Production of Slow π^+-Mesons in Photographic Emulsion Nuclei by 460 MEV Protons and Neutrons of 400 MEV Effective Energy

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The method of an emulsion chamber was used to investigate the production of charged mesons by 460 mev protons and neutrons of 400 mev effective energy.

INTRODUCTION

THE purpose of this research was the investigation of the production characteristics of slow π-mesons in photographic emulsion nuclei by 460 mev protons and neutrons of 400 mev effective energy, the analysis of stars accompanying the production of mesons and their comparison with stars not containing π-mesons. The use of the emulsion chamber technique developed by one of the authors[1] enabled one to follow the complete path of produced π-mesons from the place of their ionization terminal to the place of production, and thus gave the opportunity to observe a large number of stars with the production of charged π-mesons and to plot angular and energy distributions of π-mesons in the region of low energies.

There are many papers in the literature devoted to the study of emulsion stars produced by high energy nucleons. From the results of these papers there were obtained the fundamental characteristics of stars in agreement with the theory of nucleonic cascade, which is generated when a fast nucleon is captured by a nucleus. The investigation of stars with the production of mesons of opposite sign can give additional information about the nucleonic cascade.

1. PROCEDURE

The methodology of the emulsion chamber, previously described in detail[1], was used in the investigation. Each of the two utilized emulsion chambers consisted of 20 layers of photographic emulsion of 6 mm total thickness and 42 mm in diameter, placed in a specially constructed thin-walled cassette (plate-holder). The chambers were exposed to beams of fast neutrons and protons brought out of the synchrocyclotron of the USSR Academy of Sciences' Institute for Nuclear Problems; one chamber in a beam of neutrons with a 400 mev effective energy for the production of mesons--the other in a 460 mev proton beam.

In the investigation, emulsions prepared by D.M. Samoilovich were used, permitting the effective recording of proton tracks with energies up to 300 mev. The relationship of range-energy for these emulsions was calculated from the known percentage composition of their constituents (85% AgBr) and verified by measuring the average path of the μ-mesons, produced during the $\pi \to \mu$ decay ($\overline{R}_\mu = 590\,\mu$). The emulsion shrinkage coefficient was determined by comparing the thickness of the emulsion chamber before and after development, and also by comparing the average track lengths of μ-mesons of the $\pi \to \mu$ decay located in the plane of the emulsion and making with it an angle of nearly 90°.

The scanning of the emulsion layers was done with MBI-2 microscopes under a total magnification of 225×; the measurements under a magnification of 2025×. While scanning, the ends of π-mesons were recorded and "starless" π-mesons were identified by the grain density gradient and by the characteristic "willowiness" of the track determined by multiple scattering. Special attention was paid to the observer's accuracy in reporting the relationship between the numbers of π^+- and π^--mesons in the emulsion. As shown by special check, the effectiveness of recording the π-meson ends was nearly 100%.

The π^+ and π^--mesons found by this method were then followed to the place of their production or to the place where they emerged from the emulsion chamber. In order to facilitate the tracing of the tracks of charged particles while passing from one layer to the next, a roentgen grid coordinate system with 30-35 μ lines, 3 mm apart was superimposed on the emulsion layers.

The chambers used in the investigation allowed the study of production characteristics of π-mesons with energies up to 40 mev.

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* Deceased.

** This communication is based on the results of research carried out in 1952-1953.

2. THE ENERGY AND ANGULAR DISTRIBUTIONS OF PRODUCED π-MESONS

As a result of scanning the emulsion layers and following the tracks of π^+- and π^--mesons in neighboring layers, there were found 193 cases of π^+- and π^-- meson production by neutrons (185 π^-- and 8 π^+-mesons) and 121 cases of π^+- and π^--meson production by protons (63 π^+- and 58 π^--mesons).

