Nucleomesodynamics in Strong Coupling. III
Translational Motion, Meson-Field Mass and Magnetic Moment of the Nucleon

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We consider an extended nucleon strongly interacting with the meson field. The meson field is assumed pseudoscalar, and the interaction of pseudovector type. This paper is an immediate continuation of the work of the writers[1,2], in which the nucleon was assumed infinitely heavy (stationary). A method is developed which is based on the expansion of the solution in powers of the reciprocal of the nucleon mass. In zeroth approximation the result corresponds to an infinitely heavy nucleon and coincides with that previously found[1,2]. The method is applicable only in the case of sufficiently slow (nonrelativistic) motion of the nucleon. The wave function of the system is calculated to the accuracy of terms of first order in the reciprocal of the nucleon mass. The meson-field mass of the nucleon is calculated to the accuracy of terms quadratic in the reciprocal of the nucleon mass. In the limit of strong coupling of the meson field with the nucleon, the nucleon magnetic moment is calculated in terms of an expansion in powers of the reciprocal of the nucleon mass. The first term of the series corresponds to the approximation of the infinitely heavy nucleon, and the following terms give a correction connected with the translational motion. The results obtained indicate that the difference of the magnetic moments of proton and neutron can be explained, not by taking into account the translational motion of the nucleon, but by renouncing the assumption of the limit of strong coupling.

1. INITIAL EQUATIONS; TRANSFORMATION TO NEW VARIABLES AND SEPARATION OF TRANSLATIONAL MOTION OF SYSTEM

In previous papers[1,2] we considered an extended nucleon strongly interacting with the meson field. The latter was assumed to be pseudoscalar and to contain charged and neutral mesons symmetrically. The interaction of the meson field with the nucleon was taken to be of pseudovector type. The nucleon was assumed infinitely heavy, i.e., stationary (fixed at the origin of coordinates). The approximation method was based on the condition of strong coupling

$$(g/\mu a)^2 \gg 1, \tag{1}$$

where g is a dimensionless constant of interaction of the nucleon with the meson field, μ is the reciprocal of the Compton wavelength of the meson, and a is the effective radius of the extended nucleon.

In the present paper we retain all the initial assumptions of the earlier papers[1,2], with the exception of the assumption of an infinitely heavy nucleon. The mass of the nucleon is here taken as finite, and its translational motion is taken into account. Thus the Hamiltonian of the system has the form

$$\hat{H} = -(1/2\,M_0)\,\Delta_\xi + \hat{H}_0 + \hat{H}'(\xi;q), \tag{2}$$

where

$$\hat{H}_0 = \frac{1}{2}\sum_{\alpha=1}^{3}\int [\pi_\alpha^2 + (\nabla\varphi_\alpha)^2 + \mu^2\varphi_\alpha^2]\,dV = \frac{1}{2}\sum_{\alpha\varkappa}\omega_\varkappa\,[q_{\alpha\varkappa}^2 - \partial^2/\partial q_{\alpha\varkappa}^2] \tag{3}$$

is the energy of the oscillations of the meson field,

$$\hat{H}'(\xi, q) = -\frac{g}{\mu}\sqrt{4\pi} \times \sum_{\alpha=1}^{3}\tau_\alpha \int (\sigma, \nabla\varphi_\alpha)\,U(\mathbf{r}-\xi)\,dV \tag{4}$$

is the energy of interaction of the nucleon with the meson field, $U(\mathbf{r})$ is the form factor of the nucleon, σ and τ are the operators of spin and isotopic spin, respectively, and ξ is the coordinate of the nucleon. M_0 is the mass of the "bare" nucleon, not including the meson-field part of the mass.

The normal coordinates $q_{\alpha\varkappa}$ of the meson field are determined as the coefficients of the expansion of the meson field $\varphi_\alpha(\mathbf{r})$ in a Fourier series of the form

$$\varphi_\alpha(\mathbf{r}) = \sum_\varkappa \frac{q_{\alpha\varkappa}}{\sqrt{\omega_\varkappa}}\chi_\varkappa(\mathbf{r}), \tag{5}$$

$$\chi_\varkappa(\mathbf{r}) = \sqrt{\frac{2}{L^3}}\begin{cases}\cos\boldsymbol{\varkappa}\mathbf{r} & \text{for } \varkappa_x \leqslant 0,\\ \sin\boldsymbol{\varkappa}\mathbf{r} & \text{for } \varkappa_x > 0,\end{cases} \tag{6}$$

L^3 is the volume of the fundamental region of periodicity, and $\omega_{\varkappa} = \sqrt{\mu^2 + \varkappa^2}$ ($\omega_{\varkappa} > 0$). Here, as before[1,2], the natural system of units is used ($\hbar = c = 1$).

It is readily seen that the operator (2) for the energy of the system is invariant with respect to the translational transformation $T_{\mathbf{a}}$, which consists of the simultaneous displacement of both the nucleon and the entire meson field by the vector $\mathbf{a}$: $\xi \to \xi + \mathbf{a}$, $\varphi_\alpha(\mathbf{r}) \to \varphi_\alpha(\mathbf{r} - \mathbf{a})$. To such a transformation of the field there corresponds the following transformation of the normal coordinates:

$$q_{\alpha\varkappa} \to q_{\alpha\varkappa} \cos \varkappa\mathbf{a} + q_{\alpha,-\varkappa} \sin \varkappa\mathbf{a}.$$

The translation operator $T_{\mathbf{a}}$ commutes with $\hat{H}$, and is consequently an integral of the motion. By carrying out infinitely small translations in the directions of the three axes of the Cartesian coordinates, one readily convinces onself that the vector

$$\hat{\mathbf{k}} = -i\nabla_\xi - i \sum_{\alpha\varkappa} \varkappa\, q_{\alpha,-\varkappa}\, \partial/\partial q_{\alpha\varkappa} \tag{7}$$

is also an integral of the motion. It will be called the total momentum of the system.

In order to separate the translational motion of the system, it is expedient to go from the variables ξ, $q_{\alpha\varkappa}$ to new variables $\mathbf{R}$, $p_{\alpha\varkappa}$ by the formulas

$$\xi = \mathbf{R};\, q_{\alpha\varkappa} = p_{\alpha\varkappa} \cos \varkappa\mathbf{R} + p_{\alpha,-\varkappa} \sin \varkappa\mathbf{R};\, q_{\alpha,-\varkappa} = p_{\alpha,-\varkappa} \cos\varkappa\mathbf{R} - p_{\alpha\varkappa} \sin \varkappa\mathbf{R}. \tag{8}$$

The new variables $p_{\alpha\varkappa}$ are the coefficients of the expansion of the function $\tilde{\varphi}_\alpha(\mathbf{r}) \equiv \varphi_\alpha(\mathbf{r} + \xi)$ in a series of the form (5)

$$\tilde{\varphi}_\alpha(\mathbf{r}) = \sum_{\varkappa} \omega_\varkappa^{-1/2} p_{\alpha\varkappa} \chi_\varkappa(\mathbf{r}). \tag{9}$$

The transformation from the coordinates $q_{\alpha\varkappa}$, $q_{\alpha,-\varkappa}$ to the coordinates $p_{\alpha\varkappa}$, $p_{\alpha,-\varkappa}$ by the formula (8) is entirely analogous to the rotation of Cartesian coordinates in a plane by the angle $\varkappa\mathbf{R}$, and in just the same way leaves invariant the expressions

$$q_{\alpha\varkappa}^2 + q_{\alpha,-\varkappa}^2 = p_{\alpha\varkappa}^2 + p_{\alpha,-\varkappa}^2;$$

$$\frac{\partial^2}{\partial q_{\alpha\varkappa}^2} + \frac{\partial^2}{\partial q_{\alpha,-\varkappa}^2} = \frac{\partial^2}{\partial p_{\alpha\varkappa}^2} + \frac{\partial^2}{\partial p_{\alpha,-\varkappa}^2}.$$

Therefore, in the new variables $\hat{H}_0$ retains its previous form:

$$\hat{H}_0(q) = \hat{H}_0(p) = \frac{1}{2} \sum_{\alpha\varkappa} \omega_\varkappa (p_{\alpha\varkappa}^2 - \partial^2/\partial p_{\alpha\varkappa}^2). \tag{10}$$

Substituting into Eq. (4) the function $\tilde{\varphi}_\alpha(\mathbf{r} - \xi)$ instead of $\varphi_\alpha(\mathbf{r})$, one readily obtains

$$\hat{H}'(\xi, q) = -\frac{g}{\mu} \sqrt{4\pi} \times \sum_{\alpha=1}^{3} \tau_\alpha \int (\sigma, \nabla\tilde{\varphi}_\alpha(\mathbf{r}))\, U(\mathbf{r})\, dV = \hat{H}'(0, p). \tag{11}$$

