=84 |
| N=123 | N=124 | N=125 | N=126 | N=127 | N=128 | N=129 | N=130 | N=131 | |

FIG. 6. Scheme of "anomalies" in addition energies of neutrons and protons.

permits us to make the following conclusions:

1. The isobaric cross sections of the surface of the masses are, with sufficient accuracy, parabolas of second order for the nuclei of each of the types of parity [at least, for the values $|Z - Z^*| < (4\text{-}5)$]. Their curvatures are not dependent on the parity of $Z$, and evidently depend only slightly on the parity of $A$ ($k = 0.56$ for $A_e$ and 0.54 for $A_o$);

2. Minimal isobaric cross sections are achieved for nuclei of different parities for particular (distinct) values $Z = Z^*$. In this case the curves $Z^* = f(A)$ are higher for nuclei of the type $Z_o$, $N_o$ and $Z_o$, $N_e$ (which possibly coincide) and are lower (by $\sim 0.1$ in units of $Z$) for nuclei of the type $Z_e$, $N_e$ or $Z_e$, $N_o$ (which may also coincide);

3. The correction to the parity $\delta$ for even-even nuclei is equal in mean to +0.75 mev, for odd-odd, to -0.75 mev, for even-odd, +0.1 mev, for odd-even, -0.1 mev;

4. The character of the shells $N = 126$ and $Z = 82$ agrees with the hypothesis of spin-orbit interaction. The effect of both shells reaches a maximum at the nucleus $P^{208}$;

5. Comparison of the empirical data with the customary formula for the binding energy shows that in the mass interval under consideration its coefficients do not remain constant, but change somewhat; the character of their change agrees with the assumption on the certain volume (in comparison with the law $A^{1/3}$) of the decrease of the effective radii of heavy nuclei.

In conclusion, I take this opportunity to thank Professor D. D. Ivanenko for his constant interest in the work, and also S. I. Larin and N. A. Bekeshko for a series of valued suggestions.

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