The correct notion of energy and angular distribution of produced mesons requires edge effect corrections determined by the finite dimensions of the emulsion chamber. To calculate the edge effect each recorded case of a π-meson production was assigned to coefficient $\varkappa$ expressing the statistical weight of a meson with the given production characteristics. The value of the coefficient $\varkappa$ ($\varkappa = 1/p$, where p is the probability of recording a given meson in the emulsion chamber) was determined from the path length of the meson and its orientation in the emulsion chamber and was calculated for each case of π-meson production by means of specially constructed nomograms (see Appendix).

The energy distribution of π^+- and π^--mesons produced by protons and π^--mesons produced by neutrons is shown in Fig. 1. (The energy distribution of π^+-mesons produced by neutrons is not given because the number of cases found was small.) It is seen from the Figure that the spectra of π^+-mesons produced by protons are comparable within experimental error to the spectra of π^+-mesons produced by neutrons, as is to be expected from the principle of charge symmetry. The displacement of the spectra (ΔE = 15 mev) can be explained by the action of the nuclear Coulomb field displacing the spectra 7.5 mev in opposite directions. It is worth noting that the 7.5 mev value of the shift agrees with the value of the Coulomb barrier height for photoemulsion nuclei. The spectrum of π^--mesons produced by protons is characterized by a weaker dependence on energy in comparison with the spectra of π^+-mesons produced by protons and π^--mesons produced by neutrons.

The angular distributions of π^+- and π^--mesons produced by protons and π^--mesons produced by neutrons are shown in Fig. 2. It can be seen that in accordance with the principle of charge symmetry, the angular distributions of π^+-mesons produced by protons are similar to those of π^--mesons produced by neutrons. It is difficult to give a more detailed interpretation of the results obtained for the angular distribution of produced π-mesons because a substantial part of the energy spectrum in the region of small energies may be determined by mesons which have lost their energy at the expense of scattering by nucleons within the nucleus.

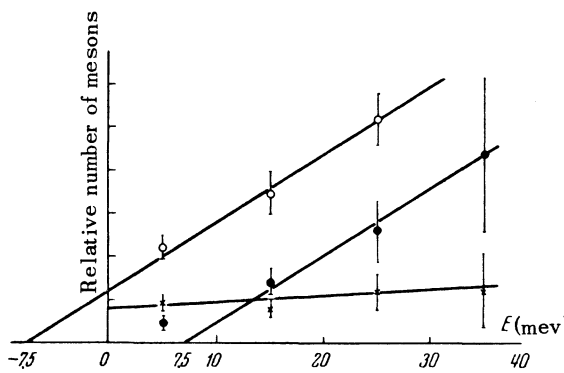

FIG. 1. The energy distribution of π-mesons produced by 460 mev protons and neutrons of 400 mev effective energy in photoemulsion nuclei. O—π^--mesons, produced by neutrons; ●—π^+-mesons produced by protons; ×—π^--mesons produced by protons.

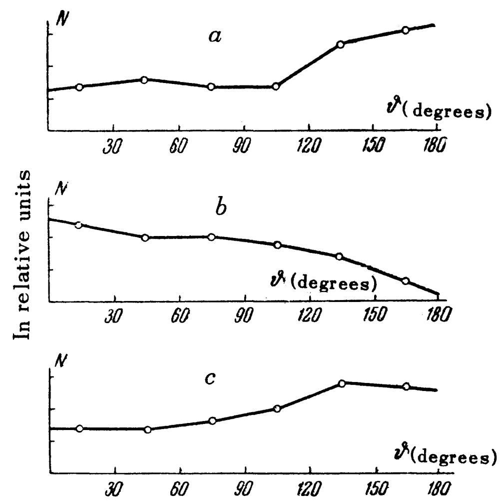

FIG. 2. The angular distributions of π-mesons produced by 460 mev protons and neutrons of 400 mev effective energy. a—π^--mesons produced by neutrons; b—π^--mesons produced by protons; c—π^+-mesons produced by protons.