In the new variables the operator for the energy of the system has the form

$$\hat{H} = \frac{1}{2M_0}(-i\nabla_{\mathbf{R}} + \hat{\Omega})^2 + \hat{H}_0(p) + \hat{H}'(0, p), \tag{12}$$

where

$$\hat{\Omega} = i \sum_{\alpha\varkappa} \varkappa p_{\alpha,-\varkappa}\, \partial / \partial p_{\alpha,\varkappa}. \tag{13}$$

In Eq. (12) the differentiation with respect to $\mathbf{R}$ is carried out with constant $p_{\alpha\varkappa}$. The momentum operator (7) has, in the new variables, the simple form

$$\hat{\mathbf{k}} = -i\nabla_{\mathbf{R}}. \tag{14}$$

Since the operators (12) and (14) commute, the wave function of the system can be chosen so that it is an eigenfunction of both operators. An eigenfunction of the operator (12) has the form

$$\chi = L^{-3/2} e^{i\mathbf{kR}} \Psi(p), \tag{15}$$

where in the general case $\Psi(p)$ is an arbitrary function of the $p_{\alpha\varkappa}$. In order that (15) be at the same time also an eigenfunction of the operator (12), it is necessary that $\Psi(p)$ satisfy the equation

$$\left[\frac{1}{2M_0} (\mathbf{k} + \hat{\Omega})^2 + \hat{H}_0(p) + \hat{H}'(0,p)\right] \Psi(p) = E\Psi(p). \tag{16}$$

In this equation the coordinate R already does not appear at all. Thus the translational motion of the system separates exactly.

2. THE APPROXIMATION METHOD

In the case of the infinitely heavy nucleon, for $M_0 \to \infty$, the first term of the left member of Eq. (16) vanishes, and the equation goes over into that for a stationary nucleon, which was solved in our earlier papers[1,2]. The difference is only that instead of the coordinates $q_{\alpha\varkappa}$ used earlier[1,2] there now appear the new variables $p_{\alpha\varkappa}$. In the case of finite but sufficiently large M_0 we shall treat the term

$$(1/2M_0)(\mathbf{k} + \hat{\Omega})^2$$

of Eq. (16) as a small perturbation. Then applying the ordinary perturbation theory we obtain the expansion of the solution of Eq. (16) in powers of $1/M_0$.

The solution of the unperturbed equation, found previously[1,2], has the form

$$\Psi = \psi_s^v \Phi(v, p), \tag{17}$$

where ψ_s^v is a four-component function describing the spin and charge motion of the nucleon and v_i are angle variables. As shown before[1,2], the angles v_i are themselves functions of the $p_{\alpha\varkappa}$, so that in its application to functions of the form (17)

$$\partial/\partial p_{\alpha\varkappa} = \partial/\partial^* p_{\alpha\varkappa} + \sum_i (\partial v_i / \partial p_{\alpha\varkappa})\, \partial/\partial v_i,$$

where $\partial/\partial^* p_{\alpha\varkappa}$ denotes differentiation only with respect to the variable $p_{\alpha\varkappa}$ not occurring in the arguments v_i. According to this,

$$\hat{\Omega} = i \sum_{\alpha\varkappa} \varkappa p_{\alpha,-\varkappa} \left[\frac{\partial}{\partial^* p_{\alpha\varkappa}} + \sum_i \frac{\partial v_i}{\partial p_{\alpha\varkappa}} \frac{\partial}{\partial v_i} \right]. \tag{18}$$

As found before[1,2], in the case of strong coupling the field $\tilde{\varphi}_\alpha(\mathbf{r})$ in the neighborhood of the nucleon remains nearly equal to the self-consistent meson field $\tilde{\varphi}_\alpha^v(\mathbf{r})$, which is determined by Eq. (60) of an earlier paper[1],

$$\tilde{\varphi}_\alpha^v(\mathbf{r}) = -(\mathbf{g}_\alpha, \nabla W), \tag{19}$$

$$W(\mathbf{r}) = \frac{g}{\mu} \frac{1}{\sqrt{4\pi}} \int U(\mathbf{r}') \frac{e^{-\mu|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} dV'; \tag{20}$$

$$\mathbf{g}_\alpha \equiv \{\psi_s^{v*}, \tau_\alpha \sigma \psi_s^v\}.$$

The coefficients of the expansion of the function $\tilde{\varphi}_\alpha^v(\mathbf{r})$ in a Fourier series of the form (9) have the form

$$p_{\alpha\varkappa}^v = (\mathbf{g}_\alpha, \varkappa) \sqrt{\omega_\varkappa} W_{-\varkappa}; \tag{21}$$

$$W_\varkappa = \int W(\mathbf{r}) \chi_\varkappa(\mathbf{r})\, dV.$$

Because of the spherical symmetry of the functions $U(\mathbf{r})$ and $W(\mathbf{r})$ the coefficients $W_\varkappa$, as is seen from Eqs. (21) and (6), are different from zero only in the half-space of $\varkappa$ in which $\varkappa_x < 0$. The coefficients $p_{\alpha\varkappa}^v$, and also, as is seen from Eq. (66) of Ref. 1, $\partial v_i / \partial p_{\alpha\varkappa}$ are different from zero only for $\varkappa_x > 0$.

Because of the small difference between the functions $\tilde{\varphi}_\alpha(\mathbf{r})$ and $\tilde{\varphi}_\alpha^v(\mathbf{r})$ in the neighborhood of the nucleon, in the second factor of the operator (18) we may replace $p_{\alpha,-\varkappa}$ approximately by $p_{\alpha\varkappa}^v$. It can be shown that this introduces into the result a fractional error of the order $(\mu a/g)^2$, which is small in the case of strong coupling, when the inequality (1) holds. After this replacement the second term of the operator (18) turns out equal to zero, since $p_{\alpha,-\varkappa}^v$ and $\partial v_i / \partial p_{\alpha\varkappa}$ are not both different from zero for any value of $\varkappa$. Thus in the further work application of the operator $\hat{\Omega}$ to functions of the form (17) will not involve differentiation with respect to the arguments v_i.

We now proceed to the calculation of the matrix elements of the perturbation. Since $\hat{\Omega}$ does not contain operators on the spin and charge degrees of freedom, and ψ_s^v does not contain the variables $p_{\alpha\varkappa}$ explicitly, in taking the matrix elements of the operator $\hat{\Omega}$, ψ_s^v is brought outside the sign of operation, forming a factor $\{\psi_{s'}^{v*}, \psi_s^v\} = \delta_{s's}$. Therefore, in the perturbation calculation we can ignore all states of the spin-charge motion that are different from the perturbed state s. In what follows it is always assumed that the spin-charge motion of the nucleon is not excited.

It is sufficient to carry out the calculation of the perturbation matrix elements with functions $\Phi(v, p)$ which, according to Ref. 2, have the form

$$\Phi = Q_\nu(\vartheta, \beta, \delta) \times \exp\left\{-\frac{1}{2}\sum_{\alpha\varkappa}\left[(p_{\alpha\varkappa} - p_{\alpha\varkappa}^v)^2 + \frac{1}{2}\ln\pi\right]\right\} \times \prod_l H_{n_l}(p_l - p_l^v), \tag{22}$$

where H_{n_l} are Chebyshev-Hermite polynomials of degree n_l, l are certain of the indices $\alpha\varkappa$ (indices of free mesons) and Q_ν is the wave function of the quasioscillator considered in Ref. 2. Here ν is a set consisting of the three quantum numbers of the above-mentioned oscillator, the half-integral positive number j [the square of the angular momentum of the rotator is equal to $j(j+1)$], the quantum number of the nucleon charge ϵ ($\epsilon = 0, 1$), and the integer quantum number of the doubled component of the nucleon spin along the axis OZ, σ. For the ground (nonisobaric) state of the rotator, i.e., for the state with $j = \frac{1}{2}$, the functions Q_ν are presented in Table I at the end of Ref. 2 ($Q_\nu = 2^{-\frac{1}{2}}\pi^{-1}C_i^\nu$).

We agree to denote mesonless states (free mesons are absent) by indices 0ν, one-meson states by indices $\alpha\varkappa\nu$, where $\alpha\varkappa$ is the index of the single free meson. States with two different free mesons will be denoted by indices $\alpha_1\varkappa_1\alpha_2\varkappa_2\nu$, where $\alpha_1\varkappa_1$ and $\alpha_2\varkappa_2$ are the indices of the free mesons. If in the function (22) $n_l = 2$ for one $l = \alpha\varkappa$, and the other $n_{l'}$ are all zero, we call such a state also a two-meson state, and give it the indices $2\alpha\varkappa\nu$.