3. THE RELATIONSHIP BETWEEN THE NUMBERS OF PRODUCED π^+- AND π^--MESONS. PRODUCTION CROSS SECTION.

The calculation of the number of small energy π^+-

and π^--mesons with corrections for the edge effect gives for the π^-/π^+ ratio in the case of meson production by neutrons a value of $(\pi^-/\pi^+)_n = 17 \pm 8$ and for the ratio π^+/π^- in the case of meson production by protons a value of $(\pi^+/\pi^-)_p = 2.2 \pm 0.7$. Since the photoemulsion nuclei consist of approximately equal numbers of protons and neutrons, then in accordance with the charge symmetry principle, the above ratios must be in agreement. The experimentally observed discrepancy in the values of the ratios $(\pi^-/\pi^+)_n$ and $(\pi^+/\pi^-)_p$ can be explained by the difference in the action of the Coulomb field on the energy spectra of π^+- and π^--mesons.

In accordance with the value of Coulomb shift in the energy spectra of π-mesons, E = 15 mev, established in Sec. 2, π^+-mesons in the energy interval of 15 to 30 mev and π^--mesons in the energy interval 0 to 15 mev were used to calculate the ratios $(\pi^-/\pi^+)_n$ and $(\pi^+/\pi^-)_p$, corrected for the Coulomb shift. With this there were obtained values of the $(\pi^-/\pi^+) = 3^{+3}_{-1.5}$ and $(\pi^+/\pi^-)_p = 2.5 \pm 0.5$ ratios which agreed within error limits; the large error in the ratio $(\pi^-/\pi^+)_n$ is associated with the small numbers of recorded cases.

The production cross section of π^+- and π^--mesons of energies under consideration ($E_\pi < 40$ mev) produced by 460 mev protons in photoemulsion nuclei were obtained by comparing the number of produced π-mesons in a definite emulsion volume with the proton beam density. The density of the proton beam was determined visually by counting with a microscope the number of tracks. With the emulsion used it was possible to make these measurements quite reliable for the horizontal tracks of the 460 mev proton beam.

The number of mesons N_0^π produced in 1 cm^3 of emulsion is determined by the expression $N_\sigma^\pi = \bar{\varkappa} N^\pi/V$, where N^π is the number of recorded π-mesons ends, V is the volume of the scanned emulsion, $\bar{\varkappa}$ is the multiplier, which takes into account the edge effect and is determined by the conditions $\bar{\varkappa} = \Sigma \varkappa / N^\pi$. The production cross section for π^--mesons is equal to:

$$\sigma = N_0^\pi / c\ I_p, \qquad (1)$$

where c is the number of atoms in 1 cm^3 of emulsion, I_p is the proton current through 1 cm^2 of the photoemulsion.

Using Eq. (1), the mean cross sections of π-meson production in photoemulsion nuclei in the energy interval $0 < E_\pi < 40$ mev were found to be equal to:

$$\sigma_{\pi^+} = (2.9 \pm 0.9)\ 10^{-27}\ \text{cm}^2;$$

$$\sigma_{\pi^-} = (1.3 \pm 0.5)\ 10^{-27}\ \text{cm}^2;$$

$$\sigma_{\pi^+ + \pi^-} = (4.2 \pm 1.4)\ 10^{-27}\ \text{cm}^2.$$

Another evaluation of π-meson production cross section was made by comparing the number of produced π-mesons with the number of stars produced by protons in the same emulsion volume, on the assumption that the star production cross section is equal to $\sigma_{\text{star}} = 0.58\,\sigma_{\text{geom}}$ and $\sigma_{\text{geom}} = 0.6 \times 10^{-24}$ cm^2 (the calculation made for the emulsion used). In this case the cross section had a value

$$\sigma(\pi^+ + \pi^-) = (4.9 \pm 1,5)\ 10^{-27}\ \text{cm}^2$$

agreeing with the first result within error limits.

The determination of the π^--meson production cross section by neutrons was made by comparing the number of produced mesons with the number of stars produced by neutrons, with the same assumptions concerning σ_{star} and σ_{geom}. Moreover, the taking into account of the nature of the neutron spectrum and the dependence of π-mesons production cross section on neutron energy, leads one to the conclusion that in practice monochromatic neutrons with an effective energy $E_n = (400 \pm 40)$ mev are involved in meson production. The value of the cross section of π^--mesons by neutrons in photoemulsion nuclei was found to be equal to:

$$(\sigma_{\pi^-})_n = (12 \pm 8)\ 10^{-27}\ \text{cm}^2$$

The large error is associated with the uncertainty in the neutron spectrum in the region of small energies.