If the initial state is mesonless, then the matrix elements $\hat{\Omega}$ are different from zero only for transitions into one-meson states. These matrix elements are equal to

$$(\alpha\varkappa\nu' \,|\, \hat{\Omega} \,|\, 0\nu) = 2^{-1/2}\, i\varkappa\sqrt{\omega_\varkappa}\, W_\varkappa(\varkappa, g_\alpha^{\nu'\nu}); \tag{23}$$

$$g_\alpha^{\nu'\nu} \equiv (\nu' \,|\, \hat{g}_\alpha | \nu) \equiv (j'\varepsilon'\sigma' \,|\, \hat{g}_\alpha \,|\, j\varepsilon\sigma) \tag{24}$$

$$\equiv \int Q_{\nu'}^* g_\alpha Q_\nu d\omega; \quad d\omega = \sin\delta\, d\vartheta d\beta d\delta .$$

The three components of the vector $\mathbf{g}_\alpha$ will be denoted, respectively, by $g_{\alpha 1}$, $g_{\alpha 2}$ and $g_{\alpha 3}$. In what follows only such matrix elements $g_{\alpha\beta}^{\nu'\nu}$ are needed in which the final state ν' as well as the initial state ν is nonisobaric ($j' = j = \frac{1}{2}$). The values of these matrix elements $g_{\alpha\beta}^{\epsilon'\sigma'|\epsilon\sigma}$ are given below:

$$g_{\alpha\beta}^{0-1|0-1} = g_{\alpha\beta}^{11|11} = \frac{1}{3}\delta_{\alpha 3}\delta_{\beta 3}; \tag{25}$$

$$g_{\alpha\beta}^{01|01} = g_{\alpha\beta}^{1-1|1-1} = -\frac{1}{3}\delta_{\alpha 3}\delta_{\beta 3};$$

$$g_{\alpha\beta}^{0-1|01} = -g_{\alpha\beta}^{1-1|11} = \frac{1}{3}(\delta_{\alpha 3}\delta_{\beta 1} + i\delta_{\alpha 3}\delta_{\beta 2});$$

$$g_{\alpha\beta}^{0-1|1-1} = -g_{\alpha\beta}^{01|11} = \frac{1}{3}(\delta_{\alpha 1}\delta_{\beta 3} + i\delta_{\alpha 2}\delta_{\beta 3});$$

$$g_{\alpha\beta}^{0-1|11} = \frac{1}{3}[\delta_{\alpha 1}\delta_{\beta 1} - \delta_{\alpha 2}\delta_{\beta 2} + i(\delta_{\alpha 1}\delta_{\beta 2} + \delta_{\alpha 2}\delta_{\beta 1})];$$

$$g_{\alpha\beta}^{01|1-1} = \frac{1}{3}[\delta_{\alpha 1}\delta_{\beta 1} + \delta_{\alpha 2}\delta_{\beta 2} + i(\delta_{\alpha 2}\delta_{\beta 1} - \delta_{\alpha 1}\delta_{\beta 2})].$$

The matrix $g_{\alpha\beta}^{\nu\nu'}$ is Hermitian: $g_{\alpha\beta}^{\nu'\nu} = (g_{\alpha\beta}^{\nu\nu'})^*$.

The nine quantities $g_{\alpha\beta}$, as is seen from Eq. (38) of Ref. 1, can be treated as a rotation matrix of a three-dimensional Cartesian coordinate system. In this connection the angles ϑ, β and δ appear as Eulerian angles. From this there follows the relations

$$\sum_{\alpha=1}^{3} g_{\alpha\beta}g_{\alpha\beta'} = \delta_{\beta\beta'}; \quad \sum_{\beta=1}^{3} g_{\alpha\beta}g_{\alpha'\beta} = \delta_{\alpha\alpha'}, \tag{26}$$

which considerably simplify the further calculations.

Further on we shall also need matrix elements of $\hat{\Omega}$ for which the initial state (right-hand index) is a one-meson state. Here, besides the transitions into mesonless states, for which the matrix element is equal to the complex conjugate of (23), there are also the following nonvanishing matrix elements:

$$(\alpha, -\varkappa\nu'' \,|\, \hat{\Omega} \,|\, \alpha\varkappa\nu') = i\varkappa\delta_{\nu''\nu'}; \tag{27}$$

$$(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'' \,|\, \hat{\Omega} \,|\, \alpha\varkappa\nu')$$

$$= 2^{-1/2}\, i\varkappa_1\sqrt{\omega_{\varkappa_1}}W_{\varkappa_1}(\varkappa_1, g_\alpha^{\nu''\nu'})\,\delta_{\alpha\alpha_2}\delta_{\varkappa\varkappa_2}$$

$$+ 2^{-1/2}\, i\varkappa_2\sqrt{\omega_{\varkappa_2}}\, W_{\varkappa_2}(\varkappa_2, g_\alpha^{\nu''\nu'})\,\delta_{\alpha\alpha_1}\delta_{\varkappa\varkappa_1};$$

$$(2\alpha\varkappa\nu'' \,|\, \hat{\Omega} \,|\, \alpha\varkappa\nu') = i\varkappa\sqrt{\omega_\varkappa}\, W_\varkappa(\varkappa, g_\alpha^{\nu''\nu'}).$$

The matrix elements of the operator $\hat{\Omega}^2$ are easily obtained from the matrix elements of $\hat{\Omega}$ given above. The nonvanishing matrix elements of $\hat{\Omega}^2$ for which the initial state is mesonless (right-hand index 0ν) are of the forms

$$(0\nu' \,|\, \hat{\Omega}^2 \,|\, 0\nu) = \frac{1}{2}\delta_{\nu'\nu}\sum_\varkappa \varkappa^4\omega_\varkappa W_\varkappa^2; \tag{28}$$

$$(\alpha\varkappa\nu' \,|\, \hat{\Omega}^2 \,|\, 0\nu) = \sqrt{\omega_\varkappa/2}\,\varkappa^2 W_{-\varkappa}(\varkappa, g_\alpha^{\nu'\nu}); \tag{29}$$

$$(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu' \,|\, \hat{\Omega}^2 \,|\, 0\nu) \tag{30}$$

$$= -\sqrt{\omega_{\varkappa_1}\omega_{\varkappa_2}}(\varkappa_1, \varkappa_2)\, W_{\varkappa_1}W_{\varkappa_2}\,[(g_{\alpha_1}, \varkappa_1)(g_{\alpha_2}, \varkappa_2)]^{\nu'\nu};$$

$$(2\alpha\varkappa\nu' \,|\, \hat{\Omega}^2 \,|\, 0\nu) = -2^{-1/2}\varkappa^2\omega_\varkappa\,[(g_\alpha, \varkappa)^2]^{\nu'\nu}. \tag{31}$$

As is well known, in order to make use of the ordinary perturbation theory it is further necessary to determine the differences of the energies of the system in the various states in zeroth approximation. According to Eqs. (12), (17) and (31) of Ref. 2, the energy differences needed in the calculation have the following values:

$$E_{\alpha\varkappa\nu'} - E_{0\nu} = \omega_\varkappa + H_{j'} - H_j; \tag{32}$$

$$E_{\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'} - E_{0\nu} = \omega_{\varkappa_1} + \omega_{\varkappa_2} + H_{j'} - H_j;$$

$$E_{2\alpha\varkappa\nu'} - E_{0\nu} = 2\omega_\varkappa + H_{j'} - H_j.$$

Here H_j is the energy of the quasioscillator, determined from Eq. (31) of Ref. 2.

On taking into account the form of the perturbation and Eq. (32), we can write the perturbed wave function of the system in first approximation in the form

$$\Psi = \psi_s^v \left\{ \Phi_{0\nu} - \frac{1}{2M_0} \sum_{\alpha\varkappa\nu'} \frac{(\alpha\varkappa\nu' \mid 2(\mathbf{k}, \hat{\Omega}) + \hat{\Omega}^2 \mid 0\nu)}{\omega_\varkappa + \Delta_{\nu'}} \Phi_{\alpha\varkappa\nu'} \right. \tag{33}$$

$$\left. - \frac{1}{2M_0} \sum_{\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'}{}' \frac{(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu' \mid \hat{\Omega}^2 \mid 0\nu)}{\omega_{\varkappa_1} + \omega_{\varkappa_2} + \Delta_{\nu'}} \Phi_{\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'} - \frac{1}{2M_0} \sum_{\alpha\varkappa\nu'} \frac{(2\alpha\varkappa\nu' \mid \hat{\Omega}^2 \mid 0\nu)}{2\omega_\varkappa + \Delta_{\nu'}} \Phi_{2\alpha\varkappa\nu'} \right\},$$

where $\Delta_{\nu'} = H_{j'} - H_j$. The matrix elements of $\hat{\Omega}$ and $\hat{\Omega}^2$ determined from Eqs. (23) and (28)-(31) are to be substituted into the expression (33). The prime on the second summation of the expression (33) indicates that terms are to be omitted in which both of the indices $\alpha_1 \varkappa_1$ coincide with the indices $\alpha_2 \varkappa_2$. It is essential to note that the quantities involved in the matrix elements of $\hat{\Omega}$ and $\hat{\Omega}^2$ are different from zero only if $j' = j-1, j, j+1$. In particular, when the initial state ν is nonisobaric ($j = 1/2$), transitions are allowed only to the states with $j' = 1/2, 3/2$.

To prove the above-mentioned selection rules for $g_{\alpha\beta}^{\nu\nu'}$ we must start with the Eqs. (24) and take into account the fact that the functions $Q_{j\epsilon\sigma}$ are linear combinations of the functions $V_{sp}^j(\vartheta, \beta, \delta)$ with different s and p. The functions V_{sp}^j are defined in Ref. 2 by Eqs. (18), (20), (24) and (25). These functions are connected with the functions T_{sp}^j used in the work of Gel'fand and Shapiro[3] (Sec.7) by the relations

$$V_{sp}^j(\vartheta, \beta, \delta) \tag{34}$$

$$= \frac{1}{2\pi} \sqrt{\frac{2j+1}{2}} (-i)^{2j-s-p} T_{sp}^{j*}(\vartheta, \pi - \delta, \beta).$$

Thus the functions $Q_{j\epsilon\sigma}(\vartheta, \beta, \delta)$ in the integrand in Eq. (24) are linear combinations of the functions $T_{sp}^{j*}(\vartheta, \pi-\delta, \beta)$ with the same index j. The selection rules mentioned above are easy to prove, if we make use of the well-known recurrence formulas for the functions $T_{sp}^j(\vartheta, \delta, \beta)$ (cf. Ref. 3, p. 88):

$$\cos \delta T_{sp}^j = c_{21}^p T_{sp}^{j+1} c_{21}^s + c_{22}^p T_{sp}^j c_{22}^s + c_{23}^p T_{sp}^{j-1} c_{23}^s; \tag{35}$$

$$2^{-1/2} i \sin \delta e^{i\vartheta} T_{s+1,p}^j = c_{21}^p T_{sp}^{j+1} c_{11}^s + c_{22}^p T_{sp}^j c_{12}^s + c_{23}^p T_{sp}^{j-1} c_{13}^s;$$

$$2^{-1/2} i \sin \delta e^{-i\vartheta} T_{s-1,p}^j = c_{21}^p T_{sp}^{j+1} c_{31}^s + c_{22}^p T_{sp}^j c_{32}^s + c_{23}^p T_{sp}^{j-1} c_{33}^s;$$

$$2^{-1/2} i \sin \delta e^{i\beta} T_{s,p+1}^j = c_{11}^p T_{sp}^{j+1} c_{21}^s + c_{12}^p T_{sp}^j c_{22}^s + c_{13}^p T_{sp}^{j-1} c_{23}^s;$$

$$2^{-1/2} i \sin \delta e^{-i\beta} T_{s,p-1}^j = c_{31}^p T_{sp}^{j+1} c_{21}^s + c_{32}^p T_{sp}^j c_{22}^s + c_{33}^p T_{sp}^{j-1} c_{23}^s.$$

For the meaning of the coefficients c_{ik}^m see p. 52 of Ref. 3. If the quantity $g_{\alpha\beta}$ in the integrand of Eq. (24) contains $\cos\delta$, the first of the formulas (35) is to be used. If $g_{\alpha\beta}$ contains $\sin\delta$ multiplied by $\sin\vartheta$ or $\cos\vartheta$, the second and third formulas are to be used, and in the remaining cases the fourth and fifth.

The perturbations $-(\mathbf{k}, \hat{\Omega})/M_0$ and $-\hat{\Omega}^2/2M_0$ do not "interfere", in the sense that they do not have nonvanishing matrix elements for one and the same transition. In particular, the matrix elements (23) and (29) are also different from zero in different half-spaces of $\varkappa$, so that the square of the absolute value of the matrix element of the total perturbation is here equal to the sum of the squares of the absolute values of the matrix elements of the separate terms. In such cases, as is well known, each of these perturbations can be introduced independently of the other, after which one can simply combine the corrections to the wave function and the energy caused by each of these perturbations separately.

Alongside the approximation method described above, we also considered ordinary adiabatic approximation: In the zeroth approximation it was assumed that the spin-charge motion of the nucleon and the oscillations of the meson field follow adiabatically the translational motion of the nucleon, and then the nonadiabatic terms were introduced as a small perturbation. Here the calculations turned out to be rather more complicated, mainly because of the fact that the matrix elements had to be calculated with more complicated wave functions, which also depended on the coordinate ξ of the translational motion. Besides this, it was also necessary in Eqs. (32) to add on the differences of the kinetic energies of the translational motions of the nucleon. The results naturally turned out to be equivalent to those obtained above, if expanded in powers of $1/M_0$.

3. ENERGY AND MESON-FIELD MASS OF THE NUCLEON

We first calculate the correction E_1 to the energy of the nucleon, caused by the perturbation $-(\mathbf{k}, \hat{\Omega})/M_0$. The first order correction is equal to zero, since $(0\nu|\hat{\Omega}|0\nu) = 0$. In the second approximation the energy correction is equal to

$$E_1^{(2)} = -\frac{1}{M_0^2}\sum_{\alpha\varkappa\nu'}\frac{|(\alpha\varkappa\nu'|(\mathbf{k},\hat{\Omega})|0\nu)|^2}{\omega_\varkappa+\Delta_{\nu'}} \tag{36}$$

$$= -\frac{1}{2M_0^2}\sum_{\alpha\varkappa\nu'}(\mathbf{k},\boldsymbol{\varkappa})^2\,W_\varkappa^2\,|(\boldsymbol{\varkappa},\mathbf{g}_\alpha^{\nu'\nu})|^2\,\frac{\omega_\varkappa}{\omega_\varkappa+\Delta_{\nu'}}.$$

It must be noted that the perturbed state is fourfold degenerate (cf. Ref. 2). However, since $(0\nu'|(\mathbf{k},\hat{\Omega})|0\nu) = 0$, the energy correction in second order is given by the same formula as in the nondegenerate case.

In Eq. (36) the summation over $\alpha\nu'$ can be carried out in the following way: first we replace $\Delta_{\nu'}$ in all terms by its value $\Delta_{3/2}$ for $j' = 3/2$. After this the sum over $\alpha\nu'$ becomes equal to

$$\sum_{\alpha\nu'}|(\boldsymbol{\varkappa},\mathbf{g}_\alpha^{\nu'\nu})|^2\frac{\omega_\varkappa}{\omega_\varkappa+\Delta_{3/2}} = \frac{\omega_\varkappa}{\omega_\varkappa+\Delta_{3/2}} \tag{37}$$

$$\times\sum_{\alpha\nu'}(\boldsymbol{\varkappa},\mathbf{g}_\alpha^{\nu\nu'})(\boldsymbol{\varkappa},\mathbf{g}_\alpha^{\nu'\nu}) = \frac{\omega_\varkappa}{\omega_\varkappa+\Delta_{3/2}}\sum_\alpha[(\boldsymbol{\varkappa},\mathbf{g}_\alpha)^2]^{\nu\nu}.$$

As was already mentioned above, in the sum (36) only the terms $g_\alpha^{\nu'\nu}$ with $j' = ½, 3/2$ are different from zero, so that the replacement of $\Delta_{\nu'}$ by $\Delta_{3/2}$ changes only the terms with $j' = ½$ by the amount

$$-|(\boldsymbol{\varkappa},\mathbf{g}_\alpha^{\nu'\nu})|^2\left\{1-\frac{\omega_\varkappa}{\omega_\varkappa+\Delta_{3/2}}\right\}.$$

Thus the exact expression of the sum (36) over α and ν' is

$$\frac{\omega_\varkappa}{\omega_\varkappa+\Delta_{3/2}}\sum_\alpha[(\boldsymbol{\varkappa},\mathbf{g}_\alpha)^2]^{\nu\nu} \tag{38}$$

$$+\sum_{\alpha s'\sigma'\,(j'=1/2)}|(\boldsymbol{\varkappa},\mathbf{g}_\alpha^{\nu'\nu})|^2\left\{1-\frac{\omega_\varkappa}{\omega_\varkappa+\Delta_{3/2}}\right\}.$$

Here the further summation over α is easy to perform, if we take account of Eqs. (25) and (26). The final result for the sum (38) is the expression

$$\sum_{\alpha\nu'}|(\boldsymbol{\varkappa},\mathbf{g}_\alpha^{\nu'\nu})|^2\frac{\omega_\varkappa}{\omega_\varkappa+\Delta_{\nu'}} \tag{39}$$

$$= \varkappa^2\left\{\frac{1}{3}+\frac{2}{3}\frac{\omega_\varkappa}{\omega_\varkappa+\Delta_{3/2}}\right\}.$$

Substituting this in Eq. (36) and noting that the average of $(\mathbf{k},\boldsymbol{\varkappa})^2$ over the angles is equal to $k^2\varkappa^2/3$, we find for the second-order correction to the energy the expression

$$E_1^{(2)} = -\frac{k^2}{6M_0^2}\sum_\varkappa \varkappa^4 W_\varkappa^2\left\{\frac{1}{3}+\frac{2}{3}\frac{\omega_\varkappa}{\omega_\varkappa+\Delta_{3/2}}\right\}. \tag{40}$$