From a comparison of $(\sigma_{\pi^-})_n$ with $(\sigma_{\pi^+})_p$ it is seen that $(\sigma_{\pi^-})_n > (\sigma_{\pi^+})_p$, although $E_p > E_n$. This is associated with the action of the nuclear Coulomb field which displaces differently the spectra of π^+- and π^--mesons.

4. ANALYSIS OF STARS ACCOMPANYING THE PRODUCTUON OF π-MESONS

In analyzing stars accompanying the production of π-mesons, the energy of the particles was determined with the aid of calibration graphs $E = f(R)$ (R--stops in flight of particles) for the paths terminating within one emulsion layer, and from the grain density $E = f(dN/dR)$ for the tracks of particles leaving a given emulsion layer. Such a treatment of the rays permitted the plotting of a rough curve for particles ejected from stars accompanying π-meson production.

The curve $E = f(dN/dR)$ was obtained experimentally for π-mesons and recalculated for protons by the formula

$$\frac{dN}{dR}(E_p) = \frac{dN}{dR}\left(\frac{E_\pi M_p}{M_\pi}\right)$$

In the current investigation, the particles ejected from stars were not identified and their energy was calculated from the curves of $E(R)$ or $E(dN/dR)$ for protons. The errors arising from such consideration were taken into account during the analysis of experimental data.

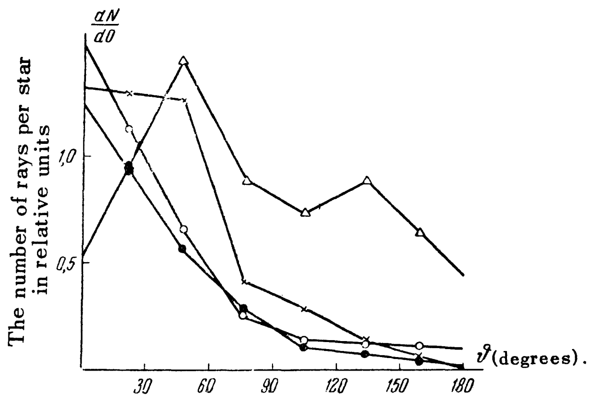

FIG. 3. The angular distribution of slow particles ($E < 30$ mev), ejected from stars under the action of 460 mev protons and neutrons of 400 mev effective energy. O -- stars with the production of π^--mesons by neutrons; ● -- stars with the production of π^+-mesons by protons; × -- stars with the production of π^--mesons by protons; Δ -- stars without production of π-mesons by protons.

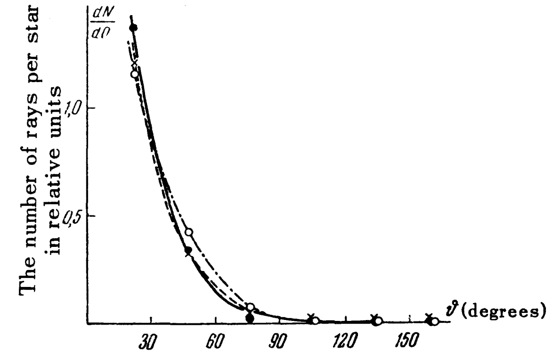

FIG. 4. The angular distribution of fast particles ($E > 30$ mev) ejected from stars with the production of mesons by 460 mev protons and neutrons of 400 mev effective energy. × -- stars with π^--mesons produced by neutrons; ● -- stars with π^+-mesons produced by protons; O -- stars with π^--mesons produced by protons.

The angular distributions and the magnitude of the anisotropy $[N(0\text{-}90°) - N(90\text{-}180°)]/[N(0\text{-}90°) + N(90\text{-}180°)]$ of slow and fast particles ejected from photoemulsion nuclei by the action of fast nucleons are shown in Figs. 3 and 4 and in Table I.