If we introduce the operator $\hat{\omega} = \sqrt{\mu^2-\Delta}$ (where Δ is the Laplace operator), the result (40) can be rewritten in the form

$$E_1^{(2)} = -\frac{k^2}{2M_0^2}S, \tag{41}$$

$$S = \frac{1}{3}\int\Delta W\left\{\frac{1}{3}+\frac{2}{3}\frac{\hat{\omega}}{\hat{\omega}+\Delta_{3/2}}\right\}\Delta W dV.$$

We now go on to the calculation of the correction to the energy due to the second perturbation term $-\Omega^2/2M_0$. By Eq. (28) this correction E_2 is in first order equal to

$$E_2^{(1)} = -\frac{1}{4M_0}\sum_\varkappa \varkappa^4\omega_\varkappa W_\varkappa^2 \tag{42}$$

$$= -\frac{1}{4M_0}\int\Delta W\hat{\omega}\Delta W dV.$$

From the formulas (28)-(31) the correction E_2 can without difficulty be found also in second approximation. But all corrections from the perturbation $-\Omega^2/2M_0$ give constant terms not dependent on $\mathbf{k}\varkappa$, which consequently have no effect on the value of the field mass of the nucleon, so that they have no particular interest for us here and will not be presented.

Just so it is pointless to calculate the corrections to the energy from the operator $-(\mathbf{k}\hat{\Omega})/M_0$ in orders higher than the second, since they contain the fourth and higher powers of $\mathbf{k}$. The correction proportional to k^2 suffices for the calculation of the field mass of the nucleon. Consideration of higher powers of $\mathbf{k}$ does not make sense, since the theory is nonrelativistic.

According to Eqs. (16), (41) and (42), the total energy of the system, which appears in Eq. (16), can be written in the form

$$E = H + \frac{k^2}{2M_0} - \frac{k^2}{2M_0^2} S - \frac{1}{4M_0}\int \Delta W \hat{\omega} \Delta W dV. \tag{43}$$

Here H is the energy of the system in the approximation of the infinitely heavy (stationary) nucleon, given by Eqs. (17) and (31) of Ref. 2.

The term in Eq. (43) proportional to k^3 can be written in the form

$$k^2/2M, \quad \text{where} \quad 1/M = 1/M_0 - S/M_0^2. \tag{44}$$

Whereas M_0 is the mass of the "bare" nucleon (not interacting with the meson field), M represents the total mass of the real nucleon (interacting with the meson field). It is, namely, with M that the experimentally determined mass of the nucleon is to be identified. With a fractional error of the order $(S/M_0)^2$ Eq. (44) can be rewritten thus: $M = M_0 + S$. From this it is seen that S is the meson-field part of the nucleon mass.

The ratio $\Delta_{3/2}/\omega_\varkappa$ is in order of magnitude equal to $(\mu a/g)^2$. In the case of strong coupling, Eq. (1), it is considerably smaller than unity, so that in the formulas obtained above one can neglect the quantity $\Delta_{3/2}$ in comparison with $\omega_\varkappa$. As a result S, for example, takes the form

$$S = \frac{1}{3}\int (\Delta W)^2 dV. \tag{45}$$

We now indicate the criterion for applicability of the method of small perturbations which is used above. The term $-\hat{\Omega}^2/2M_0$ can be regarded as a small perturbation if the energy change (42) that it causes is decidedly smaller than the energy differences (32). In order of magnitude the latter are equal (basically) to $\omega_\varkappa$, which can be arbitrarily large. But in determining the criterion of applicability of the perturbation method one must use in Eq. (32) some average $\bar{\omega}$, corresponding to the frequency of the oscillations of the meson field that interact most intensely with the nucleon. In Eq. (42) also one can give $\omega_\varkappa$ an average effective value $\bar{\omega}$ and take it in front of the sign of summation (or integration). Then, by Eqs. (42) and (45), $E_2^{(1)} \sim -3/4\, \bar{\omega}\, S/M_0$. The condition for applicability of the perturbation method can thus be written as : $S/M_0 \ll 1$.

The term $(\mathbf{k}, \hat{\Omega})/M_0$ can be regarded as a small perturbation if the energy correction (42) that it causes is much smaller than

$$k^2 S/2M_0^2 \ll \bar{\omega} \sim \sqrt{\mu^2 + a^{-2}}. \tag{46}$$

This inequality limits the nucleon momentum $\mathbf{k}$.

4. SEMI-CLASSICAL CALCULATION OF THE MESON-FIELD MASS OF THE NUCLEON

The meson-field mass of the nucleon can be calculated without recourse to the perturbation method if the translational motion of the nucleon and the vibrations of the meson field are treated classically and the spin-charge motion of the nucleon quantum-mechanically. The Hamiltonian of the system has the following form

$$E = \frac{M_0}{2}\dot{\xi}^2 + \sum_{\alpha=1}^{3}\int\Big\{\frac{1}{2}\,[\dot{\varphi}_\alpha^2 + (\nabla\varphi_\alpha)^2 + \mu^2\varphi_\alpha^2] - \frac{g}{\mu}\sqrt{4\pi}\,\tau_\alpha(\sigma, \nabla\varphi_\alpha)\,U(r)\Big\}dV. \tag{47}$$

In the state with the lowest energy of the system it is obvious that $\dot{\xi} = 0$, $\dot{\varphi}_\alpha = 0$. This corresponds to a stationary nucleon and a static meson field. The equilibrium form of the latter is determined from the condition of minimum potential energy of the system, expressed by Eq. (14) of Ref. 1. Such a minimization of the potential energy was carried out in Ref. 1 and the equilibrium static meson field so obtained turned out equal to $\tilde{\varphi}_\alpha^{(v)}(\mathbf{r})$ (cf. (19) and (20)]. By Eq. (47) the corresponding lowest energy of the system is

$$E_0 = \sum_{\alpha=1}^{3}\int\Big\{\frac{1}{2}[(\nabla\tilde{\varphi}_\alpha^v)^2 + \mu^2\tilde{\varphi}_\alpha^{v2}] - \frac{g}{\mu}\sqrt{4\pi}\tau_\alpha(\sigma, \nabla\tilde{\varphi}_\alpha^v)\,U(r)\Big\}dV. \tag{48}$$

It was shown in Ref. 1 that the potential energy does not depend on the angles ϑ, β and δ. Moreover, because of the physical homogeneity of space it does not depend on ξ. Thus the system can perform a free motion in these four degrees of freedom. For such a freely "revolving" and translationally moving nucleon the meson field has the following form

$$\varphi_\alpha(\mathbf{r}, t) = \tilde{\varphi}_\alpha^v(\mathbf{r} - \dot{\xi} t, \; \vartheta, \beta, \delta). \qquad (49)$$

Here ξ is constant, and ϑ, β and δ are functions of t.

Differentiating (49) with respect to the time, we get

$$\dot{\varphi}_\alpha = (\dot{\xi}, \nabla) \tilde{\varphi}_\alpha^v + \sum_{i=1}^{3} \frac{\partial \tilde{\varphi}_\alpha^v}{\partial v_i} \dot{v}_i,$$

where v_1, v_2 and v_3 are equal to ϑ, β and δ, respectively. Substituting this into (47) and noting that

$$\int \frac{\partial \tilde{\varphi}_\alpha}{\partial x_k} \frac{\partial \tilde{\varphi}_\alpha^v}{\partial v_i} dV \qquad (50)$$

$$= \sum_{\beta\beta'} g_{\alpha\beta} \frac{\partial}{\partial v_i} g_{\alpha\beta'} \int \frac{\partial^2 W}{\partial x_k \partial x_\beta} \frac{\partial W}{\partial x_{\beta'}} dV = 0$$

because of the spherical symmetry of the function $W(r)$, we obtain

$$E = E_0 + \frac{1}{2}(M_0 + S)\dot{\xi}^2 + T_r; \qquad (51)$$

$$T_r = \frac{1}{2} \sum_{\alpha=1}^{3} \sum_{i=1}^{3} \dot{v}_i^2 \int \left(\frac{\partial \tilde{\varphi}_\alpha^v}{\partial v_i} \right)^2 dV \qquad (52)$$

is the kinetic energy of "rotation" of the nucleon, unrelated to its translational motion. From Eq. (51) it is seen that the total inertial mass of the nucleon is equal to $M_0 + S$. This result agrees with the approximate result of the quantum calculation.

5. INTRODUCTION OF THE MAGNETIC FIELD INTO THE HAMILTONIAN OF THE SYSTEM

For the introduction of the magnetic field into the Hamiltonian of a nucleon interacting with the meson field, it is convenient first to go over to new complex variables φ, π instead of the real variables $\varphi_1, \varphi_2; \pi_1, \pi_2$; $\varphi = 2^{-1/2}(\varphi_1 - i\varphi_2)$; $\pi = 2^{-1/2}(\pi_1 + i\pi_2)$. The result is that the Hamiltonian of the system, Eqs. (2)-(4), takes the form

$$\hat{H} = -\frac{1}{2M_0}\Delta_\xi + \int \Big[\pi^*\pi + \frac{1}{2}\pi_3^2 + (\nabla\varphi^*, \nabla\varphi) \qquad (53)$$

$$+ \frac{1}{2}(\nabla\varphi_3)^2 + \mu^2 \left(\varphi^*\varphi + \frac{1}{2}\varphi_3^2\right) \Big] dV$$

$$- \frac{g}{\mu}\sqrt{4\pi} \int U(\mathbf{r} - \xi)(\sigma, \sqrt{2}\tau_+ \nabla\varphi$$

$$+ \sqrt{2}\tau_- \nabla\varphi^* + \tau_3 \nabla\varphi_3)\, dV.$$

Here

$$\tau_+ = 2^{-1/2}(\tau_1 + i\tau_2), \quad \tau_- = 2^{-1/2}(\tau_1 - i\tau_2).$$

The usual gauge-invariant introduction of the electromagnetic field leads to the replacement

$$\nabla\varphi \to \nabla\varphi - ie\mathbf{A}(\mathbf{r})\varphi, \quad \nabla\varphi^* \to \nabla\varphi^* + ie\mathbf{A}(\mathbf{r})\varphi^*,$$

$$\nabla_\xi \to \nabla_\xi - ie\frac{1 + \tau_3}{2}\mathbf{A}(\xi)$$

in all terms of the Hamiltonian. Here $\mathbf{A}$ is the vector potential of the field. The magnetic field is assumed below to be static, homogeneous and parallel to the axis OZ. The corresponding vector potential can be chosen in the form $A_x = -\frac{1}{2}\mathfrak{H} y$, $A_y = \frac{1}{2}\mathfrak{H} x$, where $\mathfrak{H}$ is the intensity of the magnetic field.

Supposing the magnetic field sufficiently weak, we retain in the Hamiltonian only terms linear in $\mathfrak{H}$. The additional terms that appear in the Hamiltonian on the introduction of the magnetic field have the form

$$\hat{H}_{\text{mag}} = \hat{H}_1 + \hat{H}_2 + \hat{H}_3 + \hat{H}_4, \qquad (54)$$

where

$$\hat{H}_1 = \frac{ie}{4M_0}(1 + \tau_3)\mathfrak{H}\left(\xi_1\frac{\partial}{\partial \xi_2} - \xi_2\frac{\partial}{\partial \xi_1}\right); \qquad (55)$$

$$\hat{H}_2 = -\frac{e}{4M_0}(1 + \tau_3)\sigma_3\mathfrak{H}; \qquad (56)$$

$$\hat{H}_3 = e\mathfrak{H}\int \varphi_1\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)\varphi_2 dV; \qquad (57)$$

$$\hat{H}_4 = \frac{eg\sqrt{\pi}}{\mu}\mathfrak{H} \qquad (58)$$

$$\times \int U(\mathbf{r} - \xi)(x\sigma_y - y\sigma_x)(\tau_1\varphi_2 - \tau_2\varphi_1)\, dV.$$

Here $\hat{H}_1$ is the operator for the interaction of the magnetic field with the orbital motion of the "bare" nucleon and $\hat{H}_2$ is the operator for the interaction of the magnetic field with the spin magnetic moment of the "bare" nucleon. This interaction is different from zero when the "bare" nucleon has charge

+ 1, and equal to zero when the charge of the "bare" nucleon is zero. Therefore, the operator factor $2^{-1/2}(1+\tau_3)$ appears in Eq. (56). $\hat{H}_3$ and $\hat{H}_4$ are the supplementary terms arising on the introduction of the magnetic field into the second and third terms of Eq. (53), respectively.

In Eqs. (57) and (58) we have again returned to the real fields φ_1 and φ_2. The further calculations will be conducted in the variables $\mathbf{R}$, $\tilde{\varphi}_\alpha(\mathbf{r})$, and $p_{\alpha\varkappa}$ introduced in accordance with Eqs. (8) and (9). In these variables $-i\nabla_\xi = -i\nabla_{\mathbf{R}} + \hat{\Omega}$, where the operator $\hat{\Omega}$ is defined by Eq. (13), and the operators (55), (57) and (58) have the forms

$$\hat{H}_1 = \frac{ie}{4M_0}(1+\tau_3)\mathfrak{H}\Big[R_1\frac{\partial}{\partial R_2} - R_2\frac{\partial}{\partial R_1} - i(R_2\hat{\Omega}_1 - R_1\hat{\Omega}_2)\Big]; \tag{59}$$

$$\hat{H}_3 = e\mathfrak{H} \times \int\Big[(x+R_1)\tilde{\varphi}_1\frac{\partial\tilde{\varphi}_2}{\partial y} - (y+R_2)\tilde{\varphi}_1\frac{\partial\tilde{\varphi}_2}{\partial x}\Big]dV; \tag{60}$$

$$\hat{H}_4 = \frac{ge\sqrt{\pi}}{\mu}\mathfrak{H}\int[(x+R_1)\sigma_2 - (y+R_2)\sigma_1](\tau_1\tilde{\varphi}_2 - \tau_2\tilde{\varphi}_1)U(r)\,dV. \tag{61}$$

6. CALCULATION OF THE MAGNETIC MOMENT OF THE NUCLEON

We now calculate the correction to the energy of the system caused by the magnetic field. The magnetic moment of the nucleon will then be defined as the coefficient of the magnetic field intensity in this correction. We shall start with the wave function of the system, Eqs. (15) and (33), which it is now convenient to rewrite in the form

$$\chi = L^{-3/2}e^{i\mathbf{kR}}\Psi, \quad \Psi = \psi_s^v(\Phi_{0\nu} + \Phi_{0\nu}^{(1)}), \tag{62}$$

where $\Phi_{0\nu}^{(1)}$ is the correction to the wave function caused by the translational motion of the nucleon (and proportional to $1/M_0$):

$$\Phi_{0\nu}^{(1)} = -\frac{1}{2M_0}\Big\{\sum_{\alpha\varkappa\nu'}\frac{(\alpha\varkappa\nu'\,|\,\hat{\Omega}^2\,|\,0\nu)}{\omega_\varkappa + \Delta_{\nu'}}\Phi_{\alpha\varkappa\nu'} + \sum_{\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'}{}'\frac{(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'\,|\,\hat{\Omega}^2\,|\,0\nu)}{\omega_{\varkappa_1}+\omega_{\varkappa_2}+\Delta_{\nu'}}\Phi_{\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'} + \sum_{\alpha\varkappa\nu'}\frac{(2\alpha\varkappa\nu'\,|\,\hat{\Omega}^2\,|\,0\nu)}{2\omega_\varkappa + \Delta_{\nu'}}\Phi_{2\alpha\varkappa\nu'}\Big\}. \tag{63}$$

Here we have already set $k = 0$, since the magnetic moment will be calculated below in the inertial system in which the total momentum of the nucleon is equal to zero.

The correction to the energy of the system caused by the weak magnetic field is given by

$$H_{\mathrm{mag}} = \int\{\chi^*\hat{H}_{\mathrm{mag}}\chi\}\,d\tau_R\,dp\,d\omega. \tag{64}$$

It must be kept in mind that Ψ does not depend on R; therefore, upon proper choice of the boundaries of the cyclic interval (in which χ is orthonormalized)

$$\int\mathbf{R}\,\{\chi^*, L\chi\}\,d\tau_R = 0,$$

where L is an arbitrary operator which does not depend on R. Thus in the integration of (64) all terms containing factors R_1 and R_2 drop out. In particular, the energy correction caused by the operator $\hat{H}_1$ is equal to zero.