As can be seen from Fig. 3 and Table I, the angular distribution of particles with energies $E < 30$ mev is peaked forward, which indicates the presence of a large number of slow cascade particles, their number being especially large in stars accompanying the production of mesons[2].

The value obtained for the anisotropy of "mesonless" stars is in good agreement with the measurements of other authors. Bernardini *et al.*[2] obtained a value of 0.25 for the anisotropy in the angular distribution of "black rays", in the case of stars produced by 400 mev protons; Blau *et al.*[3], for stars produced by 300 mev neutrons, 0.33.

TABLE I.

The value of anisotropy of slow ($E \leq 30$ mev) and fast ($E > 30$ mev) particles.

Type of Star	(p, π^+) $E_p = 460$ mev	(n, π^-) $\bar{E}_n = 400$ mev	(p, π^-) $E_p = 460$ mev	Stars without π-meson
Anisotropy Value:				
Slow Particles	0.61 ± 0.09	0.46 ± 0.06	0.47 ± 0.08	0.25 ± 0.06
Fast Particles	0.96 ± 0.04	0.80 ± 0.05	0.96 ± 0.04	—

The angular distribution of particles with energies $E > 30$ mev in stars accompanying the production of π-mesons is shown in Fig. 4. As was to be expected, these particles are ejected mostly in the forward direction, since they are mainly cascade particles, Unfortunately, the insufficient sensitivity of the emulsion used in the investigation (300 mev for protons) did not permit a reliable analysis of stars which are not accompanied by the production of π-mesons. The mean values for the numbers of rays per star which accompany the production of π^+- and π^--mesons are given in Table II; therein are also the results of Bernardini *et al.*[2] for stars not accompanying the production of mesons.

TABLE II. The mean number of rays per star.

Star Type	Mean number of rays per star	Mean number of fast particles ($E>30$ mev) per star	Mean number of slow particles ($E<30$ mev) per star
(n, π^-) . .	2.9±0.1	0.9±0.07	2.0±0.1
(p, π^-) . .	3.9±0.3	0.9±0.1	3.0±0.2
(p, π^+) . .	2.4±0.2	0.8±0.1	1.6±0.2
(p, without π)	3.4±0.2[2]	0.85±0.07[2]	2.5±0.2 [2]

It can be seen from Table II that the number of fast ($E > 30$ mev) charged particles in stars is the same for the various methods of meson production. If one assumes that the production of mesons takes place through nulceon-nucleon interaction, then the obtained results indicate the large overcharge of fast nucleons. The number of slow particles for the considered cases of π-meson production varies and is thus characterized by the fact that the slow particles compensate for the change in nuclear charge brought in by the primary protons and by the produced π^+- or π^--meson.

The general nature of the energy spectra of particles ejected from stars accompanying the meson production is illustrated by Fig. 5, where it is seen that the curves coincide in the regions of high energy and diverge regularly in the region of small energies. The calculation of the mean energy carried out by charged particles of the star gives a value of approximately 100 mev, independent of the meson producing reaction type. About 75 percent of the total energy is taken out by fast ($E > 30$ mev) particles.

If one assumes that for the stars under consideration the energy removed by the neutrons is equal to the total energy of the charged particles, then the full energy appearing in the star E_Σ is approximately equal to the energy of the nucleon which produced the star $E_\Sigma = 2(E_{\text{mean}} + QN_{\text{mean}}) + \epsilon_\pi = 2(104 + 9.4) + 160 = 440$ mev. Here E_{mean} is the mean energy of the charged particles in the star, Q is the binding energy of the nucleon in the nucleus, N_{mean} is the mean number of charged particles in the star and ϵ_π is the total energy of the meson.