Proceeding to the calculation of the correction H_2 from the operator $\hat{H}_2$, we must first of all note that according to Eqs. (7)-(9) and (29) of Ref. 1

$$\{\psi_s^{v*},\ \sigma_3\psi_s^v\} = |C_1|^2 - |C_2|^2 + |C_3|^2 - |C_4|^2 = 0.$$

Furthermore, by Eq. (25), we have

$$\int\Phi_{0\nu}^{v*}\tau_3\sigma_3\Phi_{0\nu}\,dp\,d\omega = g_{33}^{\nu\nu} \tag{65}$$

$$= \begin{cases} 1/3 & \text{for } \varepsilon = 1, \sigma = 1 \text{ or } \varepsilon = 0, \sigma = -1, \\ -1/3 & \text{for } \varepsilon = 1, \sigma = -1 \text{ or } \varepsilon = 0, \sigma = 1. \end{cases}$$

Because of the orthogonality of $\Phi_{0\nu}$ and $\Phi_{0\nu}^{(1)}$ in the coordinates $p_{\alpha\varkappa}$

$$\int\Phi_{0\nu}^{(1)*}\tau_3\sigma_3\Phi_{0\nu}\,dp\,d\omega = 0.$$

Thus

$$H_2 = -(e/4M_0)\mathfrak{H}g_{33}^{\nu\nu}. \tag{66}$$

In the calculation of H_3 one must keep in mind Eqs. (9) and (19)-(22). Besides this, it is helpful to note the following property of the quantities $g_{\alpha\beta}(\vartheta, \beta, \delta)$: in the determinant $\|g_{\alpha\beta}\|$ each element is equal to its algebraic complement, taken with sign reversed (i.e., its minor). For example,

$$g_{33} = g_{12}g_{21} - g_{11}g_{22}. \tag{67}$$

As a result of the calculation one gets

$$(0\nu\,|\,\hat{H}_3\,|\,0\nu) = -\frac{1}{2}e\mathfrak{H}I^{(0)}g_{33}^{\nu\nu}, \tag{68}$$

where, as in Ref. 1 [cf. Eq. (46))],

$$I^{(0)} = \frac{2}{3}\int\left(\frac{\partial W}{\partial r}\right)^2 dV. \tag{69}$$

Furthermore, we readily obtain

$$(\alpha\varkappa\nu')\,|\,\hat{H}_3\,|\,0\nu) = \frac{e}{\sqrt{2\omega_\varkappa}}\mathfrak{H}W_{-\varkappa} \tag{70}$$
$$\times\,[(\delta_{\alpha 1}g_{22}^{\nu'\nu} - \delta_{\alpha 2}g_{12}^{\nu'\nu})\,\varkappa_x + (\delta_{\alpha 2}g_{11}^{\nu'\nu} - \delta_{\alpha 1}g_{21}^{\nu'\nu})\,\varkappa_y],$$

$$\sum_{\alpha\varkappa\nu'}\frac{1}{\omega_\varkappa}(\alpha\varkappa\nu'\,|\,\hat{\Omega}^2\,|\,0\nu)^*(\alpha\varkappa\nu'\,|\,\hat{H}_3\,|\,0\nu) \tag{71}$$
$$= -\frac{1}{3}e\mathfrak{H}g_{33}^{\nu\nu}I_1,$$

where

$$I_1 = \sum_\varkappa \varkappa^4 W_\varkappa^2/\omega_\varkappa.$$

The following expressions are calculated in an analogous way:

$$(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'|\,\hat{H}_3\,|\,0\nu) \tag{72}$$
$$= \frac{e}{2\sqrt{\omega_{\varkappa_1}\omega_{\varkappa_2}}}\mathfrak{H}\delta_{\nu'\nu}(\delta_{\alpha_1 1}\delta_{\alpha_2 2} - \delta_{\alpha_1 2}\delta_{\alpha_2 1})$$
$$\times\int\chi_{\varkappa_1}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)\chi_{\varkappa_2}\,dV,$$

$$\sum_{\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'}{}' \frac{1}{\omega_{\varkappa_1}+\omega_{\varkappa_2}}(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'\,|\,\hat{\Omega}^2|\,0\nu)^*(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'\,|\,\hat{H}_3\,|\,0\nu) \tag{73}$$
$$= -e\mathfrak{H}\,g_{33}^{\nu\nu}\sum_{\varkappa_1\varkappa_2}\frac{(\varkappa_1,\,\varkappa_2)}{\omega_{\varkappa_1}+\omega_{\varkappa_2}}\varkappa_{1y}\varkappa_{2x}W_{\varkappa_1}W_{\varkappa_2}\int\chi_{\varkappa_1}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)\chi_{\varkappa_2}\,dV.$$

The expression (73) can be simplified by means of the following consideration. Since the operator Δ commutes with $A = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}$, integrating by parts we obtain the following relation for an arbitrary function ($f(\varkappa_1^2, \varkappa_2^2)$ expansible in power series:

$$\int f(\varkappa_1^2,\ \varkappa_2^2)\,\chi_{\varkappa_1}\hat{A}\chi_{\varkappa_2}\,dV \tag{74}$$
$$= \int[f(-\Delta,\ \varkappa_2^2)\,\chi_{\varkappa_1}]\,\hat{A}\chi_{\varkappa_2}\,dV$$
$$= \int\chi_{\varkappa_1}\hat{A}f(-\Delta,\ \varkappa_2^2)\,\chi_{\varkappa_2}\,dV$$
$$= \int f(\varkappa_2^2,\ \varkappa_2^2)\,\chi_{\varkappa_1}\hat{A}\chi_{\varkappa_2}\,dV.$$

From this it is seen that the integrals (74) are different from zero only for $\varkappa_1^2 = \varkappa_2^2$. On this basis $(\omega_{\varkappa_1} + \omega_{\varkappa_2})$ can be replaced by $2\omega_{\varkappa_2}$ in (73). Then for the sum (73) one finds the value

$$\frac{1}{6}e\mathfrak{H}\,g_{33}^{\nu\nu}\int\Delta W\frac{1}{\hat{\omega}}\Delta W dV = \frac{1}{6}e\mathfrak{H}\,g_{33}^{\nu\nu}I_1. \tag{75}$$

The matrix element

$$(2\alpha\varkappa\nu'\,|\,\hat{H}_3\,|\,0\nu) = 0, \tag{76}$$

as one can readily convince oneself by performing first the integration over p and taking into account the orthogonality of the oscillator wave functions.

Proceeding to the calculation of the energy correction H_3, in the expression (63) we drop out the $\Delta_{\nu'}$ occurring together with $\omega_\varkappa$, as was already done in the derivation of Eq. (45). Then, using Eqs. (63), (68), (71), (75) and (76), we obtain

$$H_3 = (0\nu\,|\hat{H}_3\,|\,0\nu) \tag{77}$$
$$+\int[\Phi_{0\nu}^{(1)*}\hat{H}_3\,\Phi_{0\nu} + \Phi_{0\nu}^*\hat{H}_3\,\Phi_{0\nu}^{(1)}]\,dp\,d\omega$$
$$= -{}^1\!/_2\,e\mathfrak{H}g_{33}^{\nu\nu}[I^{(0)} - (1/3M_0)I_1].$$

In the calculation of H_4 one again makes use of Eqs. (9) and (19)-(22), and also of (67). One finds

$$(0\nu\,|\,\hat{H}_4\,|\,0\nu) = -e\mathfrak{H}g_{33}^{\nu\nu}(g/3\mu)\sqrt{4\pi}K; \tag{78}$$

$$K = \int rU\frac{\partial W}{\partial r}dV. \tag{79}$$

Furthermore,

$$(\alpha\varkappa\nu'\,|\,\hat{H}_4\,|\,0\nu) = \frac{ge\sqrt{\pi}}{\mu\sqrt{2\omega_\varkappa}}\mathfrak{H} \tag{80}$$
$$\times\int[(xg_{12}^{\nu'\nu} - yg_{11}^{\nu'\nu})\,\delta_{\alpha 2} - (xg_{22}^{\nu'\nu} - yg_{21}^{\nu'\nu})\,\delta_{\alpha 1}]\,U\chi_\varkappa\,dV;$$

$$\sum_{\alpha\varkappa\nu'}\frac{1}{\omega_\varkappa}(\alpha\varkappa\nu'\,|\,\hat{\Omega}^2\,|\,0\nu)^*(\alpha\varkappa\nu'\,|\,\hat{H}_4\,|\,0\nu) \tag{81}$$
$$= \frac{ge\sqrt{\pi}}{3\mu}g_{33}^{\nu\nu}\mathfrak{H}K_1;$$

$$K_1 = \int rU\frac{\partial}{\partial r}\frac{\Delta}{\hat{\omega}}WdV.$$

On carrying out the integration over p one easily shows that

$$(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu' \mid \hat{H}_4 \mid 0\nu) = 0, \tag{82}$$

$$(2\alpha\varkappa\nu' \mid \hat{H}_4 \mid 0\nu) = 0.$$

In the calculation of H_4 we shall again drop $\Delta_{\nu'}$ in comparison with $\omega_\varkappa$. Then, using Eqs. (63), (78) and (81)-(82), we obtain

$$H_4 = -e\mathfrak{H}g_{33}^{\nu\nu}(g/3\mu)\sqrt{4\pi}\,[K + K_1/2M_0]. \tag{83}$$

Summing, finally, the expressions (66), (77) and (83), we obtain the resultant energy correction caused by the magnetic field,

$$H_{\text{mag}} = -e\mathfrak{H}g_{33}^{\nu\nu}\Big[\frac{1}{2}I^{(0)} + \frac{g}{3\mu}\sqrt{4\pi}K \tag{84}$$
$$+ \frac{1}{6M_0}\Big(\frac{3}{2} - I_1 + \frac{g}{\mu}\sqrt{4\pi}K_1\Big)\Big].$$

The magnetic moment of the nucleon, defined as the coefficient of $\mathfrak{H}$ (with sign reversed) in the expression (84), is

$$m = eg_{33}^{\nu\nu}\Big[\frac{1}{2}I^{(0)} + \frac{g}{3\mu}\sqrt{4\pi}K \tag{85}$$
$$+ \frac{1}{6M_0}\Big(\frac{3}{2} - I_1 + \frac{g}{\mu}\sqrt{4\pi}K_1\Big)\Big].$$

From this it is seen that according to (65) the magnetic moments of the proton and neutron are equal in magnitude and opposite in sign.