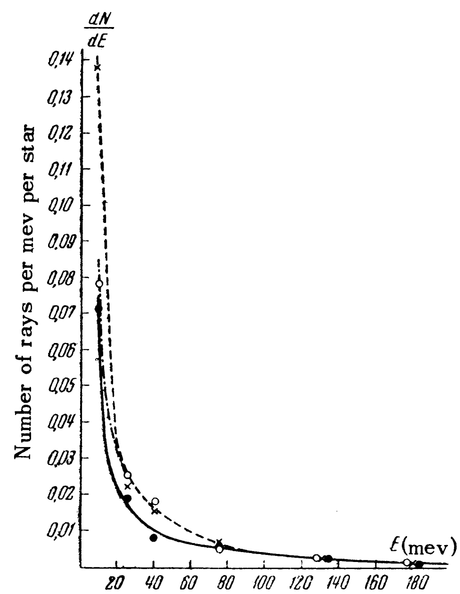

FIG. 5. Energy distributions of particles ejected from stars accompanying the production of π-mesons by 460 mev protons and neutrons of 400 mev effective energy. ×--stars with π^--mesons produced by protons; ●--stars with π^+-mesons produced by protons; O--stars with π^--mesons produced by neutrons.

In this investigation there were also measured a number of fast protons ejected from stars accompanying the production of π^+- and π^--mesons for the cases when the produced proton is ejected forward. Obviously, on the assumption of the nucleon-nucleon mechanism of π-meson production, the production of π^+- and π^--mesons by protons and neutrons must be accompanied by a different number of protons in the final state. If it is assumed that the mesons are produced on the nucleus surface, then the above-mentioned difference in the number of fast protons accompanying the production of π-mesons must be especially noticeable in the case where the produced π-meson is ejected forward. However, as follows from Table III, the number of fast protons ejected from stars is the same for the different methods of meson production.

TABLE III. Mean number of fast protons for stars accompanying the production of π-mesons ejected forward.

Production Reaction	Mean number of protons with $E > 30$ mev
(p, π^+)	0.76 ± 0.17
(p, π^-)	0.74 ± 0.11
(n, π^-)	0.86 ± 0.11

This result can be explained by the relatively large probability of π-meson production with a consequent overcharge of the nucleons participating in the reaction. The obtained results cannot be explained by assuming a strong overcharge of cascading nucleons in the nucleus up to the instant of π-meson production. Actually, with this assumption, the spectra of π^+- and π^--mesons should be identical, which is contradicted by experiment.

In conclusion, the authors express sincere thanks to Prof. M. G. Meshcheriakov and Prof. Dzhelepov for the opportunity to carry out these experiments with the synchrocyclotron of the Institute for Nuclear Problems of the Academy of Sciences, USSR, and to D. M. Samoilovich for the preparation of photoemulsions, and also to V. V. Matveev and B. B. Liubimov for assistance with the investigation.

APPENDIX

EDGE EFFECT CORRECTIONS

In order to get the correct idea of the energy and angular distribution of the produced π-mesons, corrections had to be introduced for the edge effect determined by the terminal dimensions of the emulsion chamber. We shall consider separately the edge effect associated with the finite thickness of the emulsion chamber and the edge effect determined by the finite thickness and diameter of the emulsion chamber.

a) *The edge effect associated with the finite thickness of the emulsion chamber.* Consider the mesons produced at some point O and having a given path l and given angle θ (angle relative to the direction of the proton beam). Obviously, all such mesons lie in the surface of a cone with a vertex angle θ and generatrix l (Fig. 6*a*). Moreover, if one considers angle φ (in the plane of cone generation), then it is clear from physical considerations that the mesons produced with a given l and θ must be uniformly distributed relative to this angle. However, the finite emulsion chamber thickness h leads to the conclusion that of all these mesons only those will be recorded whose ends lie on the arcs of a circle with radius $R = l \sin\theta$. The correction coefficient k is equal to the ratio of the circumference to the above-mentioned arcs.

$$k = \frac{\pi}{\varphi_1 + \varphi_2} = \frac{2k_1k_2}{k_1 + k_2}, \tag{2}$$

where $k_1 = \pi/2\varphi_1$ and $k_2 = \pi/2\varphi_2$ are the correction coefficients for mesons ejected into the upper and lower hemisphere, respectively. In practice, the corrections for the edge effect connected with the finite emulsion chamber thickness were made for each meson (with a given l, ϑ and φ) individually with the aid of the nomogram shown in the Fig. 6*b* and enabling one to find the coefficients k_1 and k_2.

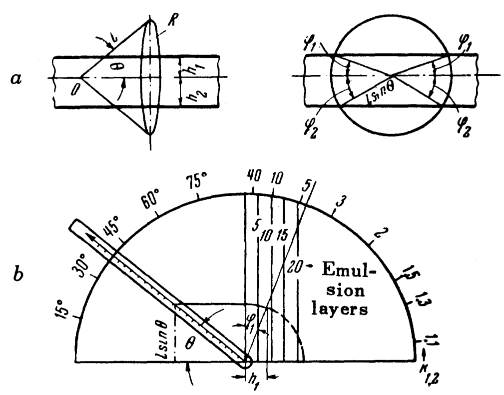

FIG. 6. Nomogram for the edge effect k.

b) *The edge effect dependent on the diameter finiteness of the emulsion chamber.* Let l and β be the respective length and penetration angle of the meson track, the projection of which onto the emulsion plane has a given direction **n** (Fig. 7a). Then it is evident that of the mesons with the given direction **n** only those will be recorded which

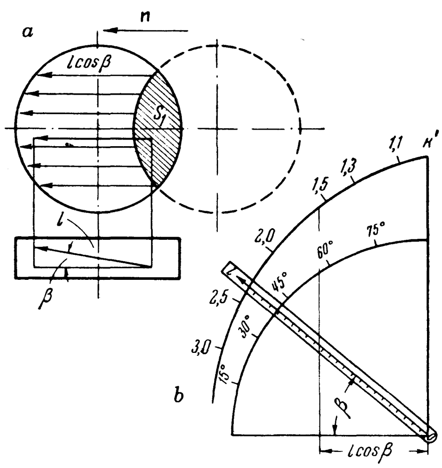

FIG. 7. Nomogram for the edge effect k'.

are produced within the limits of the cross-hatched area S_1. Since the above consideration does not depend on the choice of the direction **n**, then for all the tracks of length l and angle β the edge effect correction dependent on the finiteness of the chamber diameter will be equal to the ratio of the circle area S to the cross-hatched area S_1.

$$k' = S / S_1.$$

The determination of the value of k' was made for each track individually with the aid of the nomogram shown in Fig. 7b.

In this manner the true number of produced mesons can be calculated if to each located π-meson one assigns a statistical weight $\varkappa$ equal to the product of the coefficients k and k';

$$\varkappa = kk'.$$

[1] V. V. Alpers and A. A. Varfolomeeva, Pribori i Tekhnika Experimenta **1**, 1 (1956).

[2] Bernardini, Booth and Lindenbaum, Phys. Rev. **85**, 826 (1952).

[3] Blau, Oliver and Smith, Phys. Rev. **91**, 949 (1953).

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The Theory of a Fermi Liquid

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A theory of the Fermi liquid is constructed, based on the representation of the perturbation theory as a functional of the distribution function. The effective mass of the excitation is found, along with the compressibility and the magnetic susceptibility of the Fermi liquid. Expressions are obtained for the momentum and energy flow.

As is well known, the model of a Fermi gas has been employed in a whole series of cases for the consideration of a system of Fermi particles, in spite of the fact that the interaction among such particles is not weak. Electrons in a metal serve as a classic example, Such a state of the theory is unsatisfactory, since it leaves unclear what properties of the gas model correspond to reality and what are intrinsic to such a gas.

For this purpose we must keep in mind that the problem is concerned with definite properties of the energy spectrum ("Fermi type spectrum"), for whose existence it is necessary, but not sufficient that the particles which compose the system obey Fermi statistics, i.e., that they possess half-integer spin. For example, the atoms of deuterium interact in such a manner that they form molecules. As a result, liquid deuterium possesses an energy spectrum of the Bose type. Thus the presence of a Fermi energy spectrum is connected not only with the properties of the particles, but also with the properties of their interaction.

A liquid of the Bose type was first considered by the author of the present article in application to the properties of He II. It follows from the character of the spectrum of such a liquid that a vis-