The experimentally observed difference of the absolute magnitudes of the magnetic moments of the proton and neutron can in general not be obtained in the approximation limit of strong coupling, in which the wave function of the system has the form

$$\Psi = \psi_s\Phi(\xi, q). \tag{86}$$

To prove this in general, we assume that (86) represents the proton state of the system. In our notation the operator for the charge of the system[4] can be written in the form

$$\hat{\varepsilon} = \hat{\varepsilon}' + {}^1/_2(1 + \tau_3); \tag{87}$$
$$\hat{\varepsilon}' = -i\sum_\varkappa (q_{1\varkappa}\partial/\partial q_{2\varkappa} - q_{2\varkappa}\partial/\partial q_{1\varkappa}).$$

The function (86) is an eigenfunction of this operator and corresponds to the eigenvalue +1. From this it follows, since $\{\psi_s^*, \tau_3\psi_s\} = 0$, that

$$\overline{\varepsilon'} = \int \Phi^*\hat{\varepsilon}'\Phi d\xi\, dq = \frac{1}{2}. \tag{88}$$

Since the operator for the vibrational energy of the meson field (cf. Eq. (1) of Ref. 2] is real, the function

$$\Psi_1 = \psi_s\Phi^*(\xi, q) \tag{89}$$

is also an eigenfunction of the energy operator and corresponds to the same energy as does the function (86). It is easy to see that (89) is a neutron state of the system. Indeed, in this state

$$\overline{\varepsilon'} = \int \Phi\hat{\varepsilon}'\Phi^* d\xi\, dq \tag{90}$$
$$= -\int \Phi[\hat{\varepsilon}'\Phi]^* d\xi\, dq = -\frac{1}{2},$$

and consequently the eigenvalue of the charge operator (87) is equal to zero.

In an entirely analogous way, by considering the operator of the total angular momentum of the system, one can show that the states (86) and (89) correspond to opposite orientations of the spin of the nucleon.

If we now compare the corrections to the energy of the system, Eqs. (56)-(58), caused by the magnetic field, for the states (86) and (89), they turn out precisely identical. Consequently, the magnitudes of the magnetic moments of the proton and neutron states of the system are found to be identical.

Thus it can be possible to obtain a difference of the magnitudes of the magnetic moments of proton and neutron only by renouncing the multiplicative form of the wave function, Eq. (86), with the factor ψ_s, which was determined in Ref. 1. This is not to be achieved by a more precise consideration of the transitional motion of the nucleon, but by departing from the limit of strong coupling and introducing the corresponding corrections.

7. DISCUSSION OF RESULTS

If in Eq. (85) we drop the last two terms (containing I_1 and K_1), which appeared because of our taking into account the translational motion of the nucleon, the value of the magnetic moment $\overline{m}$ turns out to be identical with that found in the work of Pauli and Dancoff[4]. In the comparison it must be noted that our parameter g is equal to the parameter $2^{-1/2}g$ of Ref. 4. The agreement of our value of m, obtained in the symmetrical theory, with the value of m obtained in Ref. 4 in the charged meson theory assures us that the interaction of the nucleon with neutral mesons does not influence the size of the magnetic moment (at least in the case of the infinitely heavy nucleon).

It must be noted that in Ref. 4 the magnetic moment was determined from the density of the meson current, which led to the difficulties in connection with the impossibility of satisfying the equation of continuity of the current inside the extended nucleon. In our work this difficulty is not encountered, since the magnetic moment of the nucleon is determined from the expressoin for the energy of the nucleon in the magnetic field.

Investigation shows that in the case $\mu a \gg 1$ [a is the effective radius of the form factor $U(r)$] the results of the theory depend essentially on the shape of the form factor $U(r)$. Since in the present state of science there is no sound basis for choosing this shape, an impermissable arbitrariness appears in the theory in the case $\mu a \gg 1$. Therefore, we confine ourselves below to the consideration of the case $\mu a \ll 1$, in which the results are insensitive to the choice of the form factor. In this case Eq. (20) can be approximately rewritten as follows:

$$W(r) \approx \frac{g}{\mu\sqrt{4\pi}} \int \frac{U(r')}{|\mathbf{r}-\mathbf{r}'|} dV'. \tag{91}$$

Expanding the functions U and W in the integrands in Eqs. (75) and (81) in series of the form

$$U(r) = (2\pi R)^{-1/2} \sum_k U_k \frac{\sin kr}{r},$$

where R is the radius of the fundamental region in which the basis functions are orthonormal and $k = n\pi/R$ ($n = 1, 2, 3, \ldots$), and carrying out the integration over the volume term by term, we obtain

$$I_1 = \frac{4\pi g^2}{\mu^2} \sum_k \frac{U_k^2}{k}, \quad K_1 = \frac{g\sqrt{4\pi}}{\mu} \sum_k \frac{U_k^2}{k}. \tag{92}$$

From this it is seen that the sum of the two last terms in Eq. (85) is zero. Thus by Eqs. (69), (79) and (91), one gets

$$\begin{aligned} m &= eg_{33}^{\nu\nu}\left[\frac{1}{2} I^{(0)} + \frac{g\sqrt{4\pi}}{3\mu} K + \frac{1}{4M_0}\right] \\ &= \frac{1}{6} eg_{33}^{\nu\nu}\left[\left(\frac{g}{\mu}\right)^2 \iint \frac{U(r)U(r')}{|\mathbf{r}-\mathbf{r}'|} dV\, dV' + \frac{3}{2M_0}\right]. \end{aligned} \tag{93}$$

Consequently, in the case $\mu a \ll 1$ the inclusion of the translational motion of the nucleon to first approximation does not change the value of the magnetic moment of the nucleon.

By Eqs. (45) and (91) one gets for the meson-field mass of the nucleon

$$S = \frac{4\pi}{3}\left(\frac{g}{\mu}\right)^2 \int U^2 dV. \tag{94}$$

The quantities (93) and (94) can be evaluated readily if we replace the form factor by the function

$$U(r) = \begin{cases} 3/4\pi a^3 & \text{for } r < a \\ 0 & \text{for } r > a, \end{cases} \tag{95}$$

taking advantage of the above-mentioned small sensitivity of the results to the shape of the form factor (in the case $\mu a \ll 1$). Then we get

$$m = eg_{33}^{\nu\nu}[g^2/5\mu^2 a + 1/4M_0], \tag{96}$$

$$S = g^2/\mu^2 a^3. \tag{97}$$

To estimate the parameters of the theory we replace in Eq. (96) the mass M_0 of the "bare" nucleon by the mass M of the physical nucleon, and equate the quantity (96) to the experimental value of the magnetic moment of the nucleon. For the latter we take the arithmetical average of the magnetic moments of the proton and neutron, equal to 2.4 nuclear magnetons. Then for $M/\mu = 6.7$ we get

$$(g/\mu a)^2 \mu a \approx 2.5. \tag{98}$$

From this it follows that for $\mu a \ll 1$ the criterion (1) for strong coupling is satisfied.

We now pass to the consideration of the criterion for the heavy nucleon approximation ($M_0 \gg S$). By Eq. (97), it can be written

$$(g/\mu a)^2/\mu a \ll M/\mu = 6.7. \tag{99}$$

This inequality limits the strength of the coupling, and in the case $\mu a \ll 1$ can be fulfilled only for weak or intermediate coupling. Multiplying Eqs. (98) and (99), we get $(g/\mu a)^2 \ll 4$.

CONCLUSIONS

1. In the limiting case of strong coupling the magnetic moments of proton and neutron are found to be identical (in magnitude) even with inclusion of the translational motion of the nucleon.

2. In the case $\mu a \ll 1$ comparison with experiment of the nucleon magnetic moment calculated for the strong-coupling limit and the heavy nucleon confirms the criterion of strong coupling.

3. The heavy-nucleon approximation is not applicable in the limiting case of strong coupling.

4. In the case of strong coupling the meson-field mass of the nucleon is not small in comparison with M_0, and possibly even exceeds M_0. In the latter case M_0 turns out to be of the order of or smaller than the mass of the mesons playing an important part in the interaction with the nucleon, so that it is of interest to consider the relativistic motion of a nucleon in the meson field, including the possibility of the formation of nucleon pairs.

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